5) Which of the following is the general solution of the differential equation $y''' + 3y'' + y' + 3y = 0$? (c_1, c_2, and c_3 are arbitrary constants.)

a) $y = c_1e^{-x} + c_2\cos x + c_3\sin x$

b) $y = c_1e^{x} + c_2\cos x + c_3\sin x$

c) $y = c_1e^{-3x} + c_2\cos x + c_3\sin x$

d) $y = c_1e^{-3x} + e^{x}\left(c_2\cos x + c_3\sin x\right)$

e) $y = c_1e^{3x} + e^{x}\left(c_2\cos x + c_3\sin x\right)$

6) Which of the following is the solution to the initial-value problem $\begin{cases} y'' - 4y' - 5y = 0 \\ y(0) = 1,\ y'(0) = 5 \end{cases}$ whose general solution is $y = c_1e^{-x} + c_2e^{5x}$ such that c_1 and c_2 are arbitrary constants?

a) $y = e^{-x}$

b) $y = e^{5x}$

c) $y = e^{-x} + e^{5x}$

d) $y = e^{-x} + 5e^{5x}$

e) $y = e^{-x} - 5e^{5x}$

7) If the method of undetermined coefficients is used for the particular solution of the differential equation $y'' + 6y' + 5y = xe^{-x}$, what should it look like? (A and B coefficients to be determined)

a) $y_p = Ae^{-x}$ b) $y_p = Axe^{-x}$ c) $y_p = e^{-x}(Ax + B)$ d) $y_p = xe^{-x}(Ax + B)$ e) $y_p = x^2e^{-x}(Ax + B)$

8) Which of the following linear differential equation arises when solving the differential equation $-6xy' + 3(y')^2 - y = 0$?

a) $\frac{dx}{dp} + \frac{6}{7p}x = \frac{6}{7}$ b) $\frac{dx}{dp} - \frac{6}{7p}x = \frac{6p}{7}$ c) $\frac{dx}{dp} + \frac{6}{5p}x = -\frac{6}{7}$ d) $\frac{dx}{dp} - \frac{6}{5p}x = -\frac{6}{7}$ e) $\frac{dx}{dp} + \frac{7}{6p}x = \frac{6}{7}$

9) Which of the following is the particular solution of the differential equation $y^{(4)} + 2y'' + y = x^2$?

a) $y_p = x^2 - 4$ b) $y_p = x^2 + x - 4$ c) $y_p = x^2 + 2x + 4$ d) $y_p = x^2 + 4$ e) $y_p = x^2 + 2x + 4$

10) Which of the following is the particular solution of the differential equation $y''' + 5y'' + 6y' = 3e^{x}$?

a) $y_p = -\frac{1}{4}e^{x}$ b) $y_p = -4e^{x}$ c) $y_p = e^{x}$ d) $y_p = \frac{1}{4}e^{x}$ e) $y_p = 4e^{x}$

11) Which of the following is true for the differential equation $(\cos x\cos y - \cot x)dx + (\sin x\sin y)dy = 0$?

a) The differential equation is exact.

b) The differential equation is not exact whose integration factor is $\mu(x) = \sin^2 x$.

c) The differential equation is not exact whose integration factor is $\mu(x) = \frac{1}{\sin^2 x}$.

d) The differential equation is not exact whose integration factor is $\mu(y) = \sin^2 y$.

e) The differential equation is not exact whose integration factor is $\mu(y) = \frac{1}{\sin^2 y}$.

12) If a solution of the differential equation $y''' - 3y'' + 4y' - 12y = 0$ is $y_1(x) = e^{3x}$, which of the following is the general solution of the differential equation? (c_1, c_2, and c_3 are arbitrary constants.)

