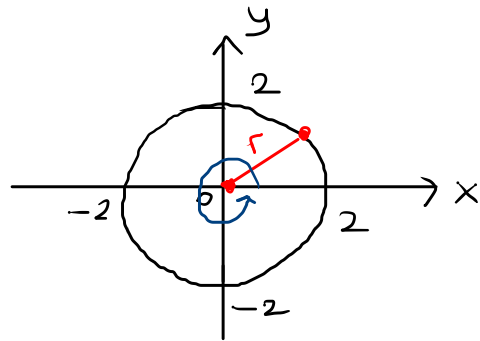
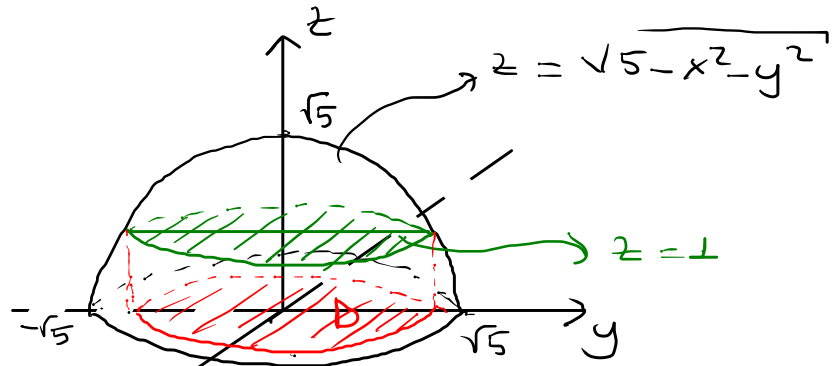


Örnekler

1) $x^2 + y^2 + z^2 = 5$ küresinin $z=1$ düzlemini üstünde kalan kısmının hacmini bulunuz.



$$\left. \begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \\ J &= r \end{aligned} \right\}$$

$$\begin{aligned} 5 - r^2 &= t \\ -2r dr &= dt \\ r dr &= -\frac{dt}{2} \end{aligned}$$

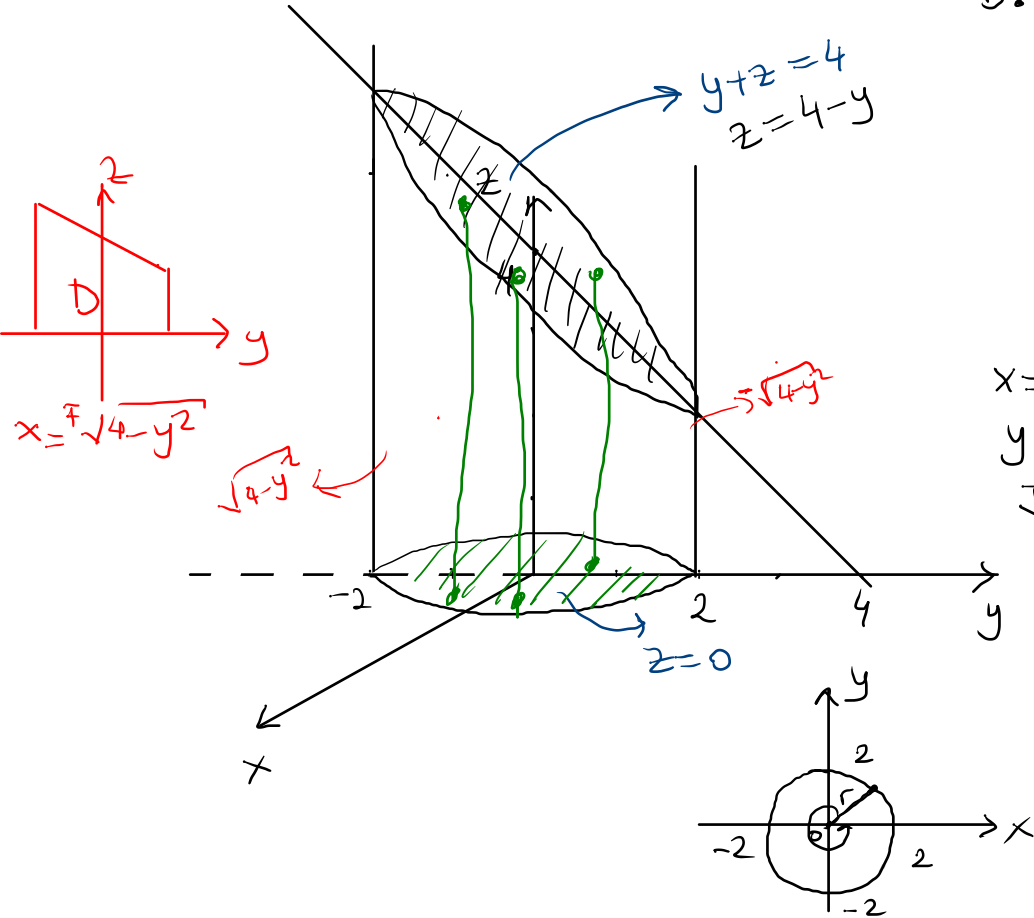
$$\begin{aligned} r=0 &\Rightarrow t=5 \\ r=2 &\Rightarrow t=1 \end{aligned}$$

