THERMODYNAMICS 2

Problem Session – I

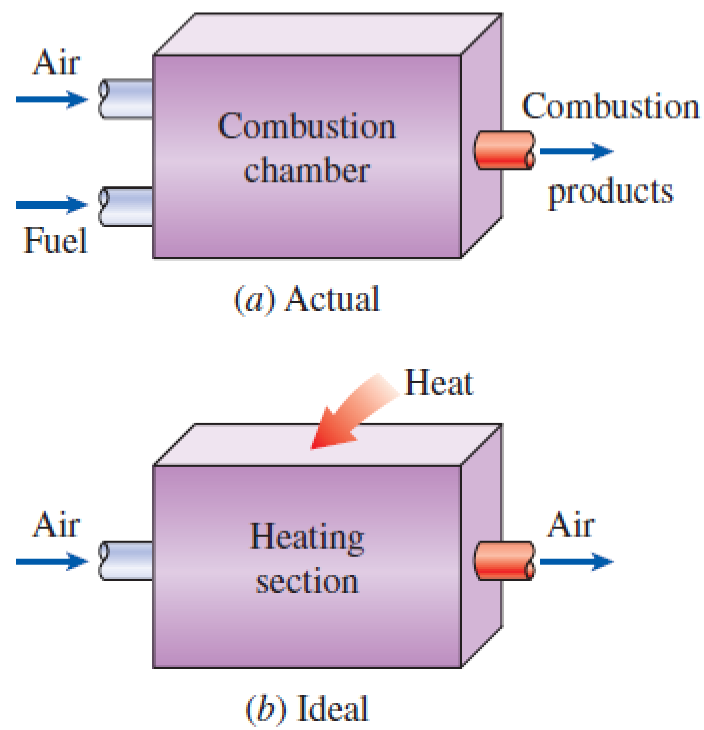
Gas Power Cycles

Air-standard assumptions:

1. The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.
2. All the processes that make up the cycle are internally reversible.
3. The combustion process is replaced by a heat-addition process from an external source.
4. The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.

Cold-air-standard assumptions: When the working fluid is considered to be air with constant specific heats at room temperature (25°C).

Air-standard cycle: A cycle for which the air-standard assumptions are applicable.



Isentropic (*s* = *constant*) Relations for Ideal Gas:

For Otto & Diesel;

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^{k}$$

For Brayton;

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

where *k* is always a **constant**!

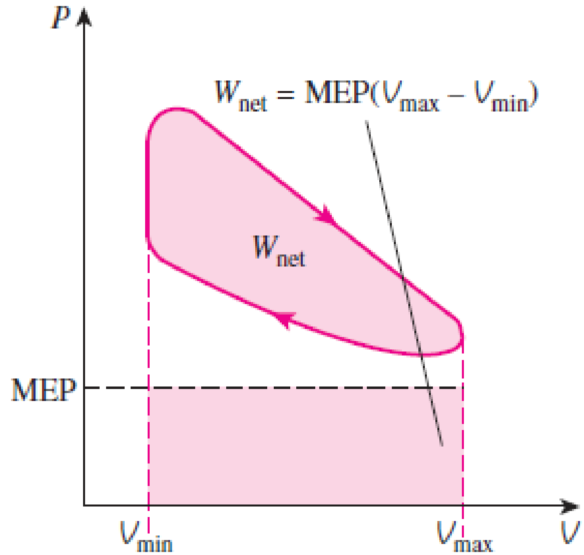
Change in Entropy for Ideal Gas:

*with **constant** specific heats:*

$$s_2 - s_1 = c_{v,ave} \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \quad \textbf{or} \quad s_2 - s_1 = c_{p,ave} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

*with **variable** specific heats: Everything (u, h, v, s°, etc.) is from Tables (Table A – 17 for air)*

$$s_2 - s_1 = s_2^0 - s_1^0 - R\ln\frac{P_2}{P_1} \quad \text{where } s_1^0 \text{ and } s_2^0 \text{ are from the Tables}$$



$$\text{MEP} = \frac{W_{net}}{V_{max} - V_{min}} = \frac{w_{net}}{v_{max} - v_{min}} \qquad (\text{kPa})$$

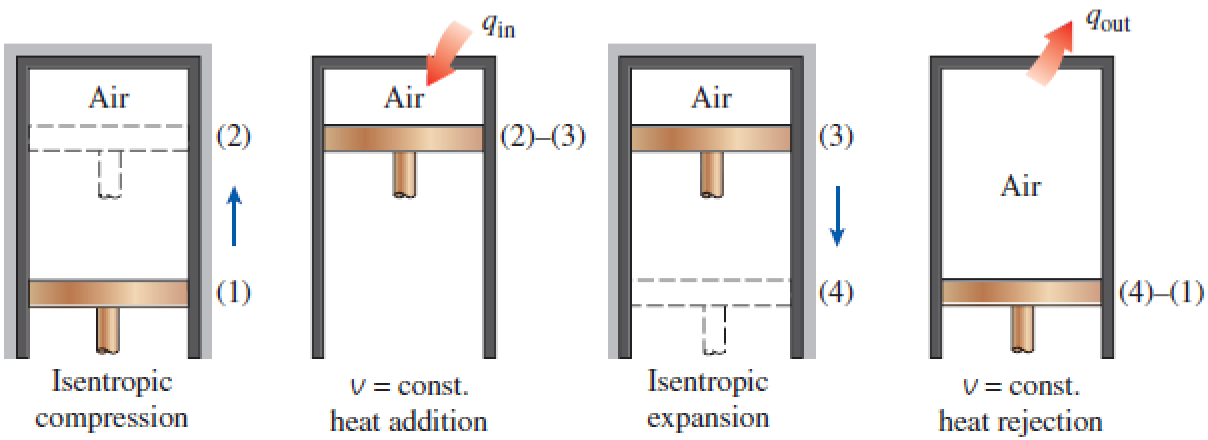
$$\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}} \qquad r = \frac{V_{max}}{V_{min}} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

$$\eta_{th,Diesel} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right] \qquad r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$

