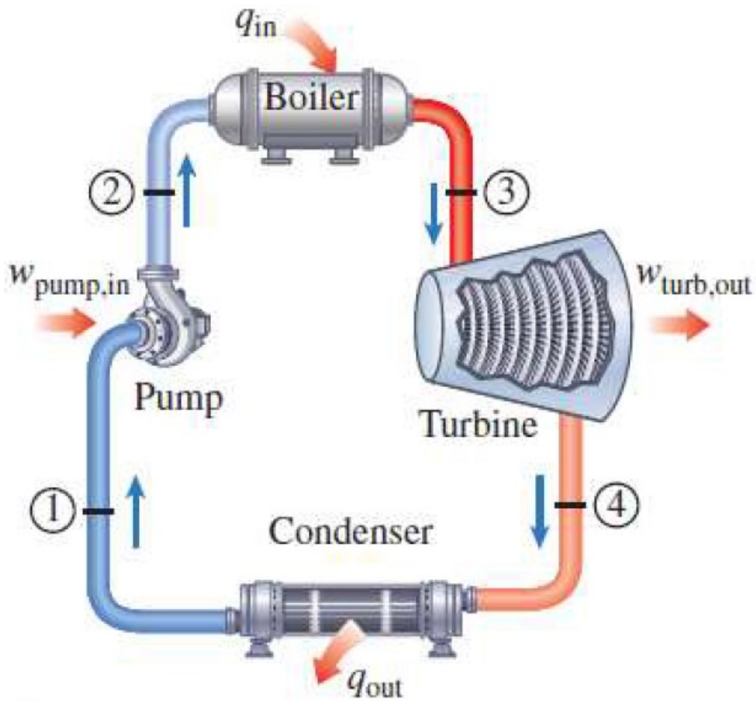
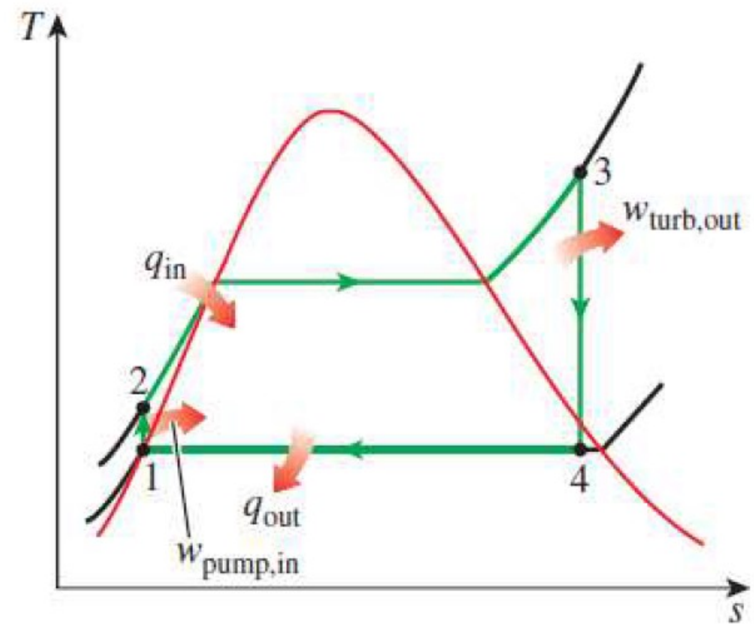
THERMODYNAMICS 2

Problem Session – II

Vapor & Combined Power Cycles

Rankine Cycle (The ideal cycle for vapor power cycles)





1-2 Isentropic compression (in a pump)
2-3 Constant-pressure heat addition (in a boiler)
3-4 Isentropic expansion (in a turbine)
4-1 Constant-pressure heat rejection (in a condenser)