$$D: \sqrt{5-x^2-y^2} = 1 \Rightarrow x^2+y^2=4$$

$$\begin{aligned} V &= \iint_D (\sqrt{5-x^2-y^2} - 1) dA = \int_0^{2\pi} \int_0^2 (\sqrt{5-r^2} - 1) r dr d\theta \\ &= \int_0^{2\pi} \int_0^2 r \sqrt{5-r^2} dr d\theta - \int_0^{2\pi} \int_0^2 r dr d\theta \\ &= \int_0^{2\pi} \int_5^1 \sqrt{t} \cdot \left(-\frac{dt}{2}\right) d\theta - \int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^2 d\theta \\ &= \frac{1}{2} \int_0^{2\pi} \int_1^5 \sqrt{t} dt d\theta - \int_0^{2\pi} 2 d\theta \\ &= \frac{1}{2} \int_0^{2\pi} \left(\frac{2}{3} t^{3/2}\right) \Big|_1^5 d\theta - (2\theta) \Big|_0^{2\pi} \\ &= \frac{1}{3} (5\sqrt{5}-1) \theta \Big|_0^{2\pi} - 4\pi \\ &= \frac{2\pi}{3} (5\sqrt{5}-1) - 4\pi = \frac{(10\sqrt{5}-14)\pi}{3} \text{ birim}^3 \end{aligned}$$

2) $x^2 + y^2 = 4$ silindiri ile $y + z = 4$, $z = 0$ düzlemleri arasında kalan cismin hacmini bulunuz.

$$D: x^2 + y^2 = 4$$



$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \\ J &= r \end{aligned}$$

$$V = \iint_D (4 - y - 0) dA = \int_0^{2\pi} \int_0^2 (4 - r \sin \theta) r dr d\theta$$

$$= \int_0^{2\pi} \int_0^2 4r dr d\theta - \int_0^{2\pi} \int_0^2 r^2 \sin \theta dr d\theta$$

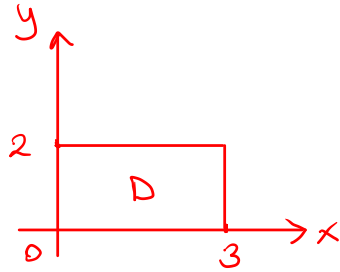
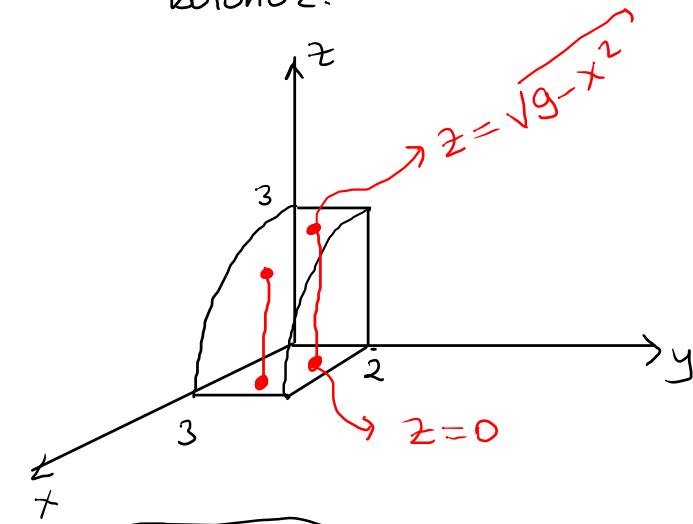
$$= \int_0^{2\pi} \left(2r^2 \Big|_0^2 \right) d\theta - \int_0^{2\pi} \left(\frac{r^3}{3} \Big|_0^2 \right) \sin \theta d\theta$$