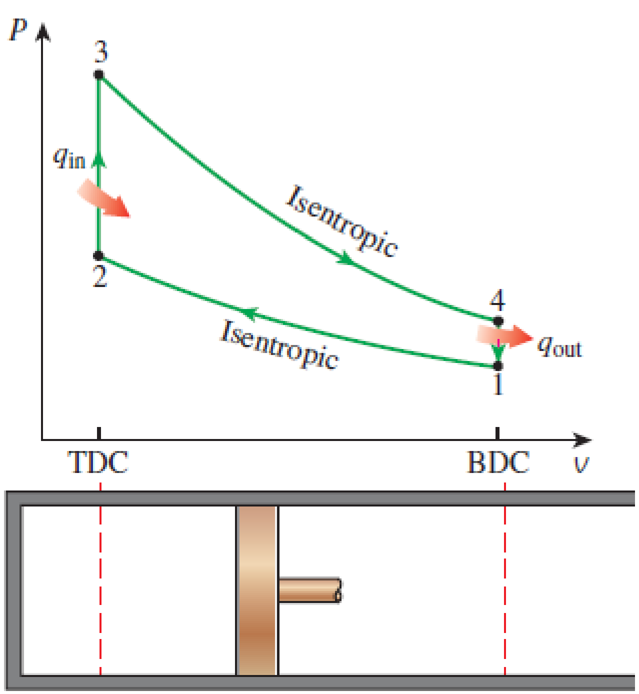
$$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}} \qquad r_p = \frac{P_2}{P_1}$$

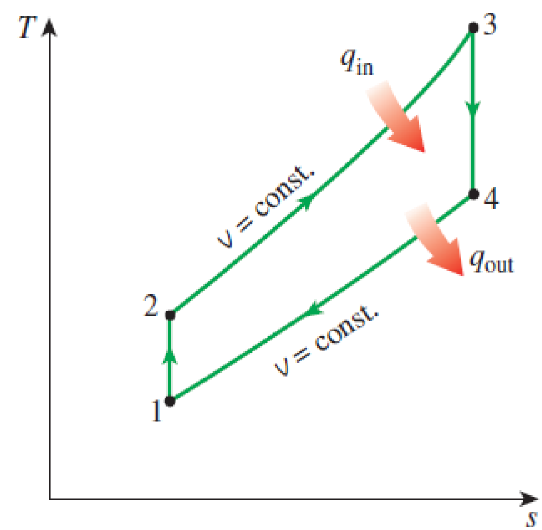
only for simple ideal cycles with constant specific heats at room temperature

OTTO CYCLE: THE IDEAL CYCLE FOR SPARK-IGNITION ENGINES



1-2 Isentropic compression
2-3 Constant-volume heat addition
3-4 Isentropic expansion
4-1 Constant-volume heat rejection





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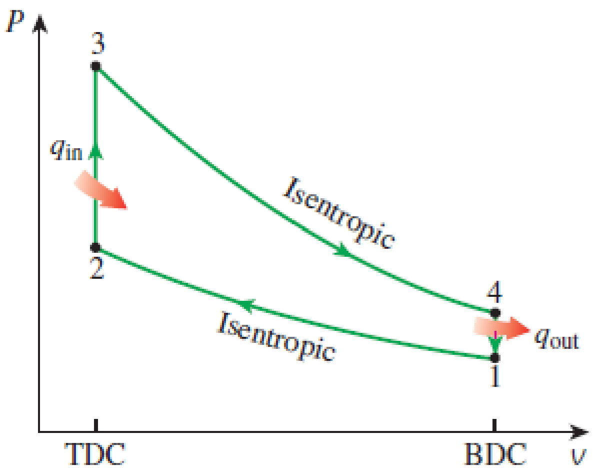
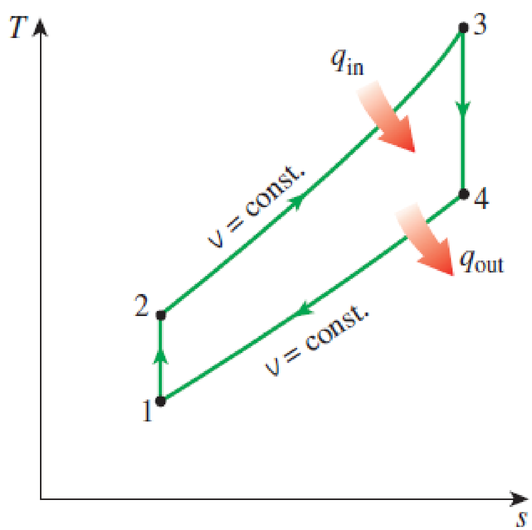




1-2 Isentropic Compression

$mBE\colon\ m_1 = m_2$

$EBE\colon\ m_1u_1 + W_C = m_2u_2$

$u_1 + w_C = u_2$

$EnBE\colon\ m_1s_1 + S_{gen} = m_2s_2$

$s_1 + s_{gen} = s_2$

$ExBE\colon m_1ex_1 + W_C = m_2ex_2 + Ex_D$ $\qquad Ex_D = T_0S_{gen}$

$ex_1 + w_C = ex_2 + ex_D$ $\qquad ex_i = (u_i - u_0) - T_0(s_i - s_0) + P_0(v_i - v_0)$

2-3 Constant-volume heat addition

$mBE\colon\ m_2 = m_3$

$EBE\colon\ m_2u_2 + Q_{in} = m_3u_3$

$u_2 + q_{in} = u_3$

$EnBE\colon\ m_2s_2 + \frac{Q_{in}}{T_s} + S_{gen} = m_3s_3$

$s_2 + \frac{q_{in}}{T_s} + s_{gen} = s_3$

$ExBE\colon m_2ex_2 + Ex^{Q_{in}} = m_3ex_3 + Ex_D$

$ex_2 + ex^{Q_{in}} = ex_3 + ex_D$ $\qquad Ex^{Q_{in}} = (1 - \frac{T_o}{T_H})Q_{in}$