1) Which of the following is the particular solution of the differential equation $y''-2y'-3y=8\sin x$?

a) $y_p=-\sin x+\cos x$

b) $y_p=8\sin x+4\cos x$

c) $y_p=-\frac{8}{5}\cos x$

d) $y_p=\frac{8}{5}\cos x+\frac{4}{5}\sin x$

e) $y_p=-\frac{8}{5}\sin x+\frac{4}{5}\cos x$

$(D^2-2D-3)y=8\sin x$

K.D: $r^2-2r-3=0$

$(r-3)(r+1)=0$

$r_1=3 \quad r_2=-1$

$y_h=c_1e^{3x}+c_2e^{-x}$

$y_0=A\sin x+B\cos x$

$y_0'=A\cos x-B\sin x$

$y_0''=-A\sin x-B\cos x$

$-A\sin x-B\cos x-2(A\cos x-B\sin x)-3(A\sin x+B\cos x)\equiv 8\sin x$

$(-4A+2B)\sin x+(-2A-4B)\cos x\equiv 8\sin x$

$-4A+2B=8$

$\underline{-2A-4B=0}$

$-10A=16$

$A=\frac{-16}{10}$

$=-\frac{8}{5}$

$B=\frac{4}{5}$

2) Since a particular solution of the differential equation $y'=-1+x^2+2xy+y^2$ is given as $y=-x$, which of the following is the general solution of the equation? (c is arbitrary constant.)

a) $y=-x+\frac{2}{c-x}$ b) $y=-x+\frac{1}{c-x}$ c) $y=-x-\frac{1}{c-x}$ d) $y=-x-\frac{x}{c-x}$ e) $y=x+\frac{2}{c-x}$

$\left\{ y=y_1+\frac{1}{u} \right.$

3) Which of the following is the general solution of the differential equation $(2xy+3y^2)dx-(2xy+x^2)dy=0$? ($c$ is arbitrary constant.)

a) $y^2-xy=cx^3$

b) $y^2+xy=cx^3$

c) $x^3+xy=cy^2$

d) $x^3-y^2=cxy$

e) $x^3+y^2=cxy$

$P(\lambda x,\lambda y)=\lambda^{2}P(x,y)$

$Q(\lambda x,\lambda y)=\lambda^{2}Q(x,y)$

$\frac{y}{x}=u$

Homojen dif. denk.

4) Since the differential equation $y'+y^2+(5\tan^2 x)y-25\sec^2 x=0$ has a particular solution $y_1=5$, which of the following is the transformation of this differential equation into Bernoulli's differential equation?

a) $\frac{dz}{dx}+(2+5\tan^2 x)z=-z^2$

b) $\frac{dz}{dx}+(3+5\tan^2 x)z=-z^2$

c) $\frac{dz}{dx}+(10+5\tan^2 x)z=-z^2$

d) $\frac{dz}{dx}+(6+3\tan^2 x)z=-z^2$

e) $\frac{dz}{dx}+(5+10\tan^2 x)z=-z^2$

$1+\tan^2x=\sec^2x$

$\tan^2x=\sec^2x-1$

1°) $y=y_1+u \rightarrow$ Bernoulli

2°) $y=y_1+\frac{1}{u} \rightarrow$ Lineer.

$y=5+u \Rightarrow y'=u'$

$u'+(5+u)^2+(5\tan^2x).(5+u)-25\sec^2x=0$

$u'+25+10u+u^2+25\tan^2x+5u\tan^2x$

$-25\sec^2x=0$

$u'+10u+u^2+\cancel{25(1+\tan^2x)}+5u\tan^2x-\cancel{25\sec^2x}=0$

$u'+5u(2+\tan^2x)=-u^2$

$\boxed{y = c_1 e^{-3x} + c_2 \cos x + c_3 \sin x}$

$(c_2+c_3)\cos x + i(c_2-c_3)\sin x$

$c_2 e^{ix} \quad c_3 e^{-ix}$

$c_2 e^{ix} = (\cos x + i \sin x) c_2$

$c_3 e^{-ix} = (\cos x - i \sin x) c_3$

$c_2 + c_3 = C_2 \quad i(c_2 - c_3) = C_3$

5) Which of the following is the general solution of the differential equation $y''' + 3y'' + y' + 3y = 0$? ($c_1$, c_2, and c_3 are arbitrary constants.)