1-2 Isentropic compression in the pump

$mBE: \quad \dot{m}_1 = \dot{m}_2$

$EBE: \dot{m}_1 h_1 + \dot{W}_{pump} = \dot{m}_2 h_2$

$h_1 + w_{pump} = h_2$

$w_{pump} = v(P_2 - P_1)\ , \quad \eta_{pump} = \frac{h_{2s}-h_1}{h_{2a}-h_1}$

$EnBE: \dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2$

$s_1 + s_{gen} = s_2 \Rightarrow s_{gen} = 0\ (ideal)$

$ExBE: \dot{m}_1 ex_1 + \dot{W}_{pump} = \dot{m}_2 ex_2 + \dot{Ex}_D$

$ex_1 + w_{pump} = ex_2 + ex_D$

2-3 Constant-pressure heat addition in the boiler

$mBE: \quad \dot{m}_2 = \dot{m}_3$

$EBE: \dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$

$h_2 + q_{in} = h_3$

$EnBE: \dot{m}_2 s_2 + \frac{\dot{Q}_{in}}{T_s} + \dot{S}_{gen} = \dot{m}_3 s_3$

$s_2 + \frac{q_{in}}{T_s} + s_{gen} = s_3$

$ExBE: \dot{m}_2 ex_2 + \dot{Ex}^{\dot{Q}_{in}} = \dot{m}_3 ex_3 + \dot{Ex}_D\ , \quad \dot{Ex}^{Q_{in}} = \left(1 - \frac{T_o}{T_s}\right) \dot{Q}_{in}$

$ex_2 + ex^{q_{in}} = ex_3 + ex_D$

3-4 Isentropic expansion in the turbine

$mBE: \dot{m}_3 = \dot{m}_4$

$EBE: \dot{m}_3 h_3 = \dot{m}_4 h_4 + \dot{W}_T$

$h_3 = h_4 + w_T \qquad \eta_{Turbine} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$

$EnBE: \dot{m}_3 s_3 + \dot{S}_{gen} = \dot{m}_4 s_4$

$s_3 + s_{gen} = s_4$

$ExBE: \dot{m}_3 ex_3 = \dot{m}_4 ex_4 + \dot{W}_T + \dot{Ex}_D$

$ex_3 = ex_4 + w_T + ex_D$

4-1 Constant-pressure heat rejection in the condenser

$mBE: \dot{m}_4 = \dot{m}_1$

$EBE: \dot{m}_4 h_4 = \dot{m}_1 h_1 + \dot{Q}_{out}$

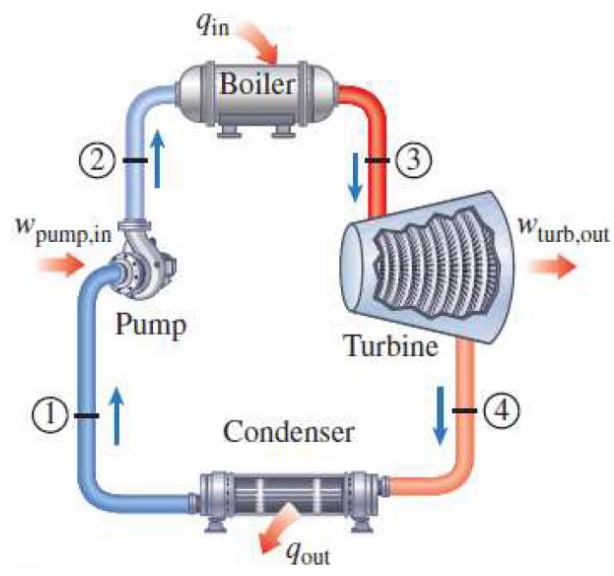
$h_4 = h_1 + q_{out}$

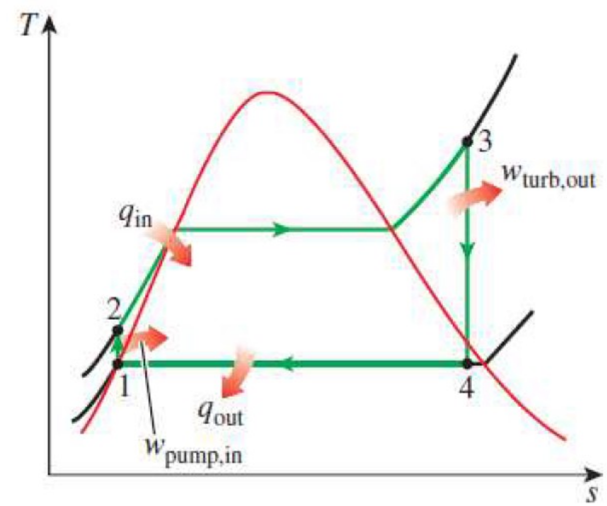
$EnBE: \dot{m}_4 s_4 + \dot{S}_{gen} = \dot{m}_1 s_1 + \frac{\dot{Q}_{out}}{T_b}$

$s_4 + s_{gen} = s_1 + \frac{q_{out}}{T_b}$

$ExBE: \dot{m}_4 ex_4 = \dot{m}_1 ex_1 + \dot{Ex}^{Q_{out}} + \dot{Ex}_D\ , \ \dot{Ex}^{Q_{out}} = \left(1 - \frac{T_o}{T_b}\right) \dot{Q}_{out}$

$ex_4 = ex_1 + ex^{Q_{out}} + ex_D$





Consider a steam power plant that operates on a simple ideal Rankine cycle and has a net power output of 45 MW. Steam enters the turbine at 7 MPa and 500°C and is cooled in the condenser at a pressure of 10 kPa by running cooling water from a lake through the tubes of the condenser at a rate of 2000 kg/s. Take T_s=700 °C.