3-4 Isentropic Expansion

$mBE\colon\ m_3 = m_4$

$EBE\colon\ m_3u_3 = m_4u_4 + W_e$

$u_3 = u_4 + w_e$

$EnBE\colon\ m_3s_3 + S_{gen} = m_4s_4$

$s_3 + s_{gen} = s_4$

$ExBE\colon m_3ex_3 = m_4ex_4 + W_e + Ex_D$

$ex_3 = ex_4 + w_e + ex_D$

4-1 Constant-volume heat rejection

$mBE\colon\ m_4 = m_1$

$EBE\colon\ m_4u_4 = m_1u_1 + Q_{out}$

$u_4 = u_1 + q_{out}$

$EnBE\colon\ m_4s_4 + S_{gen} = m_1s_1 + \frac{Q_{out}}{T_b}$

$s_4 + s_{gen} = s_1 + \frac{q_{out}}{T_b}$

$ExBE\colon m_4ex_4 = m_1ex_1 + Ex^{Q_{out}} + Ex_D$ $\qquad Ex^{Q_{out}} = (1 - \frac{T_o}{T_b})Q_{out}$

$ex_4 = ex_1 + ex^{Q_{out}} + ex_D$

The compression ratio of an air-standard Otto cycle is 9.5. Prior to the isentropic compression process, the air is at 100 kPa, 35°C, and 600 cm^3. The temperature at the end of the isentropic expansion process is 800 K. Using specific heat values at room temperature, determine

(a) the highest temperature and pressure in the cycle
(b) the amount of heat transferred in, in kJ
(c) the thermal efficiency and the thermal efficiency of a Carnot cycle operating between the same temperature limits
(d) the mean effective pressure.

Solution:

Assumptions
1 *The air-standard assumptions are applicable.*
2 *Kinetic and potential energy changes are* negligible.
3 Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K, c_v = 0.718 kJ/kg·K, R = 0.287 kJ/kg·K, and k = 1.4 (Table A-2).

Using specific heat values at room temperature

Analysis (*a*) Process 1–2: isentropic compression.

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (308\text{ K})(9.5)^{0.4} = 757.9\text{ K}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \longrightarrow P_2 = \frac{v_1}{v_2}\frac{T_2}{T_1}P_1 = (9.5)\left(\frac{757.9\text{ K}}{308\text{ K}}\right)(100\text{ kPa}) = 2338\text{ kPa}$$

Process 3–4: isentropic expansion.

$$T_3 = T_4\left(\frac{v_4}{v_3}\right)^{k-1} = (800\text{ K})(9.5)^{0.4} = \mathbf{1969\text{ K}}$$

Process 2-3: v = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow P_3 = \frac{T_3}{T_2}P_2 = \left(\frac{1969\text{ K}}{757.9\text{ K}}\right)(2338\text{ kPa}) = \mathbf{6072\text{ kPa}}$$

(*b*) $$m = \frac{P_1 V_1}{RT_1} = \frac{(100\text{ kPa})(0.0006\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(308\text{ K})} = 6.788\times10^{-4}\text{ kg}$$

<u>2-3 Constant-volume heat addition</u>

$mBE\colon m_2 = m_3 = m = 6.788\times10^{-4} kg$

$EBE\colon mu_2 + Q_{in} = mu_3$

$Q_{in} = m(u_3 - u_2) = mc_v(T_3 - T_2)$

$Q_{in} = (6.788\times10^{-4} kg)\left(0.718\frac{\text{kj}}{\text{kgK}}\right)(1969 - 757.9)\text{K} = 0.59\text{kJ}$

4-1 Constant-volume heat rejection

$mBE:\ m_4 = m_1 = m = 6.788 \times 10^{-4}\ kg$

$EBE: mu_4 = mu_1 + Q_{out}$

$Q_{out} = m(u_4 - u_1) = mc_v(T_4 - T_1)$

$Q_{out} = (6.788 \times 10^{-4} kg)\left(0.718 \frac{\mathrm{kj}}{\mathrm{kgK}}\right)(800 - 308)\mathrm{K} = 0.24\mathrm{kJ}$

$$W_{\text{net}} = Q_{\text{in}} - Q_{\text{out}} = 0.590 - 0.240 = 0.350 \text{ kJ}$$

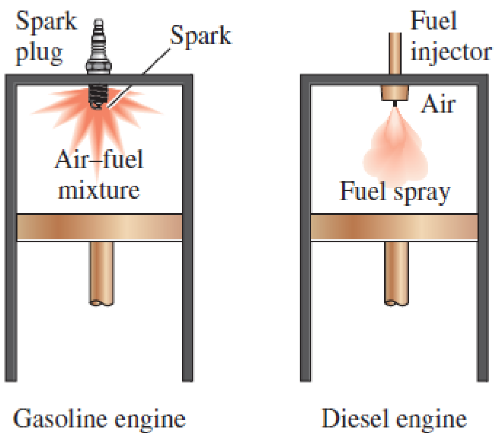
$$\eta_{\text{th}} = \frac{W_{\text{net,out}}}{Q_{\text{in}}} = \frac{0.350 \text{ kJ}}{0.590 \text{ kJ}} = \mathbf{59.4\%}$$

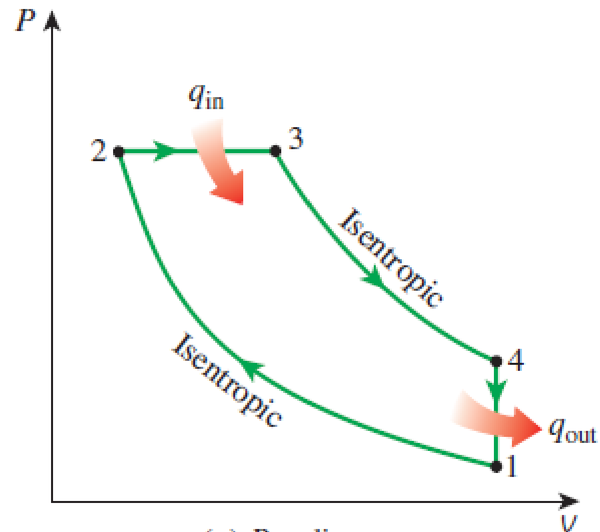
$$\eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{308 \text{ K}}{1969 \text{ K}} = 0.844 = 84.4\%$$

