TAYLOR – MCLAURIN SERİLERİ

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + f''(a)\frac{1}{2!}(x-a)^2 + \cdots$$

Taylor serisi

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$$

Mclaurin serisi

BAZI MCLAURIN SERİLERİ

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

ÖR: $\sqrt[3]{7}$'yi 3. derece Taylor Polinomunu kullanarak yaklaşık olarak hesaplayınız.

$f(x) = \sqrt[3]{x}$ fonksiyonunun $x_0 = 8$ noktasında üretilen 3. derece Taylor polinomu

$$T_3(x) = f(8) + f'(8)(x-8) + \frac{f''(8)}{2!}(x-8)^2 + \frac{f'''(8)}{3!}(x-8)^3$$

dır.

$$f(x) = \sqrt[3]{x} = x^{1/3} \Rightarrow f(8) = 2$$

$$f'(x) = \frac{1}{3}x^{-2/3} \Rightarrow f'(8) = \frac{1}{3.4}$$

$$f''(x) = \frac{-2}{9}x^{-5/3} \Rightarrow f''(8) = \frac{-2}{9.32}$$

$$f'''(x) = \frac{10}{27}x^{-8/3} \Rightarrow f'''(8) = \frac{10}{27.2^8}$$

$$\Rightarrow \sqrt[3]{7} \cong 2 + \left(\frac{1}{12}\right).(7-8) + \left(\frac{-2}{9.2^5}\right).\frac{1}{2}.(7-8)^2 + \frac{10}{27.2^8}.\frac{1}{3!}(7-8)^3$$

$$\cong 2 - \frac{1}{12} - \frac{1}{9.2^5} - \frac{10}{27.6.2^8}$$

$$\cong 1,9129$$

ÖR: 2. mertebe Taylor (Mclaurin) polinomunu kullanarak

$$\int_0^1 \frac{1-e^{-x^2}}{x^2}dx$$

integralini yaklaşık olarak hesaplayınız.

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots \quad (\text{Mclaurin serisi } e^x \text{ için})$$

$$\Rightarrow e^{-x^2} = 1+(-x^2)+\frac{(-x^2)^2}{2!}+\frac{(-x^2)^3}{3!}+\cdots$$

$$= 1-x^2+\frac{x^4}{2!}-\frac{x^6}{3!}+\cdots$$

$$\Rightarrow \int_0^1 \frac{1-e^{-x^2}}{x^2}dx \cong \int_0^1 \frac{\cancel{1}-\left[\cancel{1}-x^2+\frac{x^4}{2!}\right]}{x^2}dx$$

$$\cong \int_0^1 \frac{x^2-\frac{x^4}{2}}{x^2}dx$$

$$\cong \int_0^1 \left(1-\frac{x^2}{2}\right)dx$$

$$\cong x-\frac{x^3}{6}\Big|_0^1 = 1-\frac{1}{6}=\frac{5}{6}$$

ÖR: Mclaurin seri açılımından yararlanarak

$$\lim_{x\to 0}\frac{e^{\sin x}-2x-1}{\sin x}$$

limitinin değerini hesaplayınız.

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

$$e^{\sin x} = 1+\sin x+\frac{(\sin x)^2}{2!}+\frac{(\sin x)^3}{3!}+\cdots$$

$$\Rightarrow \lim_{x \to 0} \frac{e^{\sin x} - 2x - 1}{\sin x} = \lim_{x \to 0} \frac{\left(\cancel{1} + \sin x + \frac{\sin^2 x}{2!} + \frac{\sin^3 x}{3!} + \dots\right) - 2x - \cancel{1}}{\sin x}$$

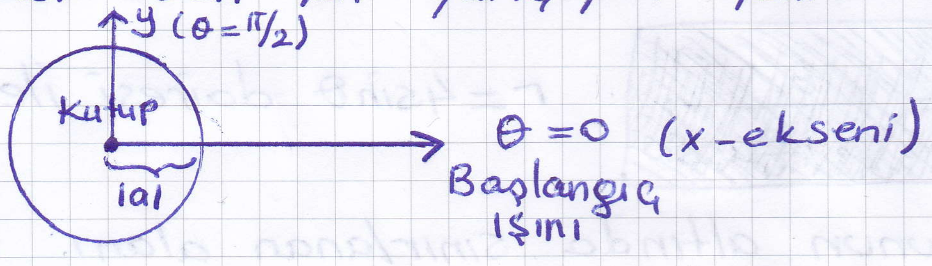
$$= \lim_{x \to 0} \left[\left(1 + \frac{\sin x}{2!} + \frac{\sin^2 x}{3!} + \dots\right) - \frac{2x}{\sin x} \right]$$

$$= 1 - 2$$

$$= -1$$

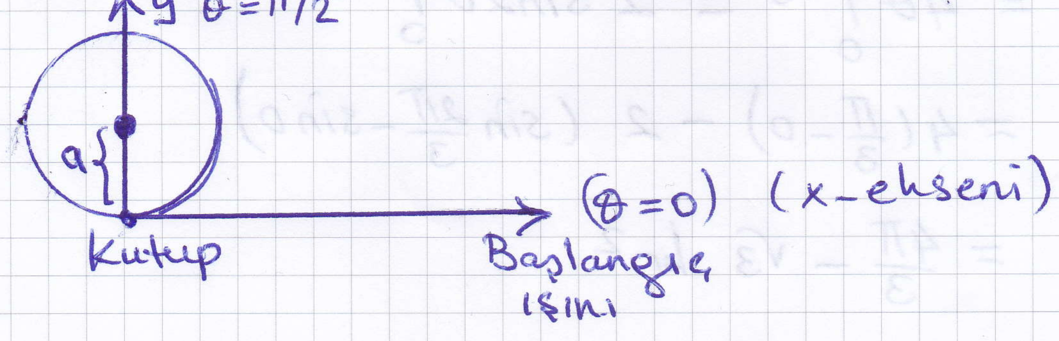
– ÇEMBERLER –

1) $f(\theta)=r=a$ şeklinde denklemi verilen kutupsal eğri, merkezi kutup (başlangıç) noktası olan $|a|$ yarıçaplı çemberdir.