$$= 8 \int_0^{2\pi} d\theta - \frac{8}{3} \int_0^{2\pi} \sin \theta d\theta$$

$$= 8 \left(\theta \Big|_0^{2\pi} \right) + \frac{8}{3} \left(\cos \theta \Big|_0^{2\pi} \right)$$

$$= 16\pi$$

3) $x^2 + z^2 = 9$ silindiri ile $y=2$, $x=0$, $y=0$, $z=0$ düzlemleri tarafından sınırlanan cismin hacmini bulunuz.



$$V = \iint_D (\sqrt{9-x^2} - 0) dA$$

$$= \int_0^3 \int_0^2 \sqrt{9-x^2} dy dx = \int_0^3 (y \cdot \sqrt{9-x^2} \Big|_0^2) dx$$

$$= 2 \int_0^3 \sqrt{9-x^2} dx$$

$$= 2 \int_0^{\pi/2} \sqrt{9-9\sin^2 t} \cdot 3\cos t dt$$

$$= 18 \int_0^{\pi/2} \cos^2 t dt$$

$$= 18 \int_0^{\pi/2} \frac{1+\cos 2t}{2} dt$$

$$= 9 \int_0^{\pi/2} dt + 9 \int_0^{\pi/2} \cos 2t dt$$

$$= 9t \Big|_0^{\pi/2} + \frac{9}{2} \sin 2t \Big|_0^{\pi/2}$$

$$= \frac{9\pi}{2} \text{ br }^3$$

$$x = 3 \sin t$$

$$dx = 3 \cos t dt$$