(d) $$V_{\min} = V_2 = \frac{V_{\max}}{r}$$

$$\text{MEP} = \frac{W_{\text{net,out}}}{V_1 - V_2} = \frac{W_{\text{net,out}}}{V_1(1 - 1/r)} = \frac{0.350 \text{ kJ}}{(0.0006 \text{ m}^3)(1 - 1/9.5)}\left(\frac{\text{kPa} \cdot \text{m}^3}{\text{kJ}}\right) = \mathbf{652 \text{ kPa}}$$

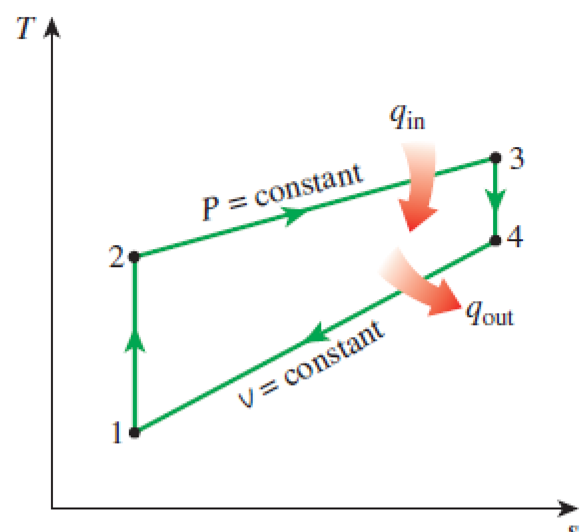
DIESEL CYCLE: THE IDEAL CYCLE FOR COMPRESSION-IGNITION ENGINES





(a) P-v diagram

- **1-2** isentropic compression
- **2-3** ***constant-pressure heat addition***
- **3-4** isentropic expansion
- **4-1** constant-volume heat rejection.



(b) T-s diagram

2-3 Constant-pressure heat addition

mBE: $m_2 = m_3$

EBE: $m_2 h_2 + Q_{in} = m_3 h_3$

$$h_2 + q_{in} = h_3$$

$EnBE$: $m_2 s_2 + \frac{Q_{in}}{T_s} + S_{gen} = m_3 s_3$

$$s_2 + \frac{q_{in}}{T_s} + s_{gen} = s_3$$

$ExBE$: $m_2 ex_2 + Ex^{Q_{in}} = m_3 ex_3 + Ex_D$

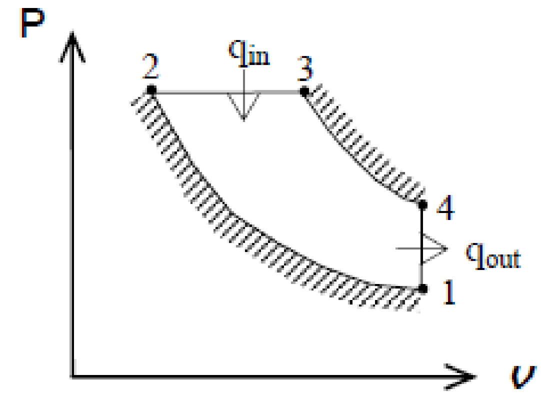
$$ex_2 + ex^{Q_{in}} = ex_3 + ex_D$$

$ex_i = (h_i - h_o) - T_o(s_i - s_o)$

An ideal diesel engine has a compression ratio of 20 and uses air as the working fluid. The state of air at the beginning of the compression process is 95 kPa and 20°C. If the maximum temperature in the cycle is not to exceed 2200 K, determine (a) the thermal efficiency and (b) the mean effective pressure. Assume constant specific heats for air at room temperature.

Solution:

***Assumptions* 1** *The air-standard assumptions are applicable.* **2** *Kinetic and potential energy changes are* negligible. **3** Air is an ideal gas with constant specific heats.



Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K, c_v = 0.718 kJ/kg·K, R = 0.287 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (a) Process 1−2: isentropic compression.

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (293 \text{ K})(20)^{0.4} = 971.1 \text{ K}$$

Process 2−3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{2200\text{K}}{971.1\text{K}} = 2.265$$

Process 3−4: isentropic expansion.

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3\left(\frac{2.265 v_2}{v_4}\right)^{k-1} = T_3\left(\frac{2.265}{r}\right)^{k-1} = (2200\text{ K})\left(\frac{2.265}{20}\right)^{0.4} = 920.6\text{ K}$$

$$q_{\text{in}} = h_3 - h_2 = c_p(T_3 - T_2) = (1.005\text{ kJ/kg}\cdot\text{K})(2200 - 971.1)\text{K} = 1235\text{ kJ/kg}$$

$$q_{\text{out}} = u_4 - u_1 = c_v(T_4 - T_1) = (0.718\text{ kJ/kg}\cdot\text{K})(920.6 - 293)\text{K} = 450.6\text{ kJ/kg}$$

$$w_{\text{net,out}} = q_{\text{in}} - q_{\text{out}} = 1235 - 450.6 = 784.4\text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net,out}}}{q_{\text{in}}} = \frac{784.4\text{ kJ/kg}}{1235\text{ kJ/kg}} = \mathbf{63.5\%}$$

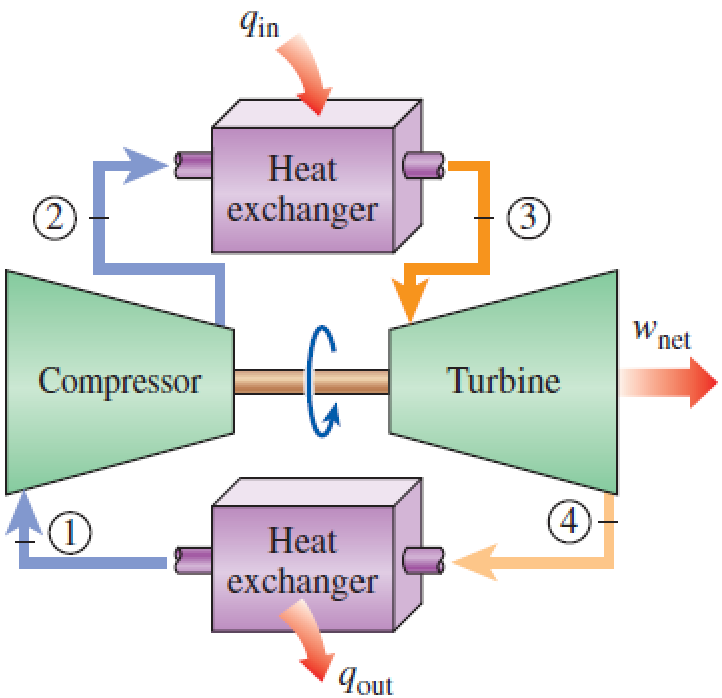
(b)

