$$y' = P(x)y^2 + Q(x)y + R(x)$$

$$\left.\begin{aligned} y &= y_1 + \frac{1}{u} \\ y' &= y_1' - \frac{u'}{u^2} \end{aligned}\right\} \quad y_1' - \frac{u'}{u^2} = P(x)\left[y_1 + \frac{1}{u}\right]^2 + Q(x)\cdot\left[y_1 + \frac{1}{u}\right] + R(x)$$

$$y_1' - \frac{u'}{u^2} = P(x)\left[y_1^2 + \frac{2y_1}{u} + \frac{1}{u^2}\right] + Q(x)y_1 + \frac{1}{u}Q(x) + R(x)$$

$$\cancel{y_1'} - \frac{u'}{u^2} = \underbrace{P(x)y_1^2 \cancel{+ Q(x)y_1} + R(x)}_{y_1'} + \frac{1}{u}\left[2y_1P(x) + Q(x)\right] + \frac{1}{u^2}P(x)$$

$$-\frac{u'}{u^2} = \frac{1}{u}\left[2y_1P(x) + Q(x)\right] + \frac{1}{u^2}P(x)$$

$$u' = -u\left[2y_1P(x) + Q(x)\right] - P(x)$$

$$\Rightarrow u' + u\left[2y_1 P(x) + Q(x)\right] = -P(x) \qquad \text{L.D.D.}$$

$$\boxed{u = \varphi(x, c)}$$

L.D.D'in genel çözümü bu şekilde elde edilir. O halde,

$$\boxed{y = y_1 + \frac{1}{\varphi(x,c)}}$$

Riccati dif. denkleminin genel çözümü olacaktır.

Ör/ $y' - \frac{y}{x} + 9x^2 = y^2$ dif. denkleminin bir özel çözümü $y_1 = 3x$ olarak verildiğine göre denklemin genel çözümünü bulunuz.

$$y' = y^2 + \frac{y}{x} - 9x^2$$

$$y = y_1 + \frac{1}{u} = 3x + \frac{1}{u} \Rightarrow y' = 3 - \frac{u'}{u^2} \Rightarrow 3 - \frac{u'}{u^2} = \left(3x + \frac{1}{u}\right)^2 + \frac{1}{x}\left(3x + \frac{1}{u}\right) - 9x^2$$

$$3-\frac{u'}{u^2}=9x^2+6x\cdot\frac{1}{u}+\frac{1}{u^2}+\frac{1}{x}\left(3x+\frac{1}{u}\right)-9x^2$$

$$\cancel{3}-\frac{u'}{u^2}=\cancel{9x^2}+\frac{6x}{u}+\frac{1}{u^2}+\cancel{3}+\frac{1}{ux}-\cancel{9x^2}$$

$$-\frac{u'}{u^2}=\frac{1}{u}\left(6x+\frac{1}{x}\right)+\frac{1}{u^2}\Rightarrow u'=-u\left(6x+\frac{1}{x}\right)-1\Rightarrow \boxed{u'+u\left(6x+\frac{1}{x}\right)=-1}$$ L.D.D

$$p(x)=6x+\frac{1}{x}\Rightarrow \lambda=e^{\int p(x)dx}=e^{\int(6x+\frac{1}{x})dx}=e^{3x^2+\ln x}=x.e^{3x^2}$$

$$x.e^{3x^2}u'+x.e^{3x^2}u\left(6x+\frac{1}{x}\right)=-x.e^{3x^2}$$

$$d(u.x.e^{3x^2})=x.e^{3x^2}du+u.e^{3x^2}dx+6x^2ue^{3x^2}dx$$

$$\underbrace{x.e^{3x^2}du+x.e^{3x^2}u.\left(6x+\frac{1}{x}\right)dx}_{\int d(u.x.e^{3x^2})}=-x.e^{3x^2}dx$$

$$\int d(u.x.e^{3x^2})=\int -x.e^{3x^2}dx\Rightarrow u.x.e^{3x^2}=-\int\frac{1}{6}e^t+k\Rightarrow u.x.e^{3x^2}=-\frac{e^t}{6}+k$$

$$3x^2=t\Rightarrow 6x\,dx=dt$$
$$x\,dx=\frac{dt}{6}$$

$$\Rightarrow u.x.e^{3x^2}=-\frac{e^{3x^2}}{6}+k$$

$$\Rightarrow \boxed{u = -\frac{1}{6x} + \frac{k}{x.e^{3x^2}}}$$ Lineer dif. denk.'in genel çözümü

$$y = y_1 + \frac{1}{u} \Rightarrow \boxed{y = 3x + \frac{1}{-\frac{1}{6x} + \frac{k}{x.e^{3x^2}}}}$$ Riccati dif. denk'nin genel çözümü

Ör/ $y' + y^2 - 1 = 0$ dif. denkleminin bir özel çözümü $y_1 = 1$ olduğuna göre denklemin genel çözümünü bulunuz.

$y' = -y^2 + 1$ Riccati dif. denk.

$$y = y_1 + \frac{1}{u} = 1 + \frac{1}{u} \Rightarrow y' = -\frac{u'}{u^2} \Rightarrow -\frac{u'}{u^2} = -\left[1 + \frac{1}{u}\right]^2 + 1 \Rightarrow -\frac{u'}{u^2} = -\left[1 + \frac{2}{u} + \frac{1}{u^2}\right] + 1$$

$$\Rightarrow -\frac{u'}{u^2} = \frac{-2}{u} - \frac{1}{u^2}$$

$\Rightarrow u' = 2u + 1 \Rightarrow \boxed{u' - 2u = 1}$ L.D.D.