2) $f(\theta)=r=2a\cos\theta$ şeklinde denklemi verilen kutupsal eğri, merkezi pozitif x-ekseni üzerinde $(a,0)$ noktasında olan yarıçapı da a olan çemberdir.

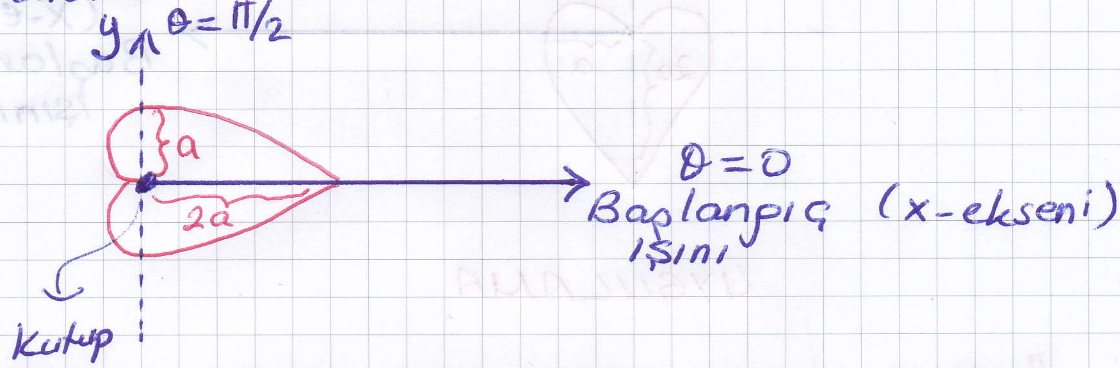
3) $f(\theta)=r=2a\sin\theta$ şeklinde denklemi verilen kutupsal eğri, merkezi pozitif y-ekseni üzerinde $(0,a)$ noktasında olan yarıçapı da a olan çemberdir.



KARDİOİDLER

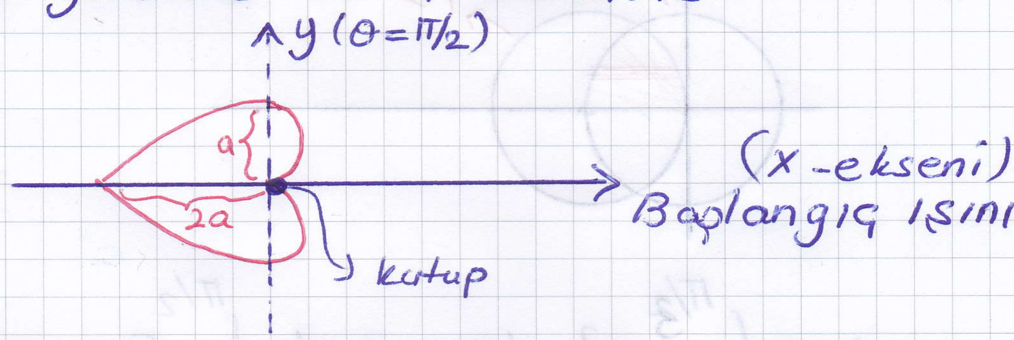
1) $f(\theta) = a(1+\cos\theta)$ $(a>0)$

x-ekseni boyunca uzanan sivri ucu x-ekseninin pozitif yönünde olan kardioid



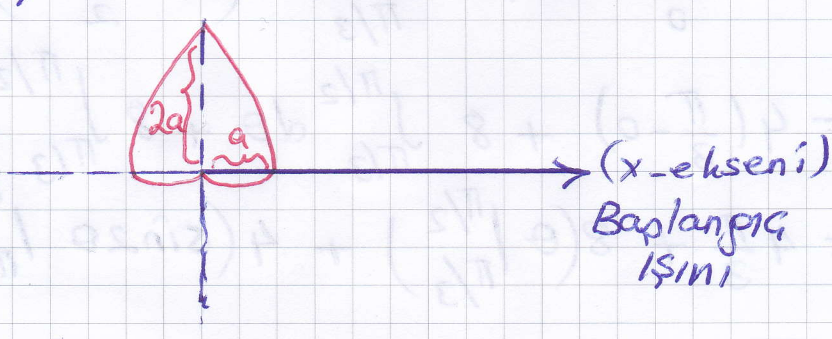
2) $f(\theta) = a(1-\cos\theta)$ $(a>0)$

x-ekseni boyunca uzanan sivri ucu x-ekseninin negatif yönünde olan kardioid



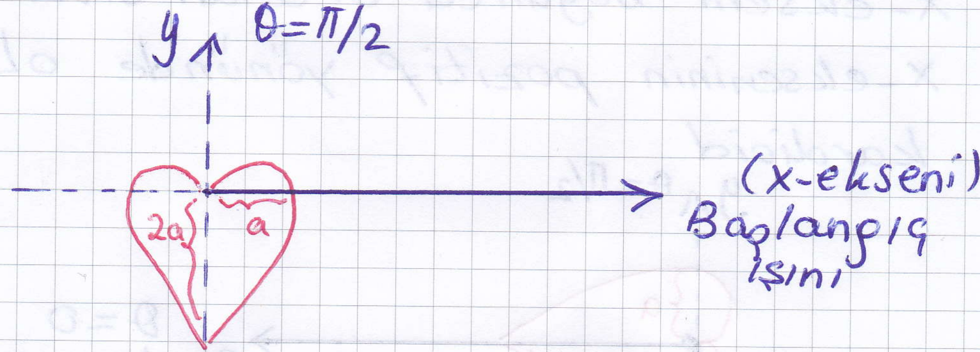
3) $f(\theta) = a(1+\sin\theta)$ $(a>0)$