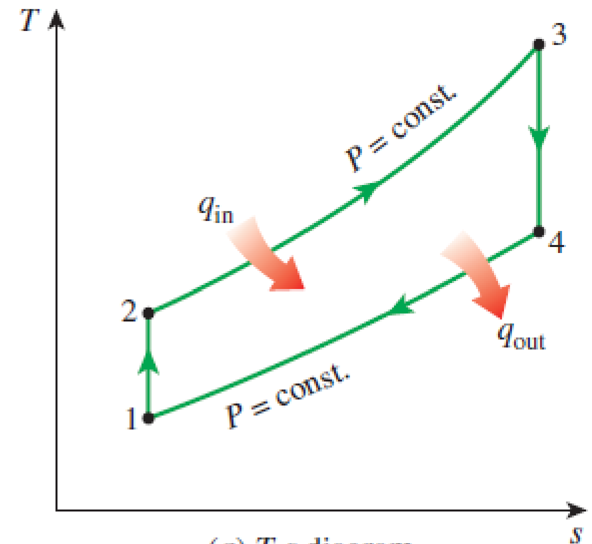
$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{ K})}{95\text{ kPa}} = 0.885\text{ m}^3/\text{kg} = v_{\max}$$

$$v_{\min} = v_2 = \frac{v_{\max}}{r}$$

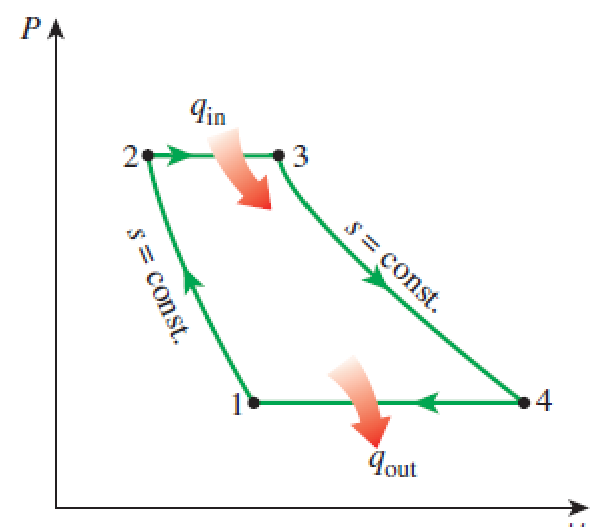
$$\text{MEP} = \frac{w_{\text{net,out}}}{v_1 - v_2} = \frac{w_{\text{net,out}}}{v_1(1 - 1/r)} = \frac{784.4\text{ kJ/kg}}{(0.885\text{ m}^3/\text{kg})(1 - 1/20)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{933\text{ kPa}}$$

BRAYTON CYCLE: THE IDEAL CYCLE FOR GAS-TURBINE ENGINES





(a) T-s diagram

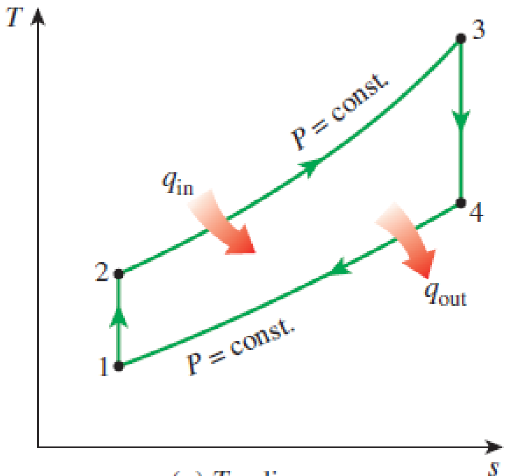


(b) P-v diagram

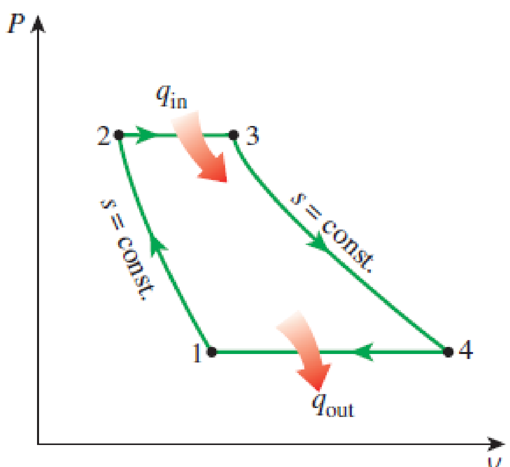
1-2 Isentropic compression (in a compressor)
2-3 Constant-pressure heat addition
3-4 Isentropic expansion (in a turbine)
4-1 Constant-pressure heat rejection

Balance Equations

1-2 Isentropic compression (in a compressor)
2-3 Constant-pressure heat addition
3-4 Isentropic expansion (in a turbine)
4-1 Constant-pressure heat rejection