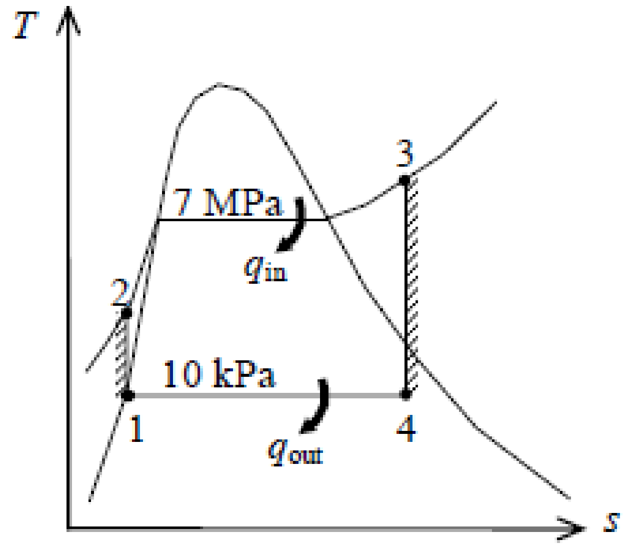
Show the cycle on a *T-s* diagram with respect to saturation lines, and determine

(a) the energy efficiency of the cycle,

(b) the exergy efficiency of the cycle,

(c) the mass flow rate of the steam,

(d) the temperature rise of the cooling water



The cycle on a *T-s* diagram
with respect to saturation lines

1-2 Isentropic compression in the pump

mBE: $\quad \dot{m}_1 = \dot{m}_2 = \dot{m}$

EBE: $\quad \dot{m}h_1 + \dot{W}_{pump} = \dot{m}h_2$

$$h_1 + w_{pump} = h_2$$

$$w_{pump} = v_1(P_2 - P_1)$$

$$h_1 = h_{f\,@\,10\text{ kPa}} = 191.81\text{ kJ/kg}$$

$$v_1 = v_{f\,@\,10\text{ kPa}} = 0.00101\ \text{m}^3/\text{kg}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)$$

$$= \left(0.00101\ \text{m}^3/\text{kg}\right)\left(7{,}000 - 10\text{ kPa}\right)\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right)$$

$$= 7.06\text{ kJ/kg}$$

$$h_2 = h_1 + w_{p,\text{in}} = 191.81 + 7.06 = 198.87\ \text{kJ/kg}$$

2-3 Constant-pressure heat addition in the boiler

$mBE: \quad \dot{m}_2 = \dot{m}_3$

$EBE: \dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$

$$h_2 + q_{in} = h_3$$

$$\boldsymbol{q_{in} = 3411.4 - 198.87 = 3212.5\text{kJ/kg}}$$

$$\left.\begin{array}{l} P_3 = 7 \text{ MPa} \\ T_3 = 500°\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3411.4 \text{ kJ/kg} \\ s_3 = 6.8000 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$h_2 = h_1 + w_{p,\text{in}} = 191.81 + 7.06 = 198.87 \text{ kJ/kg}$$

4-1 Constant-pressure heat rejection in the condenser

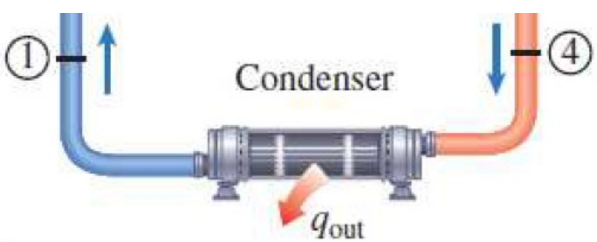
$mBE: \dot{m}_4 = \dot{m}_1$ and $\dot{m}_{cw,in} = \dot{m}_{cw,out}$

