4) $f(x)=z(x).e^{\alpha x}$ şeklinde ise;

1. yol $y=A(x).e^{\alpha x}$ şeklinde önerilir. Türevler alınır. Denklemde yerine yazıldığında A'ya bağlı yen... bir dif. denklem elde edilir. Elde edilen denklem çözülerek $A_{G.Ç}$ bulunur. y'nin istenen genel çözümü $y=A.e^{\alpha x}$'den elde edilir.

Ör: $y''' + 3y'' + 3y' + y = 5e^{x}\sin x$

$(D^3+3D^2+3D+1)\,y = 5e^{x}\sin x$

$y_h = (c_1 + c_2 x + c_3 x^2)\,e^{-x}$

K.D: $r^3+3r^2+3r+1=0$

$(r+1)^3=0$

$r_{1,2,3}=-1$

$y_ö = z_ö(x).e^{x}$

$y_ö' = z_ö' e^{x} + z_ö e^{x}$

$y_ö'' = z_ö'' e^{x} + 2z_ö' e^{x} + z_ö e^{x}$

$y_ö''' = z_ö''' e^{x} + 3z_ö'' e^{x} + 3z_ö' e^{x} + z_ö e^{x}$

$$\Rightarrow \left[(z_ö''' + 3z_ö'' + 3z_ö' + z_ö) + 3(z_ö'' + 2z_ö' + z_ö) + 3(z_ö' + z_ö) + z_ö\right] e^{x} = 5e^{x}\sin x$$

$z_ö''' + 6z_ö'' + 12z_ö' + 8z_ö = 5\sin x$ $\quad \lambda = 1$

$(D^3+6D^2+12D+8)\,z_ö = 5\sin x$

K.D: $r^3+6r^2+12r+8=0$

$(r+2)^3=0 \Rightarrow r_{1,2,3}=-2 \neq \lambda$

$$\Rightarrow \begin{rcases} z_0' = A\sin x + B\cos x \\ z_0'' = A\cos x - B\sin x \\ z_0''' = -A\sin x - B\cos x \\ z_0'''' = -A\cos x + B\sin x \end{rcases} \Rightarrow -A\cos x + B\sin x + 6(-A\sin x - B\cos x) + 12(A\cos x - B\sin x) + 8(A\sin x + B\cos x) \equiv 5\sin x$$

$$\Rightarrow (11A + 2B)\cos x + (-11B + 2A)\sin x \equiv 5\sin x$$

$$\begin{aligned} 11 \,/\, & 11A + 2B = 0 \\ 2 \,/\, & -11B + 2A = 5 \end{aligned}$$

$$125A = 10 \Rightarrow A = \frac{10}{125} = \frac{2}{25} \Rightarrow B = \frac{-11}{25}$$

$$\Rightarrow z_0 = \frac{2}{25}\sin x - \frac{11}{25}\cos x$$

$$\Rightarrow y_0 = z_0 \cdot e^x = \left(\frac{2}{25}\sin x - \frac{11}{25}\cos x\right) \cdot e^x$$

$$\Rightarrow y_{G.G} = y_h + y_0 = (c_1 + c_2 x + c_3 x^2) e^{-x} + \left(\frac{2}{25}\sin x - \frac{11}{25}\cos x\right) e^x$$

Ör/ $y''' + y'' = 3xe^{x} \rightarrow z(x).e^{\alpha x}$

$$y_{G.C.} = A(x).e^{x}$$

$$y'_{G.C} = A'e^{x} + Ae^{x}$$

$$y''_{G.C.} = A''e^{x} + 2A'e^{x} + Ae^{x}$$

$$y'''_{GC} = A'''e^{x} + 3A''e^{x} + 3A'e^{x} + Ae^{x}$$

$$\Rightarrow A'''e^{x} + 3A''e^{x} + 3A'e^{x} + Ae^{x} + A''e^{x} + 2A'e^{x} + Ae^{x} \equiv 3xe^{x}$$

$$A''' + 4A'' + 5A' + 2A \equiv 3x$$

$$(D^3 + 4D^2 + 5D + 2)A \equiv 3x$$

K.D: $r^3 + 4r^2 + 5r + 2 = 0$

$$r^3 + 4r^2 + 3r + 2r + 2 = 0$$

$$r(r^2 + 4r + 3) + 2(r+1) = 0$$

$$r(r+1)(r+3) + 2(r+1) = 0$$

$$[r+1]\underbrace{(r^2 + 3r + 2)}_{(r+1)(r+2)} = 0$$

$$r_1 = -1, \quad r_2 = -1, \quad r_3 = -2$$

$$A_h = (c_1 + c_2 x)e^{-x} + c_3 e^{-2x}$$

$$\begin{array}{r|rrrr} & 1 & 4 & 5 & 2 \\ -1 & & -1 & -3 & -2 \\ \hline & 1 & 3 & 2 & \boxed{0} \end{array}$$

$$(r^2 + 3r + 2)(r+1)$$

$$A_0 = ax + b$$

$$A'_0 = a$$

$$A''_0 = A'''_0 = 0$$

$$\Rightarrow 5a + 2(ax + b) \equiv 3x$$

$$2ax + 5a + 2b \equiv 3x$$

$2a = 3$ $\quad$ $5a + 2b = 0$

$a = 3/2$ $\quad$ $b = -15/4$

$$\Rightarrow A_{ö} = \frac{3x}{2} - \frac{15}{4} \quad \Rightarrow A_{G.Ç} = (c_1+c_2x)e^{-x} + c_3e^{-2x} + \left(\frac{3x}{2} - \frac{15}{4}\right)$$

$$y_{G.Ç} = A_{G.Ç} \cdot e^{x}$$

$$= \underbrace{(c_1+c_2x) + c_3e^{-x}}_{y_h} + \left(\frac{3x}{2} - \frac{15}{4}\right)e^{x}$$