y-ekseni boyunca uzanan sivri ucu y-ekseninin pozitif yönünde olan kardioid



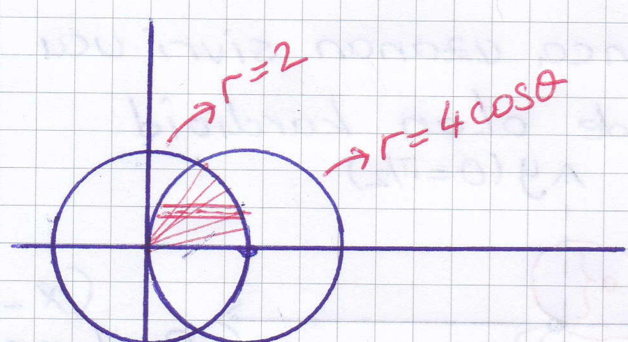
4) $f(\theta) = a(1-\sin\theta) \quad (a>0)$

y-ekseni boyunca uzanan sivri ucu y-ekseninin negatif yönünde olan kardioid



UYGULAMA

Örnek 1: $r=2$ ve $r=4\cos\theta$ eğrilerinin sınırladıkları alanı hesaplayınız.



$$A = \frac{2}{2}\int_0^{\pi/3} 2^2\, d\theta + 2\cdot\frac{1}{2}\int_{\pi/3}^{\pi/2} (4\cos\theta)^2\, d\theta$$

$$= 4\int_0^{\pi/3} d\theta + 16\int_{\pi/3}^{\pi/2} \cos^2\theta\, d\theta$$

$$= 4\theta\Big|_0^{\pi/3} + 16\int_{\pi/3}^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)d\theta$$

$$= 4\left(\frac{\pi}{3}-0\right) + 8\int_{\pi/3}^{\pi/2} d\theta + 8\int_{\pi/3}^{\pi/2}\cos 2\theta\, d\theta$$

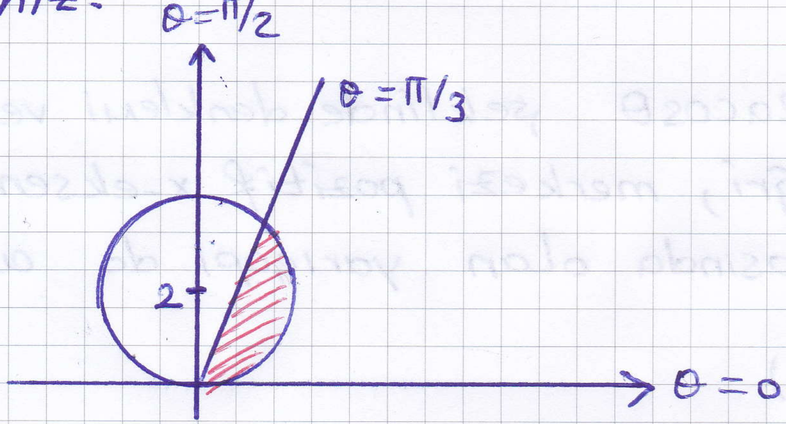
$$= 4\frac{\pi}{3} + 8\left(\theta\Big|_{\pi/3}^{\pi/2}\right) + 4\left(\sin 2\theta\Big|_{\pi/3}^{\pi/2}\right)$$

$$= 4\frac{\pi}{3} + 8\left(\frac{\pi}{2} - \frac{\pi}{3}\right) + 4\left(\sin\pi - \sin\frac{2\pi}{3}\right)$$

$$= 4\frac{\pi}{3} + \frac{8\pi}{6} - 4\cdot\frac{\sqrt{3}}{2}$$

$$= \frac{8\pi}{3} - 2\sqrt{3} \ br^2$$

ÖR 2: [redacted] $r = 4\sin\theta$ dairesi ile $\theta = \pi/3$ doğrusunun altında sınırlanan alanı hesaplayınız.



$$A = \frac{1}{2}\int_0^{\pi/3} (4\sin\theta)^2 \, d\theta$$

$$= 8\int_0^{\pi/3} \sin^2\theta \, d\theta$$

$$= 8\int_0^{\pi/3} \frac{1-\cos 2\theta}{2} \, d\theta$$

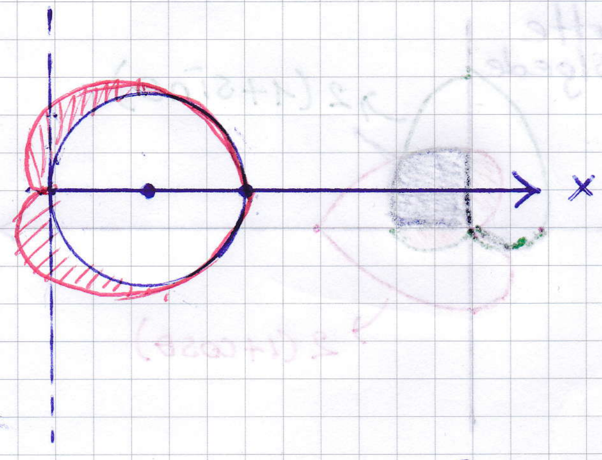
$$= 4\int_0^{\pi/3} d\theta - 4\int_0^{\pi/3} \cos 2\theta \, d\theta$$

$$= 4\theta \Big|_0^{\pi/3} - 2\sin 2\theta \Big|_0^{\pi/3}$$

$$= 4\left(\frac{\pi}{3} - 0\right) - 2\left(\sin\frac{2\pi}{3} - \sin 0\right)$$

$$= \frac{4\pi}{3} - \sqrt{3} \ br^2$$

ÖR 3: $r = a(1+\cos\theta)$ ve $r = 2a\cos\theta$ eğrileri arasında kalan alanı hesaplayınız.