(a) T-s diagram



(b) P-v diagram

1-2 Isentropic Compression

$mBE: \quad \dot m_1 = \dot m_2$

$EBE: \dot m_1 h_1 + \dot W_c = \dot m_2 h_2$

$$h_1 + w_C = h_2$$

$EnBE: \dot m_1 s_1 + \dot S_{gen} = \dot m_2 s_2$

$$s_1 + s_{gen} = s_2$$

$ExBE: \dot m_1 ex_1 + \dot W_c = \dot m_2 ex_2 + \dot{Ex}_D \qquad \dot{Ex}_D = T_0 \dot S_{gen}$

$$ex_1 + w_C = ex_2 + ex_D \qquad ex_i = (h_i - h_0) - T_0(s_i - s_0)$$

2-3 Constant-pressure heat addition

$mBE: \quad \dot m_2 = \dot m_3$

$EBE: \dot m_2 h_2 + \dot Q_{in} = \dot m_3 h_3$

$$h_2 + q_{in} = h_3$$

$EnBE: \dot m_2 s_2 + \frac{Q_{in}}{T_s} + \dot S_{gen} = \dot m_3 s_3$

$$s_2 + \frac{q_{in}}{T_s} + s_{gen} = s_3$$

$ExBE: \dot m_2 ex_2 + \dot{Ex}^{Q_{in}} = \dot m_3 ex_3 + \dot{Ex}_D \qquad \dot{Ex}^{Q_{in}} = (1 - \frac{T_0}{T_s})\dot Q_{in}$

$$ex_2 + ex^{Q_{in}} = ex_3 + ex_D$$

3-4 Isentropic Expansion

$mBE: \dot m_3 = \dot m_4$

$EBE: \dot m_3 h_3 = \dot m_4 h_4 + \dot W_T$

$$h_3 = h_4 + w_T$$

$EnBE: \dot m_3 s_3 + \dot S_{gen} = \dot m_4 s_4$

$$s_3 + s_{gen} = s_4$$

$ExBE: \dot m_3 ex_3 = \dot m_4 ex_4 + \dot W_T + \dot{Ex}_D$

$$ex_3 = ex_4 + w_T + ex_D$$

4-1 Constant-pressure heat rejection

$mBE: \dot m_4 = \dot m_1$

$EBE: \dot m_4 h_4 = \dot m_1 h_1 + \dot Q_{out}$

$$h_4 = h_1 + q_{out}$$

$EnBE: \dot m_4 s_4 + \dot S_{gen} = \dot m_1 s_1 + \frac{\dot Q_{out}}{T_b}$

$$s_4 + s_{gen} = s_1 + \frac{q_{out}}{T_b}$$

$ExBE: \dot m_4 ex_4 = \dot m_1 ex_1 + \dot{Ex}^{Q_{out}} + \dot{Ex}_D \qquad \dot{Ex}^{Q_{out}} = (1 - \frac{T_0}{T_b})\dot Q_{out}$

$$ex_4 = ex_1 + ex^{Q_{out}} + ex_D$$

A simple Brayton cycle using air as the working fluid has a pressure ratio of 8. The minimum and maximum temperatures in the cycle are 310 and 1160 K. Assuming an isentropic efficiency of 75 percent for the compressor and 82 percent for the turbine,

Take T_{source}= 1700K, $T_{boundary}$=306K and T_0=298K and determine

(a) the air temperature at the turbine exit,

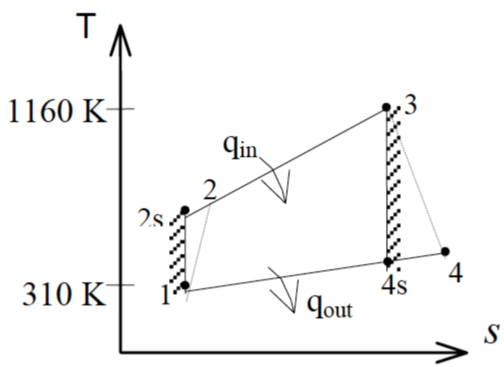
(b) the net work output,

(c) entropy generation

(d) exergy destruction

(e) the thermal efficiency, and

(f) exergy efficiency.



Assumptions
1 Steady operating conditions exist,
2 The air-standard assumptions are applicable,
3 Kinetic and potential energy changes are negligible,
4 Air is an ideal gas with variable specific heats.

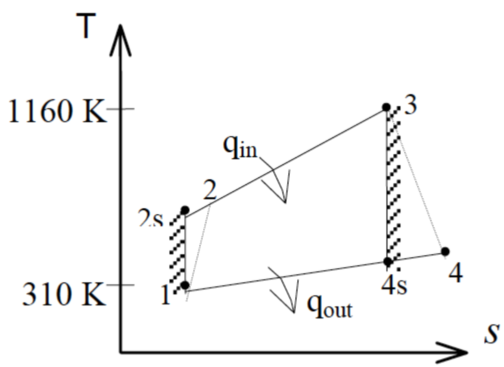
$$T_1 = 310 \text{ K} \longrightarrow \begin{array}{l} h_1 = 310.24 \text{ kJ/kg} \\ P_{r_1} = 1.5546 \end{array}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (8)(1.5546) = 12.44 \longrightarrow h_{2s} = 562.58 \text{ kJ/kg and } T_{2s} = 557.25 \text{ K}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_C}$$

$$= 310.24 + \frac{562.58 - 310.24}{0.75} = 646.7 \text{ kJ/kg}$$

Assuming an isentropic efficiency of 75 percent for the compressor



(a) the air temperature at the turbine exit (T_4),

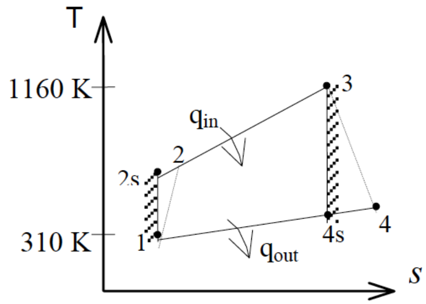
Assuming an isentropic efficiency of 82 percent for the turbine

$$T_3 = 1160\ \text{K} \longrightarrow \begin{array}{l} h_3 = 1230.92\ \text{kJ/kg} \\ P_{r_3} = 207.2 \end{array}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{8}\right)(207.2) = 25.90 \longrightarrow h_{4s} = 692.19\ \text{kJ/kg} \text{ and } T_{4s} = 680.3\ \text{K}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T (h_3 - h_{4s})$$
$$= 1230.92 - (0.82)(1230.92 - 692.19)$$
$$= 789.16\ \text{kJ/kg}$$

Thus, $T_4 = \mathbf{770.1\ K}$



b) the net work output (W_{net})=?

$$\dot{W}_{net} = \dot{W}_T - \dot{W}_C$$

$$w_{net} = w_T - w_C$$

w_C=? w_T=?