a) $y = c_1 e^{-x} + c_2 \cos x + c_3 \sin x$

b) $y = c_1 e^{x} + c_2 \cos x + c_3 \sin x$

c) $y = c_1 e^{-3x} + c_2 \cos x + c_3 \sin x$

d) $y = c_1 e^{-3x} + e^{x}(c_2 \cos x + c_3 \sin x)$

e) $y = c_1 e^{3x} + e^{x}(c_2 \cos x + c_3 \sin x)$

$(D^3 + 3D^2 + D + 3)y = 0$

k.D: $r^3 + 3r^2 + r + 3 = 0$

$r^2(r+3) + (r+3) = 0$

$(r+3)(r^2+1) = 0$

$r_1 = -3 \quad r_{2,3} = \mp i$

6) Which of the following is the solution to the initial-value problem

$$\begin{cases} y'' - 4y' - 5y = 0 \\ y(0) = 1,\ y'(0) = 5 \end{cases}$$

whose general solution is $y = c_1 e^{-x} + c_2 e^{5x}$ such that c_1 and c_2 are arbitrary constants?

a) $y = e^{-x}$

b) $y = e^{5x}$

c) $y = e^{-x} + e^{5x}$

d) $y = e^{-x} + 5e^{5x}$

e) $y = e^{-x} - 5e^{5x}$

$y = c_1 e^{-x} + c_2 e^{5x} \quad y' = -c_1 e^{-x} + 5c_2 e^{5x}$

$y(0) = 1 \Rightarrow c_1 \cdot e^0 + c_2 \cdot e^0 = 1 \Rightarrow c_1 + c_2 = 1$

$y'(0) = 5 \Rightarrow -c_1 \cdot e^0 + 5c_2 \cdot e^0 = 5 \Rightarrow -c_1 + 5c_2 = 5$

$6c_2 = 6$

$c_2 = 1$

$c_1 = 0$

7) If the method of undetermined coefficients is used for the particular solution of the differential equation $y'' + 6y' + 5y = xe^{-x}$, what should it look like? (A and B coefficients to be determined)

a) $y_p = Ae^{-x}$ b) $y_p = Axe^{-x}$ c) $y_p = e^{-x}(Ax+B)$ d) $y_p = xe^{-x}(Ax+B)$ e) $y_p = x^2 e^{-x}(Ax+B)$

$r^2 + 6r + 5 = 0$

$(r+5)(r+1) = 0$

$r_1 = -5$

$r_2 = -1$

$y_p = x.(ax+b).e^{-x}$

8) Which of the following linear differential equation arises when solving the differential equation $-6xy' + 3(y')^2 - y = 0$?

a) $\frac{dx}{dp} + \frac{6}{7p}x = \frac{6}{7}$ b) $\frac{dx}{dp} - \frac{6}{7p}x = \frac{6p}{7}$ c) $\frac{dx}{dp} + \frac{6}{5p}x = -\frac{6}{7}$ d) $\frac{dx}{dp} - \frac{6}{5p}x = -\frac{6}{7}$ e) $\frac{dx}{dp} + \frac{7}{6p}x = \frac{6}{7}$

Lagrange $y = x\varphi(y') + \psi(y')$

$y = xy' + \varphi(y')$ Clairaut

$y' = p$

$y = -6xy' + 3y'^2$

$y = -6xp + 3p^2$

$y' = -6p - 6xp' + 6pp'$

$p = -6p - 6xp' + 6pp'$

$7p = p'[-6x + 6p]$

$p' = \frac{7p}{-6x+6p}$

$\frac{dx}{dp} = \frac{-6x+6p}{7p}$

9) Which of the following is the particular solution of the differential equation $y^{(4)} + 2y'' + y = x^2$?

a) $y_p = x^2 - 4$ b) $y_p = x^2 + x - 4$ c) $y_p = x^2 + 2x + 4$ d) $y_p = x^2 + 4$ e) $y_p = x^2 + 2x + 4$

10) Which of the following is the particular solution of the differential equation $y''' + 5y'' + 6y' = 3e^x$?

a) $y_p = -\frac{1}{4}e^x$ b) $y_p = -4e^x$ c) $y_p = e^x$ d) $y_p = \frac{1}{4}e^x$ e) $y_p = 4e^x$