$$\Rightarrow \frac{A}{2} = \frac{1}{2}\int_0^{\pi} [a(1+\cos\theta)]^2 d\theta - \frac{1}{2}\int_0^{\pi/2} (2a\cos\theta)^2 d\theta$$

$$\Rightarrow \frac{A}{2} = \frac{1}{2}\int_0^{\pi} a^2(1+2\cos\theta+\cos^2\theta)d\theta - 2a^2\int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$\Rightarrow \frac{A}{2} = \frac{a^2}{2}\int_0^{\pi} d\theta + a^2\int_0^{\pi} \cos\theta\, d\theta + \frac{a^2}{2}\int_0^{\pi}\cos^2\theta\, d\theta - 2a^2\int_0^{\pi/2}\frac{1+\cos 2\theta}{2}d\theta$$

$$\Rightarrow \frac{A}{2} = \frac{a^2}{2}\theta\Big|_0^{\pi} + a^2\sin\theta\Big|_0^{\pi} + \frac{a^2}{2}\int_0^{\pi}\frac{1+\cos 2\theta}{2}d\theta - a^2\int_0^{\pi/2} d\theta - a^2\int_0^{\pi/2}\cos 2\theta\, d\theta$$

$$\Rightarrow \frac{A}{2} = \frac{a^2}{2}(\pi-0) + a^2\cancel{(\sin\pi - \sin 0)}^{0} + \frac{a^2}{4}\int_0^{\pi} d\theta + \frac{a^2}{4}\int_0^{\pi}\cos 2\theta\, d\theta$$
$$- a^2\theta\Big|_0^{\pi/2} - \frac{a^2}{2}\sin 2\theta\Big|_0^{\pi/2}$$

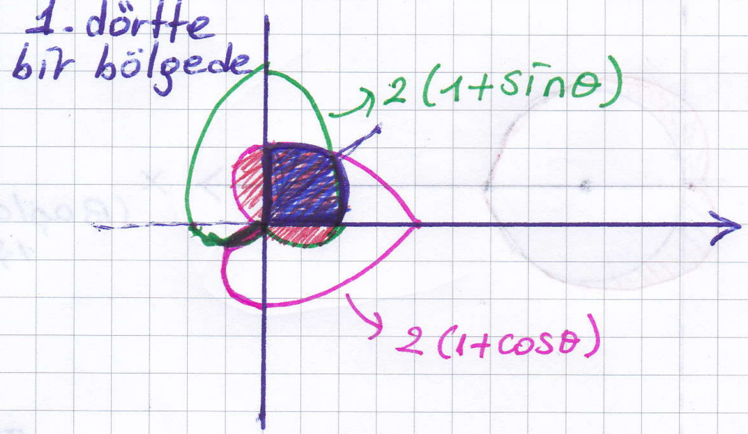
$$\Rightarrow \frac{A}{2} = \frac{a^2\pi}{2} + \frac{a^2}{4}\left(\theta\Big|_0^{\pi}\right) + \frac{a^2}{8}\sin 2\theta\Big|_0^{\pi} - a^2\left(\frac{\pi}{2}-0\right)$$
$$- \frac{a^2}{2}\cancel{(\sin\pi - \sin 0)}^{0}$$

$$\Rightarrow \frac{A}{2} = \frac{a^2\pi}{2} + \frac{a^2}{4}(\pi-0) + \frac{a^2}{8}\cancel{(\sin 2\pi - \sin 0)}^{0} - \frac{a^2\pi}{2}$$

$$\Rightarrow \frac{A}{2} = \frac{a^2\pi}{2} + \frac{a^2\pi}{4} - \frac{a^2\pi}{2}$$

$$\Rightarrow \frac{A}{2} = \frac{3a^2\pi}{4} - \frac{a^2\pi}{2} \Rightarrow A = \frac{3a^2\pi}{2} - a^2\pi = \frac{a^2\pi}{2}\, br^2$$

ÖR 4: $r = 2(1+\cos\theta)$ eğrisi ile $r = 2(1+\sin\theta)$ eğrisinin sınırladıkları bölgenin alanını bulunuz.



$2(1+\sin\theta) = 2(1+\cos\theta)$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \theta = \frac{\pi}{4}$

$$A = \frac{1}{2}\int_0^{\pi/4} \left[2(1+\sin\theta)\right]^2 d\theta + \frac{1}{2}\int_{\pi/4}^{\pi/2} \left[2.(1+\cos\theta)\right]^2 d\theta$$

$$= 2\int_0^{\pi/4} (1+2\sin\theta+\sin^2\theta)\,d\theta + 2\int_{\pi/4}^{\pi/2} (1+2\cos\theta+\cos^2\theta)\,d\theta$$

$$= 2\int_0^{\pi/4} d\theta + 4\int_0^{\pi/4} \sin\theta\, d\theta + 2\int_0^{\pi/4}\left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$+ 2\int_{\pi/4}^{\pi/2} d\theta + 4\int_{\pi/4}^{\pi/2} \cos\theta\, d\theta + 2\int_{\pi/4}^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= 2\theta\Big|_0^{\pi/4} + 4\left(-\cos\theta\Big|_0^{\pi/4}\right) + \int_0^{\pi/4} d\theta - \int_0^{\pi/4} \cos 2\theta\, d\theta$$

$$+ 2\theta\Big|_{\pi/4}^{\pi/2} + 4\sin\theta\Big|_{\pi/4}^{\pi/2} + \int_{\pi/4}^{\pi/2} d\theta + \int_{\pi/4}^{\pi/2} \cos 2\theta\, d\theta$$

$$= 2\frac{\pi}{4} - 4\left(\cos\frac{\pi}{4} - \cos 0\right) + \theta\Big|_0^{\pi/4} - \frac{1}{2}\sin 2\theta\Big|_0^{\pi/4}$$

$$+ 2\left(\frac{\pi}{2} - \frac{\pi}{4}\right) + 4\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right) + \theta\Big|_{\pi/4}^{\pi/2} + \frac{1}{2}\sin 2\theta\Big|_{\pi/4}^{\pi/2}$$