$$x=0 \Rightarrow 3 \sin t = 0 \Rightarrow \sin t = 0$$

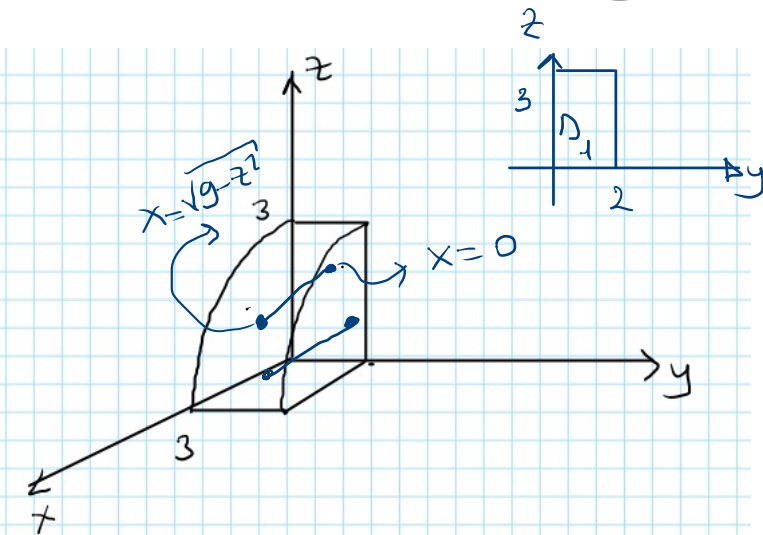
$$t=0$$

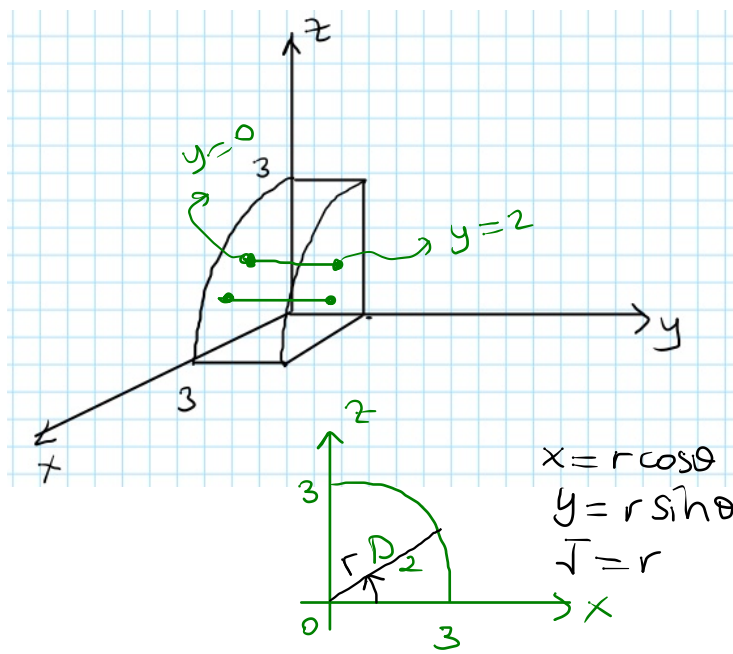
$$x=3 \Rightarrow 3 \sin t = 3 \Rightarrow \sin t = 1$$

$$t = \pi/2$$

$$V = \iint_{D_1} (\sqrt{9-z^2} - 0) dA_1$$

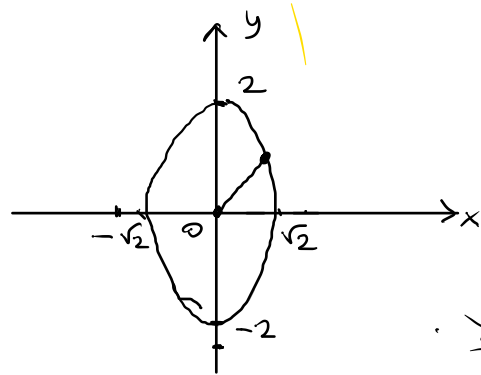
$$= \int_0^3 \int_0^2 \sqrt{9-z^2} dy dz$$





$$V = \iint_{D_2} (2-0) dA_2$$

$$= \int_0^{2\pi} \int_0^3 2r dr d\theta$$



$$x = \sqrt{2} r \cos \theta$$

$$y = 2r \sin \theta$$

$$J = 2\sqrt{2}r$$

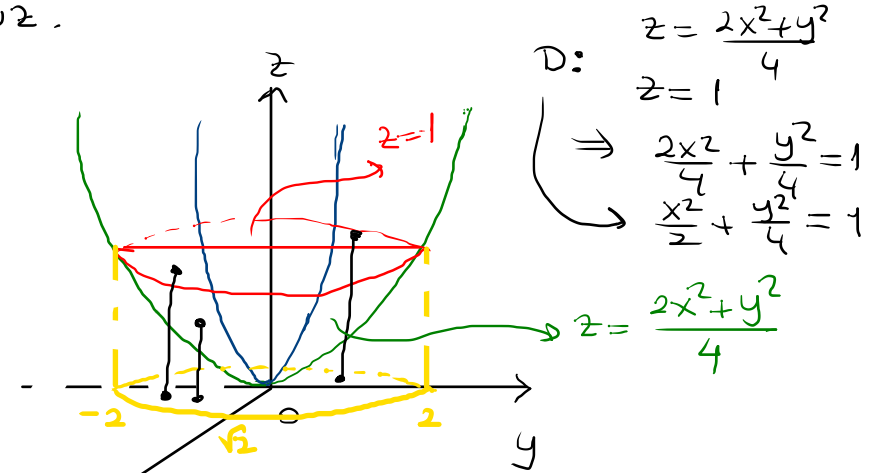
$$\frac{2r^2 \cos^2 \theta}{2} + \frac{4r^2 \sin^2 \theta}{4} = r^2$$

$$\Rightarrow V = 2\sqrt{2} \int_0^{2\pi} \int_0^1 (1-r^2) \cdot r dr d\theta$$

$$= 2\sqrt{2} \int_0^{2\pi} \left(\frac{r^2}{2} - \frac{r^4}{4} \Big|_0^1 \right) d\theta = \frac{\sqrt{2}}{2} \int_0^{2\pi} d\theta = \frac{\sqrt{2}}{2} (2\pi - 0)$$

