MAT2411- DIFFERENTIAL EQUATIONS FINAL	**B**
Full Name:	**Section No:** 2
Student ID:	**Duration:** 100 mins
Department Name:	**Date:** 08.22.2022
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1) A solution of the differential equation $x^2y''+kxy'+3y=0$ is $y=x$ such that $k\in\mathbb{R}$ and $x>0$. Which of the following is another solution to this equation?

a) $y=e^x$ b) $y=e^{3x}$ c) $y=\ln x$ d) $y=x^2$ e) $y=x^3$

y=x bir çözüm ise denklemi sağlar. $y=x \Rightarrow y'=1 \Rightarrow y''=0$

$x^2\cdot 0+kx\cdot 1+3x=0 \Rightarrow kx=-3x \Rightarrow k=-3$

$\Rightarrow x^2y''-3xy'+3y=0$ $\quad x=e^t \Rightarrow y'=e^{-t}Dy \Rightarrow y''=e^{-2t}D(D-1)y$

$e^{2t}\cdot e^{-2t}D(D-1)y-3e^t\cdot e^{-t}Dy+3y=0 \Rightarrow (D^2-D-3D+3)y=0$

$(D^2-4D+3)y=0$

k.D: $r^2-4r+3=0$, $r_1=1$, $r_2=3$

$y=c_1e^t+c_2e^{3t}$

$=c_1x+c_2x^3$

2) Which of the following is the new form of the differential equation $4y'=3y+2x\sqrt{y}$ that will be obtained as a result of applying an appropriate transformation?

a) $z'-z=x$ b) $z'-z=4x$ c) $z'-\frac{3}{8}z=\frac{x}{4}$ d) $z'-\frac{3}{8}z=x$

e) $z'+z=\frac{x}{4}$

$y'+p(x)y=q(x)y^n$

$y'-\frac{3}{4}y=\frac{x}{2}y^{1/2}$

$\frac{y'}{\sqrt{y}}-\frac{3}{4}\sqrt{y}=\frac{x}{2}$

$\sqrt{y}=z$

$\frac{y'}{2\sqrt{y}}=z' \Rightarrow \frac{y'}{\sqrt{y}}=2z'$ $\Rightarrow 2z'-\frac{3}{4}z=\frac{x}{2} \Rightarrow z'-\frac{3}{8}z=\frac{x}{4}$

3) Which of the following is B for the differential equation $(5x^3+By^3)dx+(2y-x^3)dy=0$ to be exact?

a) $-\frac{x^2}{y^2}$ b) $\frac{3x^2}{y}$ c) $-xy$ d) $\frac{x^2}{y}$ e) x^2y

$\frac{\partial}{\partial y}(5x^3+By^3)=\frac{\partial}{\partial x}(2y-x^3) \Rightarrow 3By^2=-3x^2 \Rightarrow B=-\frac{x^2}{y^2}$

4) Since the general solution of the differential equation $y''+3y'-4y=-5e^x$ is expressed as $y=C_1(x)e^x+C_2(x)e^{-4x}$, which of the following is the derivative function $C_1'=\frac{dC_1}{dx}$?

a) e^{3x} b) e^x c) e^{5x} d) 1 e) -1

$C_1'e^x+C_2'e^{-4x}=0$

$C_1'e^x-4C_2'e^{-4x}=-5e^x$

$5C_1'e^x=-5e^x$

$C_1'=-1$

5) Which of the following is the singular solution of the differential equation $xy'=y-\ln\left(\frac{1}{y'}\right)$?

a) $y=\ln x$ b) $y=\frac{1}{\ln x}$ c) $y=1+\ln x$ d) $y=\ln(1+x)$ e) $y=\ln\left(\frac{1}{x}\right)$

$y=xy'+\varphi(y')$ → Clairaut dif.

$y=xy'+\ln\left(\frac{1}{y'}\right)$ $\quad y'=p \Rightarrow$

$y=xp+\ln\left(\frac{1}{p}\right) \Rightarrow y'=p+xp'+\frac{-p'/p^2}{1/p}$

$\Rightarrow p=p+xp'-\frac{p'}{p} \Rightarrow p'\left[x-\frac{1}{p}\right]=0 \Rightarrow x=\frac{1}{p}$ ve $p=\frac{1}{x}$

$y=1+\ln(x)$

6) Which of the following differential equations has the solution $y = \frac{x}{x+1}$?

a) $xy' = y$ b) $yy' = x$ c) $y^2y' = x^2$ d) $x^2y' = y^2$ e) $y^2y' = x$

$y' = \frac{x+1-x}{(x+1)^2} \Rightarrow y' = \frac{1}{(x+1)^2} \Rightarrow x^2y' = \frac{x^2}{(x+1)^2} \Rightarrow x^2y' = y^2$

7) Which of the following is the general solution of the differential equation $(x+1)y' = y + e^x(x+1)^2$?

a) $\frac{y}{(x+1)^2} = e^x + C$ b) $\frac{y}{x+1} = e^x + C$ c) $\frac{y}{(x+1)^3} = e^{2x} + C$

d) $\frac{y}{x+1} = e^{2x} + C$ e) $\frac{y}{x+1} = e^{-2x} + C$

$y' = \frac{y}{x+1} + e^x \quad (x \neq -1)$

$y' - \frac{y}{x+1} = e^x(x+1)$ L.D.D.