$\alpha = 1$

$(D^3 + 5D^2 + 6D)y = 3e^x$

k.D: $r^3 + 5r^2 + 6r = 0$

$r(r^2 + 5r + 6) = 0$

$r_1 = 0 \quad r_2 = -2 \quad r_3 = -3$

$y_p = Ae^x$

$Ae^x + 5Ae^x + 6Ae^x = 3e^x$

$12A = 3$

$A = \frac{1}{4}$

11) Which of the following is true for the differential equation $(\cos x \cos y - \cot x)dx + (\sin x \sin y)dy = 0$?

a) ~~The differential equation is exact.~~

b) The differential equation is not exact whose integration factor is $\mu(x) = \sin^2 x$.

c) The differential equation is not exact whose integration factor is $\mu(x) = \frac{1}{\sin^2 x}$.

d) The differential equation is not exact whose integration factor is $\mu(y) = \sin^2 y$.

e) The differential equation is not exact whose integration factor is $\mu(y) = \frac{1}{\sin^2 y}$.

$P \quad Q$

$\frac{\partial P}{\partial y} = -\cos x \sin y$

$\frac{\partial Q}{\partial x} = \cos x \sin y$

$\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} = -2\cos x \sin y$

$\lambda = e^{\int \frac{-2\cos x \sin y}{\sin x \sin y}dx} = 1/\sin^2 x$

12) If a solution of the differential equation $y''' - 3y'' + 4y' - 12y = 0$ is $y_1(x) = e^{3x}$, which of the following is the general solution of the differential equation? (c_1, c_2, and c_3 are arbitrary constants.)

$\frac{dx}{dp} = -\frac{6}{7}\frac{x}{p} + \frac{6}{7} \Rightarrow \frac{dx}{dp} + \frac{6}{7}\cdot\frac{x}{p} = \frac{6}{7}$

9) $y^{(4)} + 2y'' + y = x^2$

$(D^4 + 2D^2 + 1)y = x^2$

k.D: $r^4 + 2r^2 + 1 = 0$

$r_{1,2,3,4} \neq 0 \Rightarrow y_p = ax^2 + bx + c$

$y_p' = 2ax + b \quad y_p'' = 2a$

$y_0''' = y_0^{(4)} = 0 \Rightarrow 4a + ax^2+bx+c \equiv x^2 \Rightarrow ax^2+bx+(4a+c) = x^2$

$a=1 \quad b=0 \quad 4a+c=0$

$c=-4$

$\boxed{y_0 = x^2-4}$

$(D^3-3D^2+4D-12)y=0$

K.D: $r^3-3r^2+4r-12=0$

$r^2(r-3)+4(r-3)=0$

$(r^2+4)(r-3)=0 \Rightarrow r_{2,3}=\mp 2i \quad r_1=3$

$y=c_1e^{3x}+c_2\cos 2x+c_3\sin 2x$

a) $y=c_1e^{3x}+(c_2+c_3x)e^{x}$
b) $y=c_1e^{3x}+(c_2+c_3x)e^{2x}$
c) $y=c_1e^{x}+c_2e^{2x}+c_3e^{3x}$
d) $y=c_1e^{3x}+c_2\cos x+c_3\sin x$
e) $y=c_1e^{3x}+c_2\cos 2x+c_3\sin 2x$

13) Which of the following is a homogeneous differential equation with constant coefficients whose characteristic equation is $(r-1)^3 r=0$?

$(r^3-3r^2+3r+1)r = r^4-3r^3+3r^2+r$

$\Rightarrow y^{(4)}-3y'''+3y''+y'=$

a) $y^{(4)}-3y'''-3y'=1$
b) $y^{(4)}-3y'''-3y''=1$
c) $y^{(4)}-3y'''+3y''=1$
d) $y^{(4)}-3y'''+3y''-y'=0$
e) $y^{(4)}-3y'''+3y''+y'=0$

14) What must r be for the differential equation $y=e^{rx}$, $y'''-y''-9y'+9y=0$ to has a solution?

a) -3 b) -2 c) -1 d) 0 e) 2

$D^3-D^2-9D+9=0$

$D^2(D-1)-9(D-1)=0$

$(D-1)(D^2-9)=0$

$r_1=1 \quad r_{2,3}=\mp 3$

15) Which of the following is true for the differential equation $y''=\sqrt[4]{y+(y')^2}$?