$EBE: \dot{m}_4 h_4 = \dot{m}_1 h_1 + \dot{Q}_{out}$ and

$\dot{m}_{cw,in} h_{in} + \dot{Q}_{in,cw} = \dot{m}_{cw,out} h_{out}$

$$h_4 = h_1 + q_{out}$$

$$\boldsymbol{q_{out} = 2153.6 - 191.81 = 1961.8\,kJ/kg}$$



$$s_1 = s_2 = s_{f@10\text{kPa}} = 0.6492 \text{ kJ/kg} \cdot \text{K}$$

$$s_3 = s_4 = 6.8000 \text{ kJ/kg} \cdot \text{K}$$

$$\left.\begin{array}{l} P_4 = 10 \text{ kPa} \\ s_4 = s_3 \end{array}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.8000 - 0.6492}{7.4996} = 0.8201$$

$$h_4 = h_f + x_4 h_{fg} = 191.81 + (0.8201)(2392.1) = 2153.6 \text{ kJ/kg}$$

$$\dot{W}_{net} = \dot{Q}_{in} - \dot{Q}_{out} = \dot{W}_{turbine} - \dot{W}_{pump}$$

$$w_{net} = q_{in} - q_{out} = w_{turbine} - w_{pump}$$

$$\boldsymbol{w_{net} = q_{in} - q_{out} = 1250.7\ kj/kg}$$

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{w_{net}}{q_{in}}$$

$$\eta_{ex} = \frac{\dot{W}_{net}}{\dot{E}x^{\dot{Q}_{in}}} = \frac{\dot{w}_{net}}{\dot{e}x^{q_{in}}}$$

$$\boldsymbol{ex^{q_{in}} = \left(1 - \frac{T_o}{T_s}\right) q_{in} = \left(1 - \frac{293}{973}\right) 3212.5 = 2245.118\ kj/kg}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{1250.7\ \text{kJ/kg}}{3212.5\ \text{kJ/kg}} = \mathbf{38.9\%}$$

$$\eta_{ex} = \frac{\dot{w}_{net}}{\dot{e}x^{q_{in}}} = \frac{1250.7}{2245.118} = \mathbf{55.7}\%$$

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{45{,}000\ \text{kJ/s}}{1250.7\ \text{kJ/kg}} = \mathbf{36.0\ kg/s}$$

The rate of heat rejection to the cooling water and its temperature rise are

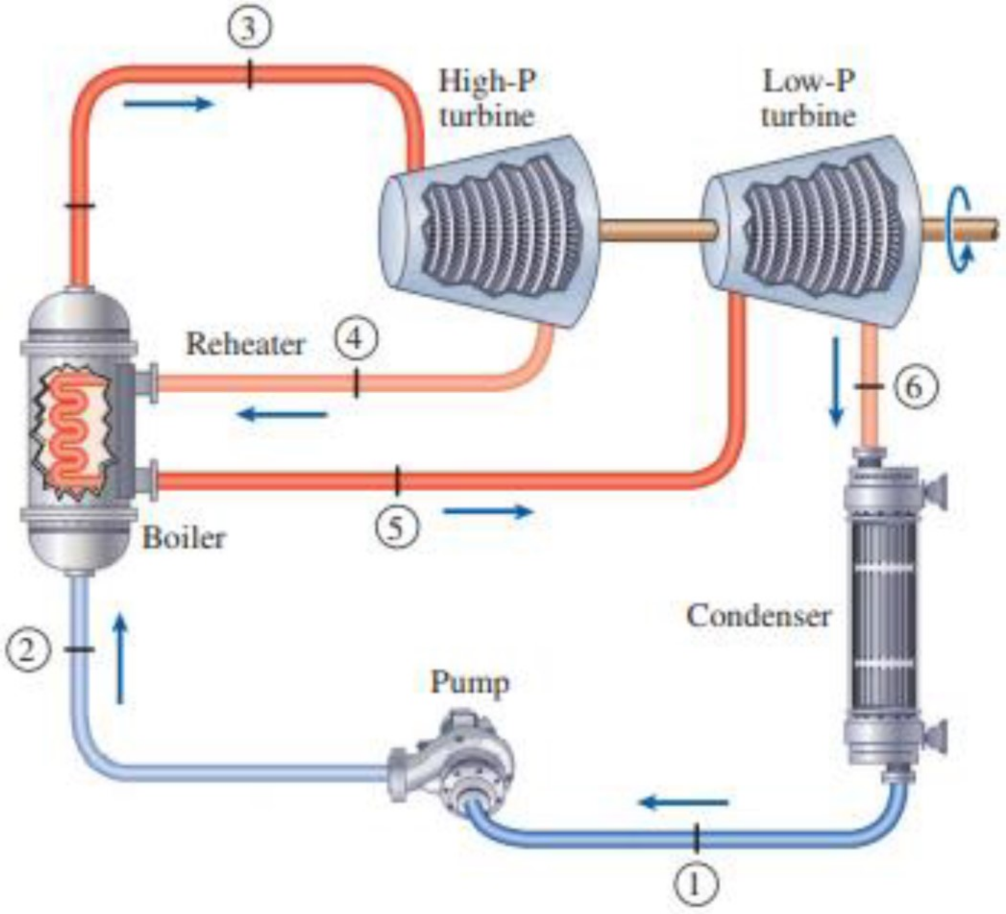
$$\dot{Q}_{out} = \dot{m}q_{out} = (36\ \text{kg/s})(1961.8\ \text{kJ/kg}) = 70{,}586\ \text{kJ/s}$$