$= \cancel{\frac{\pi}{2} - 4\left(\frac{1}{\sqrt{2}} - 1\right) + \left(\frac{\pi}{4} - 0\right) - \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin 0\right)}$

$$= \frac{\pi}{2} - 4\left(\frac{1}{\sqrt{2}} - 1\right) + \left(\frac{\pi}{4} - 0\right) - \frac{1}{2}\left(\sin\frac{\pi}{2} - \underbrace{\sin 0}_{0}\right)$$

$$+ 2\left(\frac{\pi}{4}\right) + 4\left(1 - \frac{1}{\sqrt{2}}\right) + \left(\frac{\pi}{2} - \frac{\pi}{4}\right) + \frac{1}{2}\left(\sin\frac{\pi}{2} - \underbrace{\sin 0}_{0}\right)$$

$$= \cancel{\frac{\pi}{2}} - \frac{4}{\sqrt{2}} + \cancel{4} + \cancel{\frac{\pi}{4}} - \cancel{\frac{1}{2}} + \cancel{\frac{\pi}{2}} + \cancel{4} - \frac{4}{\sqrt{2}} + \cancel{\frac{\pi}{4}} + \cancel{\frac{1}{2}}$$

$$= \frac{3\pi}{2} + 8 - \frac{8}{\sqrt{2}} \; br^2$$

ÖR 5: $r = a(1+\sin\theta)$ eğrisinin uzunluğunu hesaplayınız.

$$r' = a\cos\theta$$

$$S = 2\int_{-\pi/2}^{\pi/2} \sqrt{r^2 + r'^2}\, d\theta$$

$$= 2\int_{-\pi/2}^{\pi/2} \sqrt{[a(1+\sin\theta)]^2 + (a\cos\theta)^2}\, d\theta$$

$$= 2\int_{-\pi/2}^{\pi/2} \sqrt{a^2 + 2a^2\sin\theta + a^2\sin^2\theta + a^2\cos^2\theta}\, d\theta$$

$$= 2\int_{-\pi/2}^{\pi/2} \sqrt{2a^2 + 2a^2\sin\theta}\, d\theta$$

$$= 2a\sqrt{2}\int_{-\pi/2}^{\pi/2} \sqrt{1+\sin\theta}\, d\theta$$

$$= 2a\sqrt{2}\int_{-\pi/2}^{\pi/2} \sqrt{\underbrace{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}}_{=1} + \underbrace{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}_{=\sin\theta}}\, d\theta$$

$$= 2a\sqrt{2}\int_{-\pi/2}^{\pi/2} \sqrt{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2}\, d\theta$$

$$= 2a\sqrt{2}\int_{-\pi/2}^{\pi/2}\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)d\theta$$

$$= 2a\sqrt{2}\,.\,2\left(\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\Big|_{-\pi/2}^{\pi/2}\right)$$

$$= 4a\sqrt{2}\left[\left(\sin\frac{\pi}{4}-\cos\frac{\pi}{4}\right)-\left(\sin\left(-\frac{\pi}{4}\right)-\cos\left(-\frac{\pi}{4}\right)\right)\right.$$

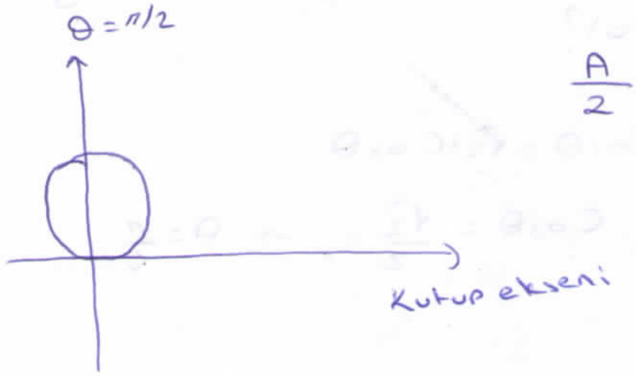
$$= 4a\sqrt{2}\left[\sin\frac{\pi}{4}-\cancel{\cos\frac{\pi}{4}}+\sin\frac{\pi}{4}+\cancel{\cos\frac{\pi}{4}}\right]$$

$$= 8a\sqrt{2}\,.\,\frac{1}{\sqrt{2}}$$

$$= 8a \text{ br}$$

ÖR 6:

(*) $r=\sqrt{2}\sin\theta$ ile sınırlı bölgenin alanı?