	Order	Degree	Linear
a)	1	2	No
b)	2	1	No
~~c)~~	2	2	Yes
d)	2	3	No
e)	2	4	No

$y''=\sqrt[4]{y+y'^2}$

$y''^4=y+y'^2$

16) Which of the following is the general solution of the differential equation $yy'=(y')^2x+\frac{1}{y'}$? (c is arbitrary constant.)

a) $y=c^2x+\frac{1}{c}$ b) $y=c^2x+c$ c) $y=cx+\frac{1}{c}$ d) $y=cx+\frac{1}{c^2}$ e) $y=c^2x+1$

$y'=p$

$y=xy'+\frac{1}{y'^2}$

$y=xp+\frac{1}{p^2}$

$y'=p+xp'-\frac{2p'}{p^3}$

$\cancel{p}=\cancel{p}+xp'-\frac{2p'}{p^3}$

$p'\left[x-\frac{2}{p^3}\right]=0$

$p'=0 \Rightarrow p=c$

$\Downarrow$

$y=xc+\frac{1}{c^2}$

17) Which of the following is the general solution of the differential equation $y'+(\cos x)y=\cos x$? (c is arbitrary constant.)

Lineer.

a) $y(x)=e^{-\sin x}+c$
b) $y(x)=e^{-\sin x}+ce^{\sin x}$
c) $y(x)=\sin x+ce^{-\sin x}$
d) $y(x)=1+ce^{-\sin x}$
e) $y(x)=e^{-\sin x}+c$

18) Which of the following is the lowest order differential equation whose solution is $y=c_1e^{x+c_2}$? (c_1, c_2 are arbitrary constants.)

a) $y'-y=0$

$y=c_1e^{x+c_2}$

$y=c_1\cdot e^x\cdot e^{c_2}$

$c=c_1e^{c_2}$

$\boxed{y=c\cdot e^x}$

$\frac{dy}{dx}-y=0$

$\int\frac{dy}{y}-\int dx=\int 0$

$\ln y-x=\ln c \Rightarrow y=c\cdot e^x$

b) $y' + y = 0$

~~c)~~ $y'' - y = 0$

~~d)~~ $y'' + y = 0$

~~e)~~ $y'' - y = 2$

19) Which of the following is the general solution of the differential equation $\boxed{\frac{dy}{dx} + xy = xe^{-x^2}y^3}$? (c is arbitrary constant.)

→ Bernoulli

a) $\frac{1}{y^2} = \frac{1}{2}e^{-x^2} + ce^{x^2}$ b) $\frac{1}{y^2} = e^{-x^2} + ce^{x^2}$ c) $\frac{1}{y^2} = \frac{1}{2}e^{x^2} + ce^{-x^2}$ d) $\frac{1}{y^2} = e^{x^2} + ce^{-x^2}$ e) $\frac{1}{y^2} = 2e^{-x^2} + ce^{-x^2}$

20) The function $y^2 = 2e^x + 3$ is a solution of which of the following differential equations?