$$\Delta T_{coolingwater} = \frac{\dot{Q}_{out}}{(\dot{m}c)_{coolingwater}} = \frac{70{,}586\ \text{kJ/s}}{(2000\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot{}^\circ\text{C})} = \mathbf{8.4^\circ C}$$

Reheat Rankine Cyle

A steam power plant operates on the reheat Rankine cycle. Steam enters the high-pressure turbine at 12.5 MPa and 550°C at a rate of 7.7 kg/s and leaves at 2 MPa. Steam is then reheated at constant pressure to 450°C before it expands in the low-pressure turbine. The isentropic efficiencies of the turbine and the pump are 85 percent and 90 percent, respectively. Steam leaves the condenser as a saturated liquid. If the moisture content of the steam at the exit of the turbine is not to exceed 5 percent, determine (*a*) the condenser pressure, (*b*) the net power output, and (*c*) the thermal efficiency. *Answers:* (*a*) 9.73 kPa, (*b*) 10.2 MW, (*c*) 36.9 percent

(*d*) Find the exergy efficiency.



1-2 Isentropic compression in the pump

$$mBE: \quad \dot{m}_1 = \dot{m}_2$$

$$EBE: \dot{m}_1 h_1 + \dot{W}_{pump} = \dot{m}_2 h_2$$

$$h_1 + w_{pump} = h_2$$

$$w_{pump} = v(P_2 - P_1)$$

$$EnBE: \dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2$$

$$s_1 + s_{gen} = s_2$$

$$ExBE: \dot{m}_1 ex_1 + \dot{W}_c = \dot{m}_2 ex_2 + \dot{E}x_D$$

$$ex_1 + w_C = ex_2 + ex_D$$

Isentropic efficiency for pump

$$\eta_{pump} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

2-3 and 4-5 Constant-pressure heat addition in the boiler

$$mBE: \quad \dot{m}_2 = \dot{m}_3, \dot{m}_4 = \dot{m}_5$$

$$EBE: \dot{m}_2 h_2 + \dot{m}_4 h_4 + \dot{Q}_{in} = \dot{m}_3 h_3 + \dot{m}_5 h_5$$

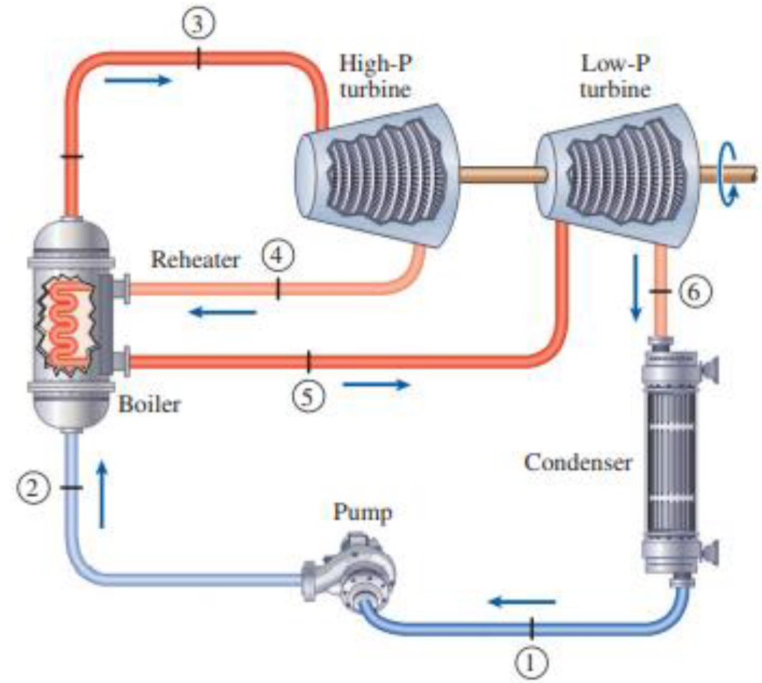
$$h_2 + h_4 + q_{in} = h_3 + h_5$$

$$EnBE: \dot{m}_2 s_2 + \dot{m}_4 s_4 + \frac{Q_{in}}{T_s} + \dot{S}_{gen} = \dot{m}_3 s_3 + \dot{m}_5 s_5$$