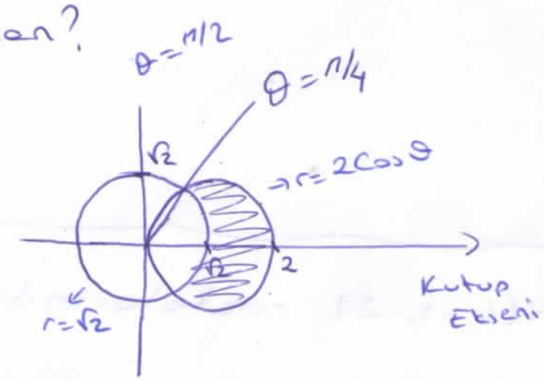


$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/2}(\sqrt{2}\sin\theta)^2\,d\theta$$

$$=\frac{1}{2}\int_0^{\pi/2}2\sin^2\theta\,d\theta$$

$$=\int_0^{\pi/2}\frac{1-\cos 2\theta}{2}\,d\theta=\frac{\pi}{4}\qquad \boxed{A=\frac{\pi}{2}}$$

(*) a) $r=2\cos\theta$ eğrisinin içinde $r=\sqrt{2}$ nin dışında kalan alan?



$$2\cos\theta=\sqrt{2}\ \rightarrow\ \boxed{\theta=\frac{\pi}{4}}$$

$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/4}(2\cos\theta)^2\,d\theta-\frac{1}{2}\int_0^{\pi/4}(\sqrt{2})^2\,d\theta$$

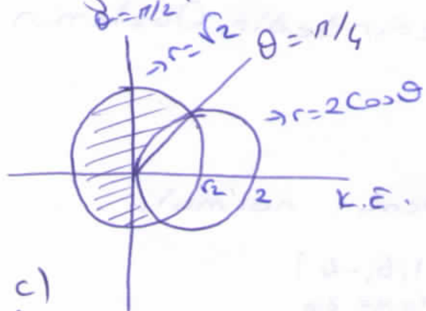
$$=\frac{1}{2}\left[\int_0^{\pi/4}(4\cos^2\theta-2)\,d\theta\right]$$

$$=\frac{1}{2}\int_0^{\pi/4}2\cos 2\theta\,d\theta=\frac{\sin 2\theta}{2}\Big|_0^{\pi/4}=\frac{1}{2}$$

$$\boxed{A=1}$$

b)

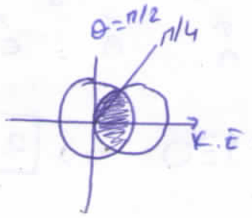
☆ $r=2\cos\theta$ nın dışında, $r=\sqrt{2}$ nin içinde kalan alanı veren integral:



$$\frac{A}{2}=\frac{1}{2}\int_{\pi/4}^{\pi}(\sqrt{2})^2\,d\theta-\frac{1}{2}\int_{\pi/4}^{\pi/2}(2\cos\theta)^2\,d\theta$$

c)

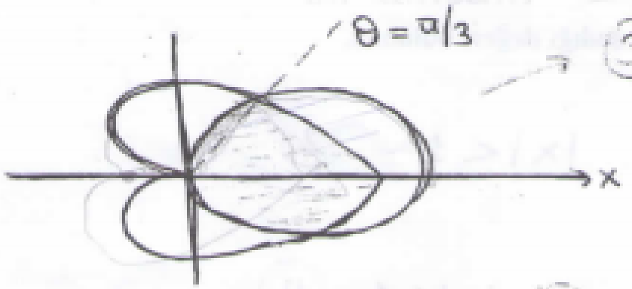
☆ Ortak Alanı veren integral:



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/4}(\sqrt{2})^2\,d\theta+\frac{1}{2}\int_{\pi/4}^{\pi/2}(2\cos\theta)^2\,d\theta$$

15

4. a) $r = 3\cos\theta$ ve $r = 1+\cos\theta$ eğrilerinin içinde kalan ortak bölgenin alanını veren integrali yazınız. (İntegral hesaplanmayacak.)



a)

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/3} (1+\cos\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2} (3.\cos\theta)^2\, d\theta$$

2 2 2 2 2 2

b) Kardiyoid içi, çember dışı

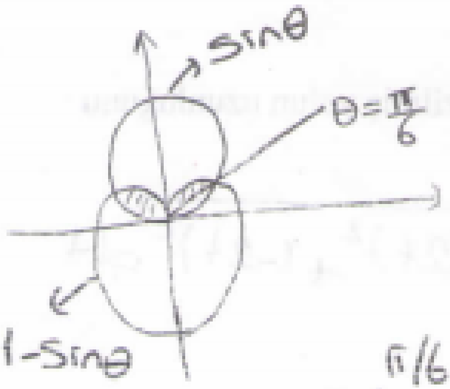
$$\frac{A}{2} = \frac{1}{2}\int_{\pi/3}^{\pi} (1+\cos\theta)^2\, d\theta - \frac{1}{2}\int_{\pi/3}^{\pi/2} (3\cos\theta)^2\, d\theta$$

c) Çember içi kar. dışı

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/3} (3\cos\theta)^2 - (1+\cos\theta)^2\, d\theta$$

10.

b) $\rho = 1 - \sin\theta$ kardiyoidi ve $\rho = \sin\theta$ çemberinin her ikisinin de içinde kalan bölgenin alanını bulunuz .(12p)



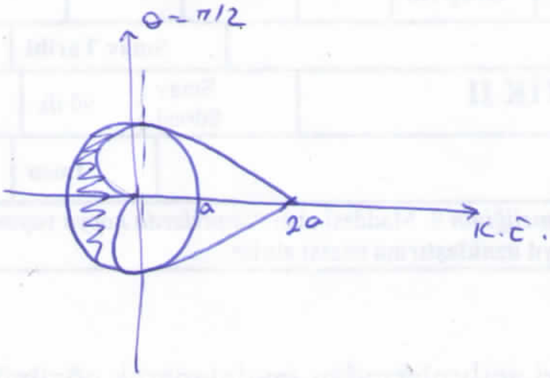
$$1-\sin\theta = \sin\theta \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/6} (\sin\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} (1-\sin\theta)^2\, d\theta$$

$$A = \int_0^{\pi/6} \frac{1-\cos 2\theta}{2}\, d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{2}(3 - 4\sin\theta - \cos 2\theta)\, d\theta$$

$$A = \left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) + \left(\frac{\pi}{2} - \frac{7\sqrt{3}}{8}\right) = \frac{7\pi}{12} - \sqrt{3}\ \text{br}^2$$