a) $y' + y^2 = 0$

b) $2yy' - y^2 + 3 = 0$

c) $2yy' + y^2 + 3 = 0$

d) $2yy' - y + 3 = 0$

e) $2yy' + y + 3 = 0$

$$\frac{y'}{y^3} + \frac{x}{y^2} = xe^{-x^2}$$

$$\frac{1}{y^2} = z$$

$$-\frac{2y'}{y^3} = z'$$

$$\frac{y'}{y^3} = \frac{-z'}{2}$$

$$-\frac{z'}{2} + xz = xe^{-x^2}$$

$$z' - 2xz = -2xe^{-x^2} \quad \text{L.D.D}$$

$$\frac{dz}{dx} - 2xz = -2xe^{-x^2}$$

$p(x) = -2x$

$\lambda = e^{\int p(x)dx}$

$= e^{-x^2}$

$$e^{-x^2}\frac{dz}{dx} - 2xe^{-x^2}z = -2xe^{-2x^2}$$

$$\underbrace{e^{-x^2}dz - 2xze^{-x^2}dx}_{} = -2xe^{-2x^2}dx$$

$-2x^2 = u$

$-4x\,dx = du$

$-2x\,dx = \frac{du}{2}$

$$\int d(e^{-x^2}z) = \int -2xe^{-2x^2}dx$$

$$z \cdot e^{-x^2} = \int e^u \cdot \frac{du}{2}$$

$$ze^{-x^2} = \frac{1}{2}e^u + c$$

$$ze^{-x^2} = \frac{1}{2}e^{-2x^2} + c$$

$$z = \frac{1}{2}e^{-x^2} + ce^{x^2}$$

$$\frac{1}{y^2} = \frac{e^{-x^2}}{2} + ce^{x^2}$$

$y^2 = 2e^x + 3$

$\cancel{2}yy' = \cancel{2}e^x$

$yy' = e^x$

$y^2 = 2yy' + 3$

$2yy' - y^2 + 3 = 0$

1) In solution of differantial equation

$$y''' - 3y'' + 3y' - y = \frac{1}{x}\sin 2x$$

by the method of variation parameters, the general solution is found by using which equation below?

$(D^3 - 3D^2 + 3D - 1)y = 0$

k.D: $r^3 - 3r^2 + 3r - 1 = 0$

$(r-1)^3 = 0$

$r_{1,2,3} = 1$

A) $y = c_1(x)e^{3x} + c_2(x)xe^{3x} + c_3(x)x^2e^{3x}$

B) $y = c_1(x)e^{x} + c_2(x)xe^{x} + c_3(x)x^2e^{x}$

C) $y = c_1(x)e^{-x} + c_2(x)e^{2x} + c_3(x)e^{3x}$

D) $y = c_1(x)e^{x} + c_2(x)e^{-x} + c_3(x)xe^{x}$

E) $y = c_1(x)e^{-x} + c_2(x)xe^{-x} + c_3(x)x^2e^{-x}$

$y = c_1e^x + c_2xe^x + c_3x^2e^x$

$y = c_1(x)e^x + c_2(x).xe^x + c_3(x).x^2e^x$

2) Which of the following is a separable differential equation obtained by applying the appropriate transformation to the differential equation $\frac{dy}{dx} = \frac{(x+y)^2}{2x^2}$?

Homojen dif. denk

$\frac{y}{x} = u \Rightarrow y = ux$

$y' = u'x + u$

A) $(1+u^2)dx - 2x^2du = 0$

B) $(1+u+u^2)dx - 2xdu = 0$

C) $(1+u)dx + 2xdu = 0$

D) $(1+u^2)du - 2xdx = 0$

E) $(1+u^2)dx - 2xdu = 0$

$u'x + u = \frac{(x+ux)^2}{2x^2}$

$u'x + u = \frac{(1+u)^2}{2} \Rightarrow 2u'x + \cancel{2u} = u^2 + \cancel{2u} + 1$

$2u'x = u^2 + 1$

$2\frac{du}{dx}.x = u^2 + 1$

$(u^2+1)\,dx - 2x\,du = 0$

3) Which of the following is a linear differential equation with constant coefficients formed using the appropriate transformation of the differential equation

$$y''x + 3y' = \frac{\ln x}{x^2} - \frac{1}{x}y$$

$x^2y'' + 3xy' = \frac{\ln x}{x} - y \Rightarrow x^2y'' + 3xy' + y = \frac{\ln x}{x}$

Cauchy-Euler

A) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = te^t$

$t = \ln x \Leftarrow x = e^t,\ y' = e^{-t}Dy$

B) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-t}$

$y'' = e^{-2t}D(D-1)y$

C) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = \ln t$

$\underbrace{e^{2t}.e^{-2t}}_{1}D(D-1)y + 3\underbrace{e^{t}.e^{-t}}_{1}Dy + y = \frac{t}{t.e}$

D) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = 8\ln t$

$(D^2 - D + 3D + 1)y = te^{-t}$

E) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} = 8t$

$(D^2 + 2D + 1)y = te^{-t}$

4) What is $c_2(x)$ if the general solution of the $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ differential equation is $y = [c_1(x) + c_2(x)x]e^{3x}$?