$$= \sqrt{2}\pi br^3$$

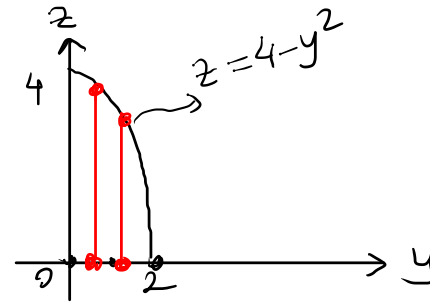
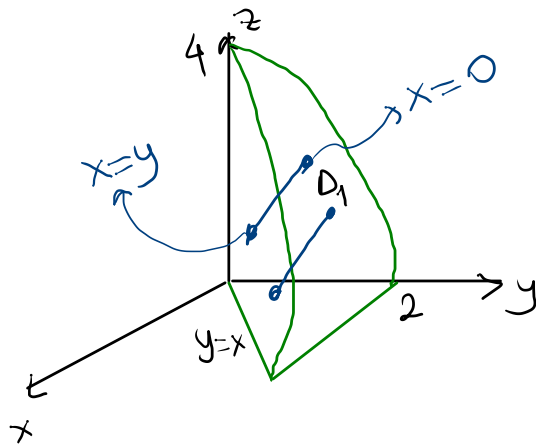
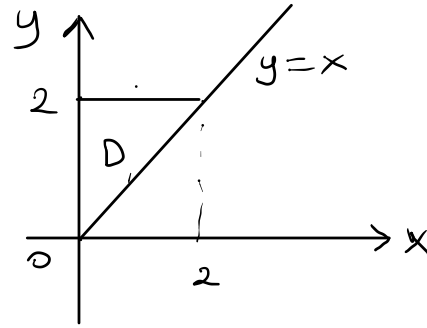
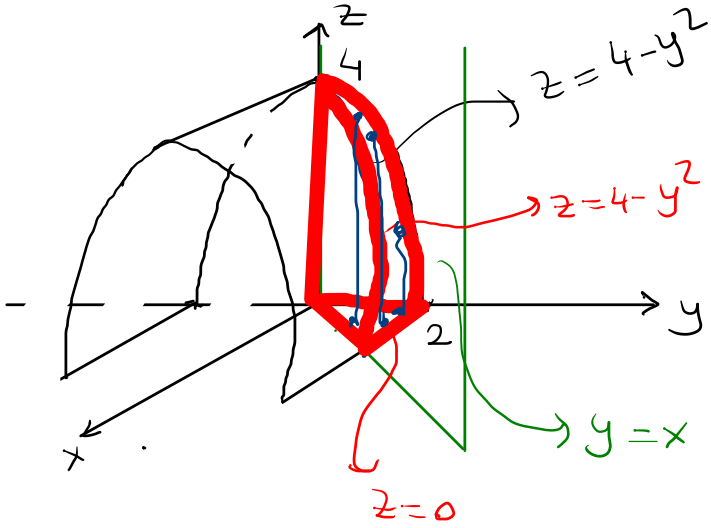
4) $2x^2 + y^2 = 4z$ paraboloidi ile $z=1$ düzleminin arasında kalan cismin hacmini bulunuz.



$$V = \iint_D \left(1 - \frac{2x^2 + y^2}{4} \right) dA$$

5) $x \geq 0, y = x, z = 0$ düzlemleri ile $z = 4 - y^2$ parabolik silindrinin birinci bölgede sınırlandığı cismin hacmini bulunuz.

$$V = \iiint_D (4 - y^2 - 0) dA = \int_0^2 \int_0^2 (4 - y^2) dy dx \quad (\text{xoy'ye izdüşüm ile hesaplama})$$



$$\begin{aligned} V &= \iint_{D_1} (y - 0) dA_1 = \int_0^2 \int_0^{4-y^2} y dz dy \\ &= \int_0^2 (yz \Big|_0^{4-y^2}) dy \\ &= \int_0^2 (4y - y^3) dy = 2y^2 - \frac{y^4}{4} \Big|_0^2 \\ &= 8 - 4 = 4 \text{ birim}^3 \end{aligned}$$

6)
öđ/

$x^2 + y^2 = 4 - z$ paraboloidi ile $z \geq 0$ ve

$3z^2 = x^2 + y^2$ konisinin sınırladığı cismin hacmini bulunuz.