$u' - 2u = 0 \Rightarrow \frac{du}{u} - 2\,dx = 0$

$\ln u - 2x = \ln c$

$\Rightarrow u = c.e^{2x}$

$c = c(x) \Rightarrow \left.\begin{array}{l} u = c(x).e^{2x} \\ u' = c'e^{2x} + 2ce^{2x} \end{array}\right\} \Rightarrow c'e^{2x} + \cancel{2ce^{2x}} - \cancel{2ce^{2x}} = 1$

$c' = e^{-2x} \Rightarrow c = -\frac{1}{2}e^{-2x} + k \Rightarrow \boxed{u = -\frac{1}{2} + ke^{2x}}$

Lineer dif. denk.in genel çöz.

$y = y_1 + \frac{1}{u} \Rightarrow \boxed{y = 1 + \frac{1}{-\frac{1}{2} + ke^{2x}}}$

Riccati dif. denk. G.Ç.

$y = 1 + \frac{2}{2ke^{2x} - 1} = \frac{2ke^{2x} + 1}{2ke^{2x} - 1}$

Ör/ $x^3 y' = x^2 y + y^2 - x^2$ dif. denkleminin genel çözümünü bulunuz.

$$y' = \frac{y^2}{x^3} + \frac{y}{x} - \frac{1}{x} \quad \text{Riccati dif. denk.}$$

$$\boxed{y_1 = x} \Rightarrow y_1' = 1 \qquad \Rightarrow 1 = \cancel{\frac{x^2}{x^3}} + \frac{x}{x} - \cancel{\frac{1}{x}} \Rightarrow 1 = 1$$

özel çözümü

$$\left.\begin{array}{l} y = y_1 + \frac{1}{u} \Rightarrow y = x + \frac{1}{u} \\ \qquad\qquad y' = 1 - \frac{u'}{u^2} \end{array}\right\} \Rightarrow 1 - \frac{u'}{u^2} = \frac{1}{x^3}\left[x + \frac{1}{u}\right]^2 + \frac{1}{x}\cdot\left[x + \frac{1}{u}\right] - \frac{1}{x}$$

$$\cancel{1} - \frac{u'}{u^2} = \frac{1}{x^3}\left[x^2 + \frac{2x}{u} + \frac{1}{u^2}\right] \cancel{+1} + \frac{1}{ux} - \frac{1}{x}$$

$$\Rightarrow \frac{-u'}{u^2} = \cancel{\frac{1}{x}} + \frac{2}{ux^2} + \frac{1}{u^2x^3} + \frac{1}{ux} - \cancel{\frac{1}{x}}$$

$$u' = -u\left[\frac{2}{x^2} + \frac{1}{x}\right] - \frac{1}{x^3} \Rightarrow u' + u\left[\frac{2}{x^2} + \frac{1}{x}\right] = \frac{-1}{x^3} \quad \text{L.D.D.}$$

$$p(x) = \frac{2}{x^2} + \frac{1}{x} \Rightarrow \lambda = e^{\int\left(\frac{2}{x^2} + \frac{1}{x}\right)dx} = e^{-\frac{2}{x} + \ln x} = x \cdot e^{-\frac{2}{x}}$$

$$\Rightarrow x \cdot e^{-\frac{2}{x}}\left[u' + u \cdot \left(\frac{2}{x^2} + \frac{1}{x}\right)\right] = -\frac{e^{-\frac{2}{x}}}{x^2}$$

$$\Rightarrow \underbrace{u \cdot x \cdot e^{-\frac{2}{x}} du + u \cdot \left(\frac{2}{x^2} + \frac{1}{x}\right) \cdot x e^{-\frac{2}{x}} dx}_{\int d\left(u \cdot x \cdot e^{-\frac{2}{x}}\right)} = -\frac{1}{x^2} e^{-\frac{2}{x}} dx$$

$$\int d\left(u \cdot x \cdot e^{-\frac{2}{x}}\right) = \int -\frac{1}{x^2} e^{-\frac{2}{x}} dx$$

$$-\frac{2}{x} = t$$

$$\frac{2}{x^2} dx = dt \Rightarrow \frac{dx}{x^2} = \frac{dt}{2}$$

$$\Rightarrow u \cdot x \cdot e^{-\frac{2}{x}} = -\int e^t \cdot \frac{dt}{2} + k$$

$$\Rightarrow u \cdot x \cdot e^{-\frac{2}{x}} = \frac{-1}{2} e^t + k$$

$$\Rightarrow u \cdot x \cdot e^{-\frac{2}{x}} = \frac{-1}{2} e^{-\frac{2}{x}} + k$$

$$\boxed{u = \frac{-1}{2x} + \frac{k}{x} e^{\frac{2}{x}}}$$

$$y = y_1 + \frac{1}{u} \Rightarrow y = x + \frac{1}{-\frac{1}{2x} + \frac{k}{x} e^{-2/x}} = x + \frac{2x}{2k e^{-\frac{2}{x}} - 1}$$

$$\Rightarrow \boxed{y = \frac{2kx e^{-\frac{2}{x}} + x}{2k e^{-\frac{2}{x}} - 1}}$$

Riccati dif. denk.inin
genel çözümü