$$s_2 + s_4 + \frac{q_{in}}{T_s} + s_{gen} = s_3 + s_5$$

$$ExBE: \dot{m}_2 ex_2 + \dot{m}_4 ex_4 + \dot{E}x^{Q_{in}} = \dot{m}_3 ex_3 + \dot{m}_5 ex_5 + \dot{E}x_D$$

$$ex_2 + ex_4 + ex^{Q_{in}} = ex_3 + ex_5 + ex_D$$



3-4 and 5-6 Isentropic expansion in the turbine

$mBE: \dot{m}_3 = \dot{m}_4 = \dot{m}_5 = \dot{m}_6$

$EBE: \dot{m}_3 h_3 + \dot{m}_5 h_5 = \dot{m}_4 h_4 + \dot{m}_6 h_6 + \dot{W}_T$

$h_3 + h_5 = h_4 + h_6 + w_T$

$EnBE: \dot{m}_3 s_3 + \dot{m}_5 s_5 + \dot{S}_{gen} = \dot{m}_4 s_4 + \dot{m}_6 s_6$

$s_3 + s_5 + s_{gen} = s_4 + s_6$

$ExBE: \dot{m}_3 ex_3 + \dot{m}_5 ex_5 = \dot{m}_4 ex_4 + \dot{m}_6 ex_6 + \dot{W}_T + \dot{E}x_D$

$ex_3 + ex_5 = ex_4 + ex_6 + w_T + ex_D$

6-1 Constant-pressure heat rejection in the condenser

$mBE: \dot{m}_6 = \dot{m}_1$

$EBE: \dot{m}_6 h_6 = \dot{m}_1 h_1 + \dot{Q}_{out}$

$h_6 = h_1 + q_{out}$

$EnBE: \dot{m}_6 s_6 + \dot{S}_{gen} = \dot{m}_1 s_1 + \frac{\dot{Q}_{out}}{T_b}$

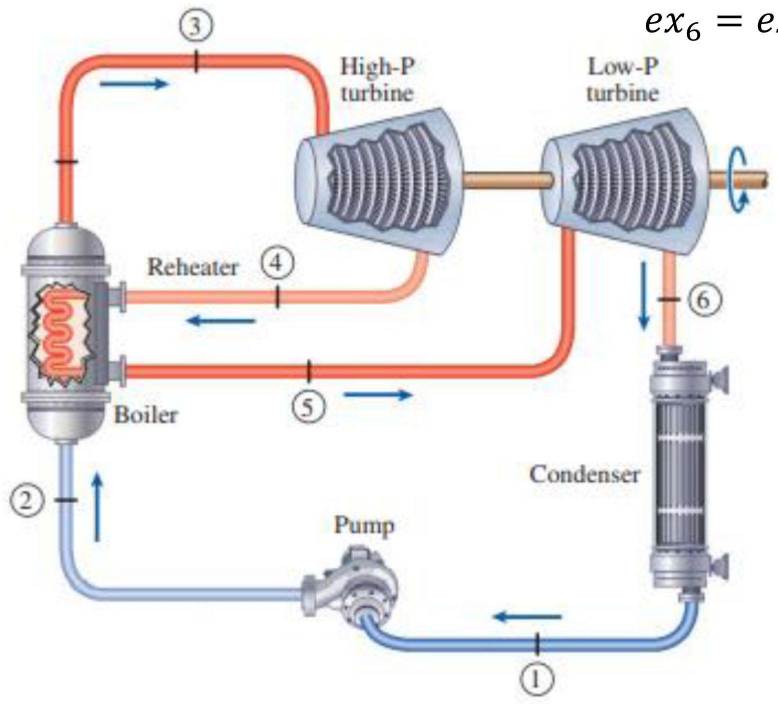
$s_6 + s_{gen} = s_1 + \frac{q_{out}}{T_b}$