<u>1-2 Isentropic Compression</u>

$$EBE: \dot{m}_1 h_1 + \dot{W}_c = \dot{m}_2 h_2$$
$$h_1 + w_C = h_2$$

h_1=310.24kJ/kg
h_2 =646.7kJ/kg

$h_1 + w_C = h_2$
w_C =336.36 kJ/kg

<u>3-4 Isentropic Expansion</u>

$$EBE: \dot{m}_3 h_3 = \dot{m}_4 h_4 + \dot{W}_T$$
$$h_3 = h_4 + w_T$$

h_3=1230.92kJ/kg
h_4 =789.16kJ/kg

$h_3 = h_4 + w_T$
w_T =441.76 kJ/kg

$w_{net} = w_T - w_C$=105.4kJ/kg

Compressor

$$EnBE: \dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2$$
$$s_1 + s_{gen} = s_2$$

$s_1{}^o = 1.73498\ kJ/kgK$
$s_2{}^o = 2.47319\ kJ/kgK$

$$s_2 - s_1 = s_2{}^o - s_1{}^o - Rln\frac{P_2}{P_1}$$
$$s_{gen,compressor} = (2.47319 - 1.73498)kj/kgK - (0.287kJ/kgK)ln8$$
$$s_2 - s_1 = s_{gen} = 0.141\ kj/kgK$$

Heat Exchanger or Combustion Chamber

$$EnBE: \dot{m}_2 s_2 + \frac{\dot{Q}_{in}}{T_s} + \dot{S}_{gen} = \dot{m}_3 s_3$$
$$s_2 + \frac{q_{in}}{T_s} + s_{gen} = s_3$$

$s_2{}^o$ =2.47319 kJ/kgK
$s_3{}^o$=3.13916 kJ/kgK

$$h_2 + q_{in} = h_3 \Rightarrow q_{in} = 584.22\ \text{kj/kg}$$
$$s_3 - s_2 = s_3{}^o - s_2{}^o - Rln\frac{P_3}{P_2}\ ,\ P_3 = P_2$$
$$assuming\ T_s = 1700\ K$$
$$s_{gen} = 0.322\ kJ/kgK$$

Turbine

$$EnBE: \dot{m}_3 s_3 + \dot{S}_{gen} = \dot{m}_3 s_4$$

$$s_3 + s_{gen} = s_4$$

$s_3{}^o$=*3.13916* kJ/kgK

$s_4{}^o$=*2.67601* kJ/kgK

$$s_4 - s_3 = s_4{}^o - s_3{}^o - Rln\frac{P_4}{P_3}$$

$$s_4 - s_3 = s_4{}^o - s_3{}^o - 0.287\left(\frac{kj}{kgK}\right)ln\frac{1}{8} = 0.1339 = s_{gen}$$

Heat Exchanger (in Closed Cycle)

$$EnBE: \dot{m}_4 s_4 + \dot{S}_{gen} = \dot{m}_1 s_1 + \frac{\dot{Q}_{out}}{T_b}$$

$$s_4 + s_{gen} = s_1 + \frac{q_{out}}{T_b}$$

$s_4{}^o$=*2.67601* kJ/kgK

$s_1{}^o = 1.73498\ kJ/kgK$

$$s_1 - s_4 = s_1{}^o - s_4{}^o - Rln\frac{P_1}{P_4} \quad , \qquad P_1 = P_4$$

$$h_4 = h_1 + q_{out} \rightarrow q_{out} = 479.36\text{kJ/kg}$$

$$T_1 = 310K$$

$$T_0 = 298K$$

$$assuming\ T_b = 306K$$

$$s_{gen} = s_1 - s_4 + \frac{q_{out}}{T_b} = 0.636kJ/kgK$$

<u>Exergy Destruction</u> (Ex_D, ex_D)

$$\dot{E}x_D = T_0 \dot{S}_{gen}$$
$$ex_D = T_0 s_{gen}$$

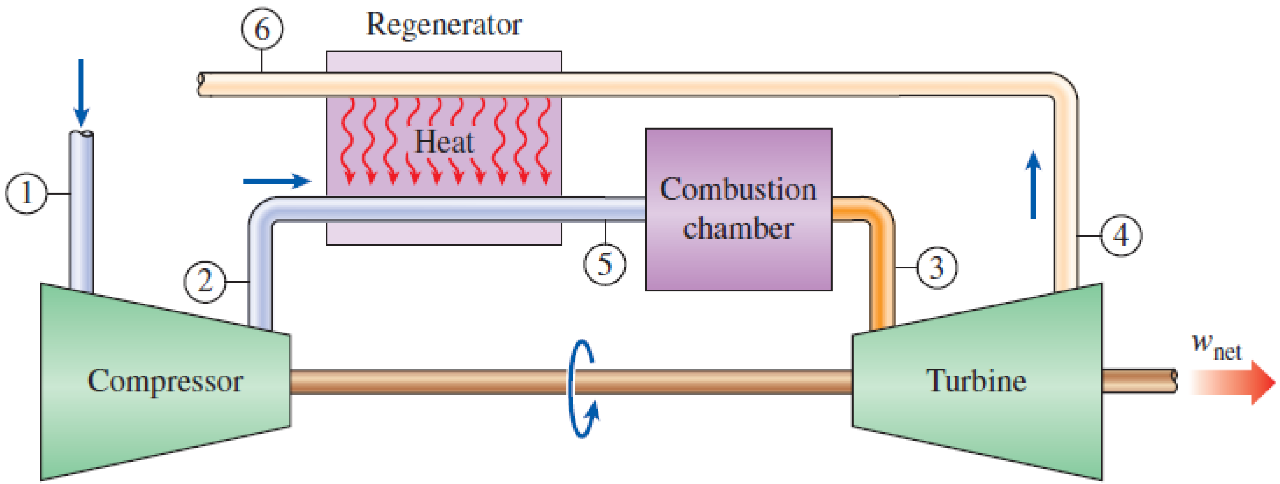
$$ex_{D,compressor} = 298K \times 0.141kJ/kgK = 42.018kJ/kg$$
$$ex_{D,combustion\ chamber} = 298K \times 0.322kJ/kgK = 95.956kJ/kg$$
$$ex_{D,turbine} = 298K \times 0.1339kJ/kgK = 39.902kJ/kg$$
$$ex_{D,heat\ exchanger} = 298K \times 0.636kJ/kgK = 189.53kJ/kg$$