$e^{\alpha x}$

	α K.D'in kökü değil	α, K.D.'in p katlı kökü
n. derece polinom $\times e^{\alpha x}$	n. derece pol $\cdot e^{\alpha x}$	x^{p}. (n. derece pol) $\cdot e^{\alpha x}$
trigonometrik $\times e^{\alpha x}$	trigonometrik $\cdot e^{\alpha x}$	x^{p}. (trigonometrik) $\cdot e^{\alpha x}$

2. yol

$$y''' + y'' = 3xe^{x}$$

$$(D^3+D^2)y = 3xe^{x}$$

$$K.D: \; r^3 + r^2 = 0$$

$$r^2(r+1) = 0$$

$$r_{1,2} = 0 \qquad r_3 = -1 \qquad \alpha = 1$$

$$y_h = (c_1+c_2x) + c_3e^{-x}$$

$$y_{ö} = (ax+b)e^{x}$$

$$y_{ö}' = ae^{x} + (ax+b)e^{x} = [ax+(a+b)]e^{x}$$

$$y_{ö}'' = ae^{x} + [ax+(a+b)]e^{x} = [ax+(2a+b)]e^{x}$$

$$y_{ö}''' = ae^{x} + [ax+(2a+b)]e^{x} = [ax+(3a+b)]e^{x}$$

$$[ax+(3a+b)]e^{x} + [ax+(2a+b)]e^{x} \equiv 3xe^{x}$$

$$2ax + (5a+2b) \equiv 3x \Rightarrow \quad 2a = 3 \Rightarrow a = \frac{3}{2}, \quad 5a+2b = 0, \quad b = -\frac{15}{4}$$

$$y_{ö} = \left(\frac{3}{2}x - \frac{15}{4}\right)e^{x}$$

$$y_{G.Ç} = y_h + y_ö = (c_1 + c_2 x) + c_3 e^{-x} + \left(\frac{3x}{2} - \frac{15}{4}\right)e^{x}$$

5) $f(x) = \underbrace{M(x)}_{m.der.} \cdot \cos\lambda x + \underbrace{N(x)}_{n.der.} \cdot \sin\lambda x$ şeklinde ise

$(m > n)$

$\mp \lambda i$ k.D'in kökü değilse o zaman $y_ö = (m.\text{der. pol}) \cdot \cos\lambda x + (m.\text{der. pol}) \cdot \sin\lambda x$

$\mp \lambda i$ k.D'in p katlı kökü ise o zaman $y_ö = x^{p} \cdot (m.\text{der. pol}) \cdot \cos\lambda x + x^{p} \cdot (m.\text{der. pol}) \cdot \sin\lambda x$ şeklinde önerilir.

Ör/ $y'' + 2y' - 3y = x\cos x$

$(D^2 + 2D - 3)y = x\cos x$

k.D: $r^2 + 2r - 3 = 0$

$(r+3)(r-1) = 0$

$r_1 = -3 \quad r_2 = 1$

$y_h = c_1 e^{-3x} + c_2 e^{x}$

$y_ö = (ax+b) \cdot \cos x + (cx+d) \cdot \sin x$

$y_ö' = a\cos x - (ax+b)\sin x + c\sin x + (cx+d)\cos x$

$= (a + cx + d)\cos x + (c - ax - b)\sin x$

$y_ö'' = c\cos x - (a + cx + d)\sin x - a\sin x + (c - ax - b)\cos x$

$= (2c - ax - b)\cos x - (2a + cx + d)\sin x$

$$\Rightarrow (2c-ax-b)\cos x - (2a+cx+d)\sin x + 2\left[(2c-ax-b)\cos x - (2a+cx+d)\sin x\right]$$

$$-3\left[(ax+b)\cos x + (cx+d)\sin x\right] \equiv x\cos x$$

$$\Rightarrow \left[(2c-ax-b+4c-2ax-2b-3ax-3b)\cos x + (-2a-\cancel{cx}-d-4a-2cx-2d-3cx-3d)\sin x\right] \equiv x\cos x$$

$$\Rightarrow \left[-6ax-6b+6c\right]\cos x + \left[-6cx-6a-6d\right]\sin x \equiv x\cos x$$

$$\left[-6ax+(-6b+6c)\right] \equiv x \quad , \quad -6cx-(6a+6d) \equiv 0$$

$$-6a = 1 \qquad -6b+6c=0 \qquad -6c=0 \qquad 6a+6d=0$$

$$a = -1/6 \qquad b=0 \qquad c=0 \qquad d=1/6$$

$$\Rightarrow y_{\ddot{o}} = -\frac{x}{6}\cos x + \frac{1}{6}\sin x$$

$$y_{G.Ç} = y_h + y_{\ddot{o}} = c_1 e^{-3x} + c_2 e^{x} - \frac{x}{6}\cos x + \frac{1}{6}\sin x$$