2) $a>0$ olmak üzere $r=a(1+\cos\theta)$ kardiyoidinin dışında, $r=a$ çemberinin içinde kalan bölgenin alanını hesaplayınız. (Şekil çiziniz)



$$\frac{A}{2}=\int_{\pi/2}^{\pi} a^2-(a+a\cos\theta)^2\,d\theta=\int_{\pi/2}^{\pi}\left(-2a^2\cos\theta-a^2\underbrace{\cos^2\theta}_{\frac{1+\cos 2\theta}{2}}\right)d\theta$$

$$=-2a^2\sin\theta-a^2\frac{\theta}{2}-\frac{a^2}{4}\sin 2\theta\Big|_{\pi/2}^{\pi}$$

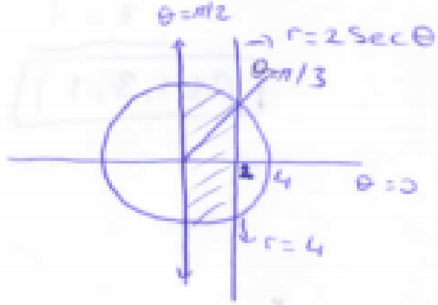
$$=-a^2\frac{\pi}{2}-\left(-2a^2-a^2\frac{\pi}{4}\right)=-\frac{a^2\pi}{2}+2a^2+\frac{a^2\pi}{4}$$

$$=2a^2-\frac{\pi}{4}a^2$$

---

5) $r=4$, $\theta=\frac{\pi}{2}$, $r=2\sec\theta$ arasında kalan bölgenin alanını veren integral?

$r=2\sec\theta=\dfrac{2}{\cos\theta}$ $\Rightarrow$ $\underbrace{r\cos\theta}_{x}=2$ $\Rightarrow$ $\boxed{x=2}$ doğrusu

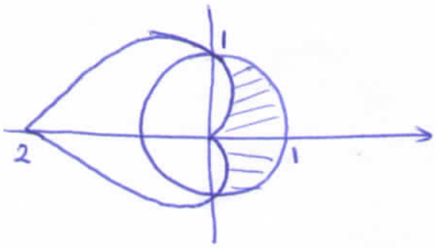


$2\sec\theta=4 \Rightarrow \sec\theta=2 \Rightarrow \cos\theta=\frac{1}{2}$

$\theta=\pi/3$

$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/3}(2\sec\theta)^2\,d\theta+\frac{1}{2}\int_{\pi/3}^{\pi/2}(4)^2\,d\theta$$

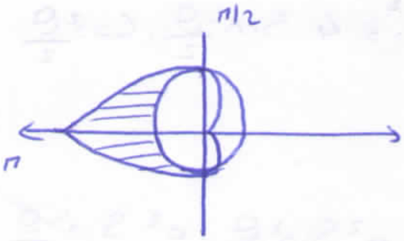
(*) $r=1$ çemberinin içinde, $r=1-\cos\theta$ kardiyoidinin dışında kalan bölgenin alanını veren integral?



$$\frac{A}{2}=\int_0^{\pi/2}\frac{1}{2}\,d\theta-\frac{1}{2}\int_0^{\pi/2}(1-\cos\theta)^2\,d\theta$$

$$A=\int_0^{\pi/2}\left(1-(1-\cos\theta)^2\right)d\theta$$

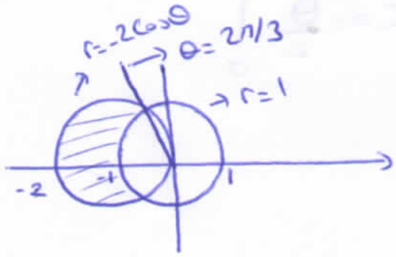
b) çemberin dışı, kardiyoidin içi:



$$\frac{A}{2}=\int_{\pi/2}^{\pi}\frac{1}{2}(1-\cos\theta)^2\,d\theta-\int_{\pi/2}^{\pi}\frac{1}{2}\,d\theta$$

$$A=\int_{\pi/2}^{\pi}\left((1-\cos\theta)^2-1\right)d\theta$$

(*) $r=-2\cos\theta$ çemberinin içinde, $r=1$ çemberinin dışında kalan alan?



$$-2\cos\theta=1 \Rightarrow \theta=\frac{2\pi}{3}$$

$$\frac{A}{2}=\frac{1}{2}\int_{2\pi/3}^{\pi}(-2\cos\theta)^2\,d\theta-\frac{1}{2}\int_{2\pi/3}^{\pi}1^2\,d\theta$$

$$A=\int_{2\pi/3}^{\pi}(4\underbrace{\cos^2\theta}_{\frac{1+\cos2\theta}{2}}-1)\,d\theta=\int_{\frac{2\pi}{3}}^{\pi}(1+2\cos2\theta)\,d\theta$$

$$=\theta+\sin2\theta\Big|_{2\pi/3}^{\pi}=\pi-\frac{2\pi}{3}-\sin\frac{\pi}{3}=\frac{\pi}{3}-\frac{\sqrt{3}}{2}$$