$ExBE: \dot{m}_6 ex_6 = \dot{m}_1 ex_1 + \dot{E}x^{Q_{out}} + \dot{E}x_D$

$ex_6 = ex_1 + ex^{Q_{out}} + ex_D$

Isentropic efficiency for turbine

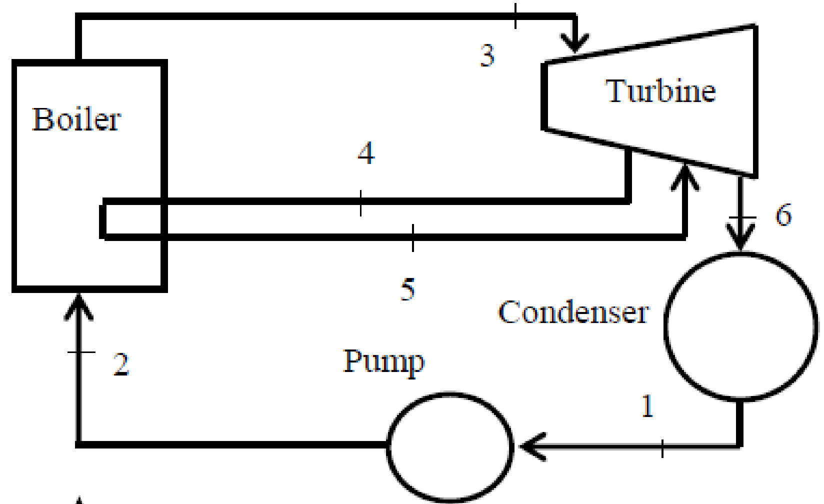
$$\eta_{Turbine} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}} = \frac{h_5 - h_{6a}}{h_5 - h_{6s}}$$



10-36 A steam power plant that operates on a reheat Rankine cycle is considered. The condenser pressure, the net power output, and the thermal efficiency are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),

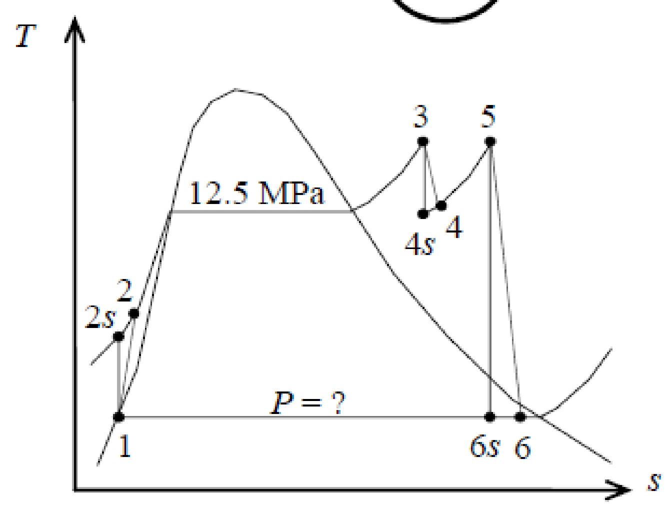


$$\left.\begin{array}{l} P_3 = 12.5 \text{ MPa} \\ T_3 = 550°\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3476.5 \text{ kJ/kg} \\ s_3 = 6.6317 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_4 = 2 \text{ MPa} \\ s_{4s} = s_3 \end{array}\right\} h_{4s} = 2948.1 \text{ kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\begin{aligned} \rightarrow h_4 &= h_3 - \eta_T (h_3 - h_{4s}) \\ &= 3476.5 - (0.85)(3476.5 - 2948.1) \\ &= 3027.3 \text{ kJ/kg} \end{aligned}$$



$$\left.\begin{array}{l} P_5 = 2 \text{ MPa} \\ T_5 = 450°\text{C} \end{array}\right\} \begin{array}{l} h_5 = 3358.2 \text{ kJ/kg} \\ s_5 = 7.2815 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_6 = ? \\ x_6 = 0.95 \end{array}\right\} h_6 = \quad \text{(Eq. 1)}$$

$$\left.\begin{array}{l} P_6 = ? \\ s_6 = s_5 \end{array}\right\} h_{6s} = \quad \text{(Eq. 2)}$$

$$\eta_T = \frac{h_5 - h_6}{h_5 - h_{6s}} \longrightarrow h_6 = h_5 - \eta_T (h_5 - h_{6s}) = 3358.2 - (0.85)(3358.2 - h_{6s}) \quad \text{(Eq. 3)}$$