→ sabitlerin değişimleri yöntemiyle çözülecek

A) $c_2 = xe^{3x} + K_2$

B) $c_2 = e^{3x} + K_2$

C) $c_2 = \frac{1}{x^2} + K_2$

D) $c_2 = -\frac{1}{x} + K_2$

E) $c_2 = \ln x + K_2$

$y = c_1(x)e^{3x} + c_2(x)xe^{3x}$

$3/\ c_1'e^{3x} + c_2'xe^{3x} = 0$ (keyfi)

$- \quad 3c_1'e^{3x} + c_2'e^{3x} + 3c_2'xe^{3x} = \frac{e^{3x}}{x^2}$

$c_2'e^{3x} = \frac{e^{3x}}{x^2}$

$c_2' = \frac{1}{x^2}$

$c_2 = \int \frac{dx}{x^2} + k_2$

$c_2 = -\frac{1}{x} + k_2$

$a_0y^{(n)} + a_1y^{(n-1)} + \dots + a_{n-1}y' + a_ny = f(x)$

$y_h = c_1y_1 + c_2y_2 + \dots + c_ny_n$

$c_1 = c_1(x),\ c_2 = c_2(x), \dots, c_n = c_n(x)$

$y = c_1(x)y_1 + c_2(x)y_2 + c_n(x)y_n$

$c_1'y_1 + c_2'y_2 + \dots + c_n'y_n = 0$ (keyfi) 1

$c_1'y_1' + c_2'y_2' + \dots + c_n'y_n' = 0$ (keyfi) 2

$\vdots$

$c_1'y_1^{(n-2)} + c_2'y_2^{(n-2)} + \dots + c_n'y_n^{(n-2)} = 0$ (keyfi) (n-1)

$c_1'y_1^{(n-1)} + c_2'y_2^{(n-1)} + \dots + c_n'y_n^{(n-1)} = \frac{f(x)}{a_0}$ n.

$x = e^t \Rightarrow t = \ln x$

$y' = e^{-t} Dy$

$y'' = e^{-2t} D(D-1)y$

5) What is the general solution of the differential equation $y'' - \frac{3}{x}y' + \frac{4}{x^2}y = x$?

A) $y = c_1 x + c_2 \ln x + \frac{1}{4}x^2$

B) $y = c_1 + c_2 x^2 - x$

C) $y = c_1 x^2 + c_2 x^2 \ln x + x^3$

D) $y = c_1 + c_2 x^2 + e^x$

E) $y = c_1 + c_2 x^2 + x$

$x^2 y'' - 3xy' + 4y = x^3$

$e^{2t} \cdot e^{-2t} D(D-1)y - 3e^t e^{-t} Dy + 4y = e^{3t}$

$(D^2 - D - 3D + 4)y = e^{3t}$

$(D^2 - 4D + 4)y = e^{3t}$

k.D: $r^2 - 4r + 4 = 0$

$(r-2)^2 = 0$

$r_{1,2} = 2$

$y_h = c_1 e^{2t} + c_2 t e^{2t}$

$y_ö = A \cdot e^{3t}$

$y_ö' = 3Ae^{3t}$

$y_ö'' = 9Ae^{3t}$

$9Ae^{3t} - 12Ae^{3t} + 4Ae^{3t} = e^{3t}$

$Ae^{3t} = e^{3t}$

$A = 1$

$y_ö = e^{3t}$

$y_{G.Ç} = y_h + y_ö$

$y_{G.Ç} = c_1 e^{2t} + c_2 t e^{2t} + e^{3t}$

$= c_1 x^2 + c_2 x^2 \ln x + x^3$

6) Which of the following is the derivative of the solution function of the differential equation $\frac{d^2y}{dx^2} = \frac{1}{y}\left[\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^2\right]$?