$\Rightarrow \frac{y'}{x+1} - \frac{y}{(x+1)^2} = e^x \Rightarrow \frac{dy}{x+1} - \frac{y\,dx}{(x+1)^2} = e^x\,dx$

$p(x) = \frac{-1}{x+1} \Rightarrow \lambda = e^{\int p(x)dx} = e^{-\int \frac{dx}{x+1}} = e^{-\ln(x+1)} = \frac{1}{x+1}$

$\int d\left(\frac{y}{x+1}\right) = \int e^x dx \Rightarrow \frac{y}{x+1} = e^x + C$

8) Which of the following is the general solution of the differential equation $yy'' + (y')^2 = 0$?

a) $y = C_1x + C_2$ b) $y = C_1x^2 + C_2$ c) $y = C_1e^x + C_2$

d) $y^2 = C_1x + C_2$ e) $y^2 = C_2e^{C_1x}$

$yy'' + y'^2 = 0$

$y' = p$, $y'' = p\frac{dp}{dy}$ $\Rightarrow$ $yp\frac{dp}{dy} + p^2 = 0$, $p\left[y\frac{dp}{dy} + p\right] = 0$

1°) $p = 0 \Rightarrow y' = 0 \Rightarrow y = c$ T.C.

2°) $y\frac{dp}{dy} + p = 0 \Rightarrow \int \frac{dp}{p} + \int \frac{dy}{y} = 0$

$\ln p + \ln y = \ln c_1 \Rightarrow p = \frac{c_1}{y} \Rightarrow y' = \frac{c_1}{y}$

$\int y\,dy = \int c_1\,dx \Rightarrow \frac{y^2}{2} = c_1x + c_2$ $\quad (2c_1 = c_1,\ 2c_2 = c_2)$

9) Which of the following is the recurrence relation obtained for the power series solution of the differential equation $y'' + 2y' = 0$ around the point $x = 0$?

a) $a_n = \frac{2}{n}a_{n-1}$ b) $a_n = \frac{1}{n}a_{n-1}$ c) $a_n = \frac{3}{n}a_{n-1}$ d) $a_n = -\frac{2}{n}a_{n-1}$ e) $a_n = -\frac{1}{n}a_{n-1}$

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10) Which of the following is true about the differential equation $y^{(4)} - y''' - 3y'' + 5y' - 2y = 0$?

a) The characteristic equation has a coincident root.
b) The characteristic equation has a complex root.
c) The general solution has 3 constants.
d) The characteristic equation has no real roots.
e) One of the roots of the characteristic equation is 2.

$(D^4 - D^3 - 3D^2 + 5D - 2)y = 0$

k.D: $r^4 - r^3 - 3r^2 + 5r - 2 = 0$

$r^4 - r^3 - 3r^2 + 3r + 2r - 2 = 0$

$r^3(r-1) - 3r(r-1) + 2(r-1) = 0$

$(r-1)[r^3 - 3r + 2] = 0$

$r_1 = 1$ $\quad r^3 - 3r + 2 \;\big|\; r-1$, $r^2 + r - 2$

$(r-1)^2 \cdot (r^2 + r - 2) = 0$

$(r-1)^2(r+2)(r-1) = 0$

11) The general solution of the differential equation $y'' + ay' + by = 0$ is $y = e^{2x}(c_1 \sin 2x + c_2 \cos 2x)$. Which of the following is a+b?

a) 2
b) 4
c) 6
d) 8
e) 10

$r_{1,2} = 2 \mp 2i$

$(r-2+2i)(r-2-2i)$

$r^2 - 2r - 2ir - 2r + 4 + 4i + 2ir - 4i + 4$

$r^2 - 4r + 8 \qquad a = -4$

$b = 8$

12) Which of the following is the function $x_1(t)$ in the system of differential equations $\dfrac{dx_1}{dt} = x_1 + x_2 + e^{-2t}$, $\dfrac{dx_2}{dt} = 4x_1 - 2x_2$?

a) $x_1(t) = c_1 e^{2t} + c_2 e^{-3t}$
b) $x_1(t) = c_1 e^{-2t} + c_2 e^{-3t}$
c) $x_1(t) = c_1 e^{-2t} + c_2 e^{3t}$
d) $x_1(t) = c_1 e^{2t} + c_2 e^{3t}$
e) $x_1(t) = c_1 e^{2t} + c_2 e^{4t}$

13) Which of the following is the solution function group that forms the general solution of the 3rd order differential equation?