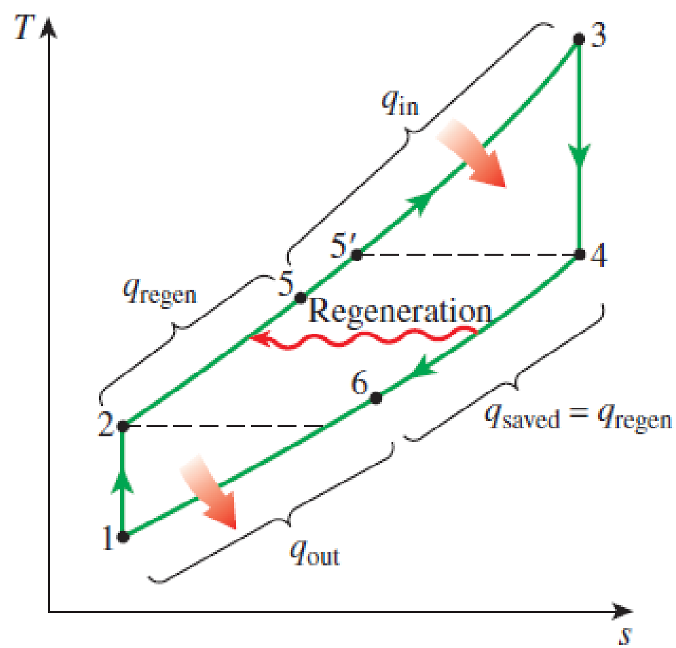
d) the thermal efficiency, and
e) exergy efficiency.

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{105.4}{584.22} = 18\%$$

$$\eta_{ex} = \frac{w_{net}}{ex^{Q_{in}}} = \frac{105.4}{631.806} = 16.68\%$$

THE BRAYTON CYCLE WITH REGENERATION





An ideal Brayton cycle with **regeneration** has a pressure ratio of 10. Air enters the compressor at 300 K and the turbine at 1200 K. Account for the variation of specific heats with temperature. If the effectiveness of the regenerator is 100 percent, determine

a) the net work output,

b) the thermal efficiency of the cycle,

c) exergy efficiency of the cycle

d) entropy generation in combustion chamber

Take T_0 = 300 K and $T_{source =}$ 1500 K

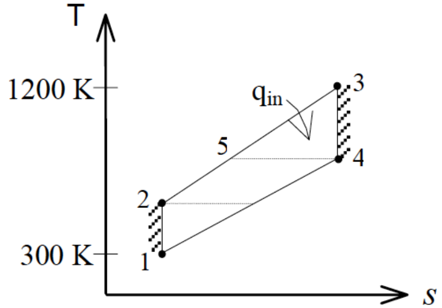
Assumptions
1 The air standard assumptions are applicable.
2 Air is an ideal gas with variable specific heats.
3 Kinetic and potential energy changes are negligible.

$$T_1 = 300\text{ K} \longrightarrow \begin{array}{l} h_1 = 300.19\text{ kJ/kg} \\ P_{r_1} = 1.386 \end{array}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (10)(1.386) = 13.86 \longrightarrow h_2 = 579.87\text{ kJ/kg}$$

$$T_3 = 1200\text{ K} \longrightarrow \begin{array}{l} h_3 = 1277.79\text{ kJ/kg} \\ P_{r_3} = 238 \end{array}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{10}\right)(238) = 23.8 \longrightarrow h_4 = 675.85\text{ kJ/kg}$$



1-2 Isentropic Compression

$mBE\colon\ m_1 = m_2$

$EBE\colon\ m_1 h_1 + W_C = m_2 h_2$

$$h_1 + w_C = h_2$$

$$w_C = h_2 - h_1$$

3-4 Isentropic Expansion

$mBE\colon\ m_3 = m_4$

$EBE\colon\ m_3 h_3 = m_4 h_4 + W_T$

$$h_3 = h_4 + w_T$$

$$h_3 - h_4 = w_T$$

$$w_{\mathrm{C,in}} = h_2 - h_1 = 579.87 - 300.19 = 279.68 \text{ kJ/kg}$$

$$w_{\mathrm{T,out}} = h_3 - h_4 = 1277.79 - 675.85 = 601.94 \text{ kJ/kg}$$

Thus,

$$w_{\mathrm{net}} = w_{\mathrm{T,out}} - w_{\mathrm{C,in}} = 601.94 - 279.68 = \mathbf{322.26\ kJ/kg}$$

Combustion Chamber

$$mBE: \dot{m}_5 = \dot{m}_3$$

$$EBE: \dot{m}_5 h_5 + \dot{Q}_{in} = \dot{m}_3 h_3$$

$$h_5 + q_{in} = h_3$$

$$q_{in} = h_3 - h_5$$

$$EnBE: \dot{m}_5 s_5 + \frac{\dot{Q}_{in}}{T_s} + \dot{S}_{gen} = \dot{m}_3 s_3$$

$$s_5 + \frac{q_{in}}{T_s} + s_{gen} = s_3$$

$$ExBE: \dot{m}_5 ex_5 + \dot{E}x^{Q_{in}} = \dot{m}_3 ex_3 + \dot{E}x_D$$

$$ex_5 + ex^{Q_{in}} = ex_3 + ex_D$$

Also, $\varepsilon = 100\% \longrightarrow h_5 = h_4 = 675.85 \text{ kJ/kg}$

$$q_{in} = h_3 - h_5 = 1277.79 - 675.85 = 601.94 \text{ kJ/kg}$$

and

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{322.26 \text{ kJ/kg}}{601.94 \text{ kJ/kg}} = \mathbf{53.5\%}$$

$$\eta_{ex} = \frac{w_{net}}{ex^{Q_{in}}} = ?$$

$$ex^{q_{in}} = \left(1 - \frac{T_0}{Ts}\right) q_{in} = \left(1 - \frac{300}{1500}\right) 601.94 = 481.55\ kJ/kg$$

$$\eta_{ex} = \frac{322.26}{481.55} = 0.67\ (67\%)$$