(*) $r = a\sin^2\frac{\theta}{2}$ eğrisinin $0 \le \theta \le \pi$ aralığındaki uzunluğu? $(a>0)$

$$S = \int_0^{\pi} \sqrt{r^2+(r')^2}\,d\theta$$

$$r^2 = a^2\sin^4\frac{\theta}{2}$$

$$r' = a.2.\sin\frac{\theta}{2}.\cos\frac{\theta}{2}.\frac{1}{2} = a.\sin\frac{\theta}{2}.\cos\frac{\theta}{2}$$

$$(r')^2 = a^2.\sin^2\frac{\theta}{2}.\cos^2\frac{\theta}{2}$$

$$r^2+(r')^2 = a^2.\sin^4\frac{\theta}{2} + a^2\sin^2\frac{\theta}{2}.\overbrace{\cos^2\frac{\theta}{2}}^{1-\sin^2\frac{\theta}{2}} = \cancel{a^2\sin^4\frac{\theta}{2}} + a^2\sin^2\frac{\theta}{2} - \cancel{a^2\sin^4\frac{\theta}{2}}$$

$$= a^2\sin^2\frac{\theta}{2}$$

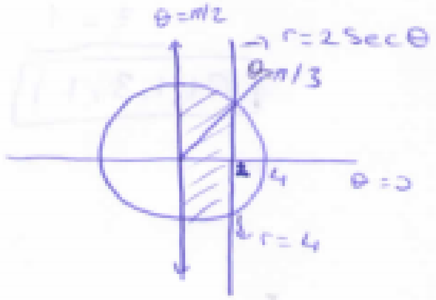
$$\sqrt{r^2+(r')^2} = \sqrt{a^2.\sin^2\frac{\theta}{2}} = a.\left|\sin\frac{\theta}{2}\right|$$

$$S = \int_0^{\pi} a.\left|\sin\frac{\theta}{2}\right|d\theta = \int_0^{\pi} a.\sin\frac{\theta}{2}\,d\theta = -2a\cos\frac{\theta}{2}\Big|_0^{\pi}$$

$$= \boxed{2a}$$

(*) $r=4$, $\theta=\frac{\pi}{2}$, $r=2\sec\theta$ arasında kalan bölgenin alanını veren integral?

$r = 2\sec\theta = \frac{2}{\cos\theta}$ $\Rightarrow$ $\underbrace{r\cos\theta}_{x} = 2$ $\Rightarrow$ $\boxed{x=2}$ doğrusu



$2\sec\theta = 4 \Rightarrow \sec\theta = 2 \Rightarrow \cos\theta = \frac{1}{2}$

$\theta = \pi/3$

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/3}(2\sec\theta)^2\,d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2}(4)^2\,d\theta$$

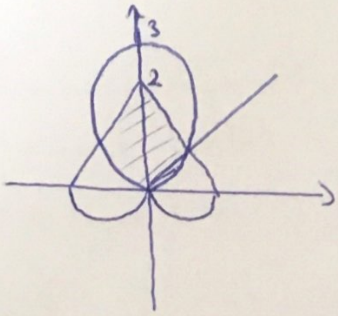
⊛ a) $r=3\sin\theta$, $r=1+\sin\theta$ ortak alan?

b) $r=3\sin\theta$ içi, $r=1+\sin\theta$ dışı alan?

c) $r=3\sin\theta$ dışı, $r=1+\sin\theta$ içi alan?

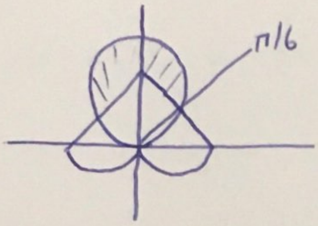
$$3\sin\theta = 1+\sin\theta \Rightarrow \sin\theta=\frac{1}{2} \qquad \boxed{\theta=\frac{\pi}{6}}$$

a)



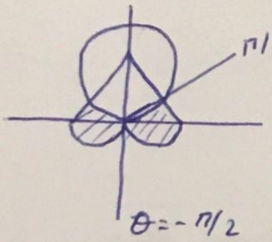
$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/6}(3\sin\theta)^2d\theta+\frac{1}{2}\int_{\pi/6}^{\pi/2}(1+\sin\theta)^2d\theta$$

b)



$$\frac{A}{2}=\frac{1}{2}\int_{\pi/6}^{\pi/2}(3\sin\theta)^2-(1+\sin\theta)^2\,d\theta$$

c)



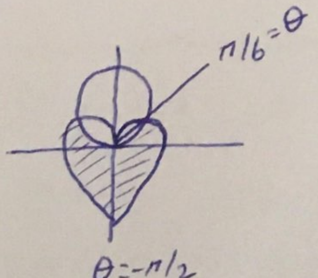
$$\frac{A}{2}=\frac{1}{2}\int_{-\pi/2}^{\pi/6}(1+\sin\theta)^2d\theta-\frac{1}{2}\int_0^{\pi/6}(3\sin\theta)^2d\theta$$

⊛ a) $r=1-\sin\theta$ içi $r=\sin\theta$ dışı alan?

b) $r=1-\sin\theta$ dışı $r=\sin\theta$ içi alan? $1-\sin\theta=\sin\theta$

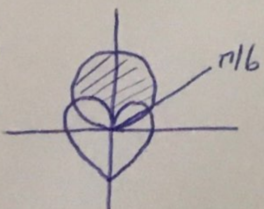
$\sin\theta=\frac{1}{2}$ $\theta=\frac{\pi}{6}$

a)



$$\frac{A}{2}=\frac{1}{2}\int_{-\pi/2}^{\pi/6}(1-\sin\theta)^2d\theta-\frac{1}{2}\int_0^{\pi/6}(\sin\theta)^2d\theta$$

b)



$$\frac{A}{2}=\frac{1}{2}\int_{\pi/6}^{\pi/2}\left((\sin\theta)^2-(1-\sin\theta)^2\right)d\theta$$