A) $\frac{dy}{dx} = ce^{\frac{y^2}{2}} - 1$

B) $\frac{dy}{dx} = ce^{y^{2x}} - 1$

C) $\frac{dy}{dx} = \frac{1 - cy}{cy}$

D) $\frac{dy}{dx} = ce^{\frac{x^2}{2}} - 1$

E) None

$y'' = \frac{1}{y}(y' + y'^2)$

$\Rightarrow yy'' = y' + y'^2$

x'in bulunmadığı

$y' = p \quad y'' = p\frac{dp}{dy}$

$yp\frac{dp}{dy} = p + p^2$

$p\left[y\frac{dp}{dy} - 1 - p\right] = 0$

$1°)\ p = 0 \Rightarrow y' = 0 \Rightarrow y = c$

$2°)\ y\frac{dp}{dy} - 1 - p = 0$

$\int \frac{dp}{1+p} - \int \frac{dy}{y} = \int 0$

$\ln(1+p) - \ln y = \ln c$

$1 + p = c_1 \cdot y$

$p = c_1 y - 1$

$y' = c_1 y - 1$

7) What is the constant **b** if the differantial equation $3ye^{2xy}dx - 4bxe^{2xy}dy = xdx$ is the exact differantial equation?

A) $b = -\frac{3}{2}$

B) $b = -\frac{2}{3}$

C) $b = -\frac{3}{4}$

D) $b = \frac{4}{3}$

E) None

$\underbrace{(3ye^{2xy} - x)}_{\frac{\partial}{\partial y}}dx - \underbrace{4bxe^{2xy}}_{\frac{\partial}{\partial x}}dy = 0$

$\frac{\partial}{\partial y}(3ye^{2xy} - x) = 3e^{2xy} + 6xye^{2xy}$

$\frac{\partial}{\partial x}(-4bxe^{2xy}) = -4be^{2xy} - 8bxye^{2xy}$

$-4b = 3$

$b = \frac{-3}{4}$

8) Which of the following is the solution of differantial equation $(y')^2 - yy'' = 0$ that satisfies the conditions $y(0) = y'(0) = 1$?

A) $y = \frac{c_1}{x} + c_2$

B) $y = \frac{1}{x} + 1$

C) $y = e^x$

D) $y = \frac{e^{x^2}}{x} + \ln x$

E) $y = \frac{\ln x}{x}$

$y' = p \quad y'' = p\frac{dp}{dy}$

$p^2 - yp\frac{dp}{dy} = 0$

$p\left[p - y\frac{dp}{dy}\right] = 0$

$1°)\ p = 0 \Rightarrow y' = 0 \Rightarrow y = c$

$2°)\ p - y\frac{dp}{dy} = 0 \Rightarrow \frac{dy}{y} - \frac{dp}{p} = 0$

$\ln y - \ln p = \ln c_1$

$p = \frac{y}{c_1}$

$y' = \frac{y}{c_1}$

$\frac{dy}{dx} = \frac{y}{c_1}$

$\int \frac{dy}{y} = \int \frac{dx}{c_1}$

$\ln y = \frac{x}{c_1} + \ln c_2$

$y = c_2 \cdot e^{\frac{x}{c_1}}$

$y' = \frac{c_2}{c_1}e^{\frac{x}{c_1}}$

$y(0) = 1$

$c_2 \cdot e^0 = 1$

$c_2 = 1$

$y'(0) = 1 \Rightarrow$

$\frac{1}{c_1} \cdot e^0 = 1$

$c_1 = 1$

$y = e^x$

9) Which of the following is the Cauchy-Euler differential equation whose general solution is $y = c_1\cos(\ln(x)) + c_2\sin(\ln(x))$?

A) $x^2\frac{d^2y}{dx^2} + y = 0$

B) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = 0$

C) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

D) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

E) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x = e^t \Rightarrow t = \ln x$

$x^2y'' + xy' + y$

$D(D-1)y + Dy + y$

$(D^2 + 1)y = 0$

$r = \mp i$

$y = c_1\cos t + c_2\sin t$

10) What is the value of the Wronskian determinant (W) of the linearly independent solutions of the differential equation $y'' + 4y = \csc 2x$?

A) 2

B) 4

C) $\cos 2x$

D) $\cos 2x \sin 2x$

E) $\sin 2x$

$(D^2 + 4)y = \csc 2x$

$r^2 + 4 = 0$

$r_{1,2} = \mp 2i$

$y_h = c_1\cos 2x + c_2\sin 2x$

$y_1 = \cos 2x$

$y_2 = \sin 2x$

$W(y_1, y_2) = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2$