a) $y_1 = 2x^2 + 1,\ y_2 = x^2 - 1,\ y_3 = 1$
b) $y_1 = 1,\ y_2 = \sin x,\ y_3 = -2\sin x$
c) $y_1 = e^{-2x},\ y_2 = e^{x},\ y_3 = 4e^{-2x}$
d) $y_1 = e^{2x},\ y_2 = xe^{2x},\ y_3 = e^{-x}$
e) $y_1 = \cos x,\ y_2 = e^{x},\ y_3 = 4\cos x$

$c_1 y_1 + c_2 y_2 + c_3 y_3 = 0$

$c_1(2x^2+1) + c_2(x^2-1) + c_3 = 0$

$c_1 = 1 \qquad c_2 = -2 \qquad c_3 = -3$

$2x^2 + 1 - 2x^2 + 2 - 3 = 0$

14) If a and b are real numbers and the solution of the differential equation $ay'' + by' + 3y = 0$ is $y = -2xe^{-3x}$, which of the following is $a + b$?

a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{5}{3}$ d) $\frac{7}{3}$ e) $\frac{8}{3}$

15) When the Laplace transform is applied to the linear differential equation with constant coefficients at the time $t = 0$, $Y(s) = \dfrac{3s}{s^2 + 4s + 8}$ is calculated such that $L^{-1}\{Y(s)\} = y(t)$. Accordingly, which of the following is the solution function $y(t)$?

a) $y(t) = 3e^{-2t}(\cos 2t + \sin 2t)$
b) $y(t) = 3e^{2t}(\cos 2t - \sin 2t)$
c) $y(t) = 3e^{-2t}(\cos 2t - \sin 2t)$
d) $y(t) = 3e^{2t}(\cos 2t + \sin 2t)$
e) $y(t) = 3e^{-2t}(\cos 2t - 2\sin 2t)$

$2r^2 - r = 0$ $r_1 = 0$ $y_1 = 1$ $y_1' = 0$

$r(2r-1) = 0$ $r_2 = 1/2$ $y_2 = e^{x/2}$ $y_2' = \frac{1}{2}e^{x/2}$

16) Which of the following is the Wronskian determinant of the solution set of the differential equation $2y''-y'=0$?

a) $\frac{1}{2}e^{\frac{x}{2}}$ b) $e^{\frac{x}{2}}$ c) 0 d) $\frac{1}{2}e^{-\frac{x}{2}}$ e) $e^{-\frac{x}{2}}$

17) When the Laplace transform is applied to the initial value problem $y'''+ay''+by'+cy=0;\ y(0)=1,\ y'(0)=0,\ y''(0)=0$, the expression $Y(s)=\frac{s^2+5s}{(s+1)(s^2+4s-4)}$ is obtained. So which of the following is $a+b+c$?

a) -1
b) 0
c) 1
d) 2
e) -2

18) Which of the following is the solution of the system of differential equations $\begin{cases} u' = 4u - v \\ v' = -4u + 4v \end{cases}$ such that $u = u(x)$ and $v = v(x)$?

a) $u(x)=c_1e^{2x}+c_2e^{6x},\ v(x)=2c_1e^{2x}-2c_2e^{6x}$
b) $u(x)=c_1e^{-2x}+c_2e^{6x},\ v(x)=2c_1e^{-2x}-2c_2e^{6x}$
c) $u(x)=c_1e^{2x}+c_2e^{-6x},\ v(x)=2c_1e^{2x}-2c_2e^{-6x}$
d) $u(x)=c_1e^{-2x}+c_2e^{-6x},\ v(x)=2c_1e^{-2x}-2c_2e^{-6x}$
e) $u(x)=c_1e^{2x}-2c_2e^{6x},\ v(x)=c_1e^{2x}+c_2e^{6x}$

19) Which of the following is the general solution of the differential equation $y^{(5)}+2y'''+y'=0$?

a) $y=c_1x+(c_2x+c_3)\sin x++(c_4x+c_5)\cos x$
b) $y=c_1+(c_2x+c_3)\sin x++(c_4x+c_5)\cos x$
c) $y=c_1+(c_2x+c_3)\sin 2x++(c_4x+c_5)\cos 2x$
d) $y=c_1x^2+(c_2x+c_3)\sin x++(c_4x+c_5)\cos x$
e) $y=c_1x^2+(c_2x+c_3)\sin 2x++(c_4x+c_5)\cos 2x$

20) The differential equation $2y'''+6y''+8y'+4y=3\sin 2x-4e^{-x}+8x^2+5+e^{-x}\cos x$ is given. Which of the following cannot be the particular solution of the nonhomogeneous part?

a) $y_p=A\cos 2x+B\sin 2x$
b) $y_p=Axe^{-x}$
c) $y_p=e^{-x}(A\cos x+B\sin x)$
d) $y_p=A$
e) $y_p=Ax^2+Bx+C$