The pressure at state 6 may be determined by a trial-error approach from the steam tables or by using EES from the above three equations:

$$P_6 = \mathbf{9.73\ kPa}, \quad h_6 = 2463.3\ \text{kJ/kg},$$

(*b*) Then,

$$h_1 = h_{f @ 9.73\ \text{kPa}} = 189.57\ \text{kJ/kg}$$

$$\boldsymbol{v}_1 = \boldsymbol{v}_{f @ 10\ \text{kPa}} = 0.001010\ \text{m}^3/\text{kg}$$

$$\begin{aligned} w_{p,\text{in}} &= \boldsymbol{v}_1(P_2 - P_1)/\eta_p \\ &= (0.00101\ \text{m}^3/\text{kg})(12{,}500 - 9.73\ \text{kPa})\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right)/(0.90) \\ &= 14.02\ \text{kJ/kg} \end{aligned}$$

$$h_2 = h_1 + w_{p,\text{in}} = 189.57 + 14.02 = 203.59\ \text{kJ/kg}$$

Cycle analysis:

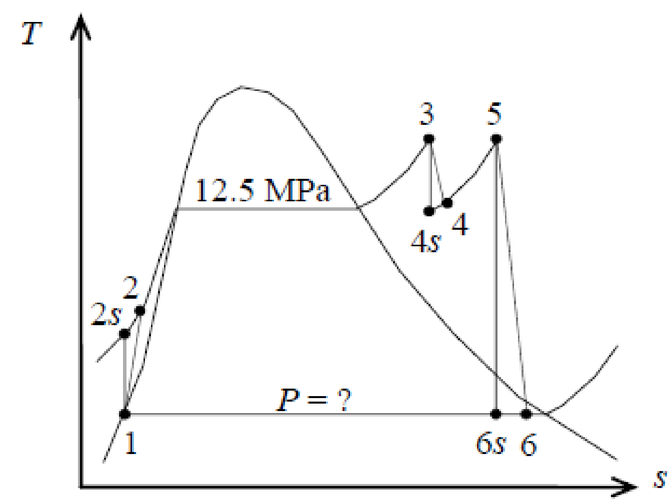
$$q_{\text{in}} = (h_3 - h_2) + (h_5 - h_4) = 3476.5 - 203.59 + 3358.2 - 2463.3 = 3603.8\ \text{kJ/kg}$$

$$q_{\text{out}} = h_6 - h_1 = 2463.3 - 189.57 = 2273.7\ \text{kJ/kg}$$

$$\dot{W}_{\text{net}} = \dot{m}(q_{\text{in}} - q_{\text{out}}) = (7.7\ \text{kg/s})(3603.8 - 2273.7)\text{kJ/kg} = \mathbf{10{,}242kW}$$

(*c*) The thermal efficiency is

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2273.7\ \text{kJ/kg}}{3603.8\ \text{kJ/kg}} = 0.369 = \mathbf{36.9\%}$$



$$(d)\ \eta_{ex} = \frac{\dot{W}_{net}}{\dot{Ex}^{Q_{in}}} = \frac{\dot{W}_{net}}{(1-\frac{T_o}{T_s})\dot{Q}_{in}} = \frac{10242\ kW}{(1-\frac{298}{1298})(7.7\ kg * 3703.8\frac{kJ}{kg})} = 0.479 = 47.9\%$$

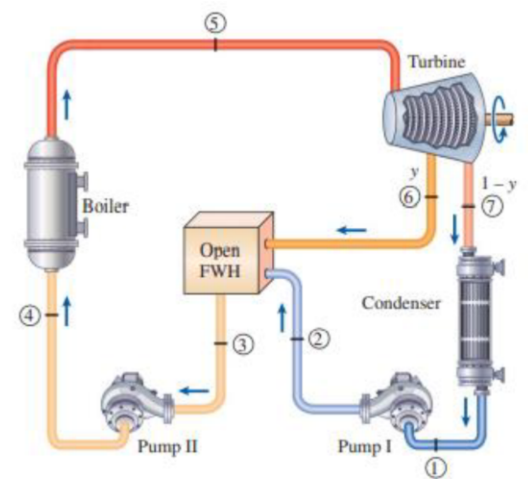
Regenerative Rankine Cyle

Consider a steam power plant that operates on a regenerative Rankine cycle and has a net power output of 150 MW. Steam enters the turbine at 10 MPa and 500 °C and the condenser at 10 kPa. The isentropic efficiency of the turbine is 80 percent, and that of the pumps is 95 percent. Steam is extracted from the turbine at 0.5 MPa to heat the feedwater in an open feedwater heater. Water leaves the feedwater heater as a saturated liquid. Show the cycle on a T-s diagram, and determine;

(a) the mass flow rate of steam through the boiler,

(b) and the thermal efficiency of the cycle.

(c) Also, determine the exergy destruction associated with the regeneration process. Assume a source temperature of 1300 K a sink temperature of 303 K.



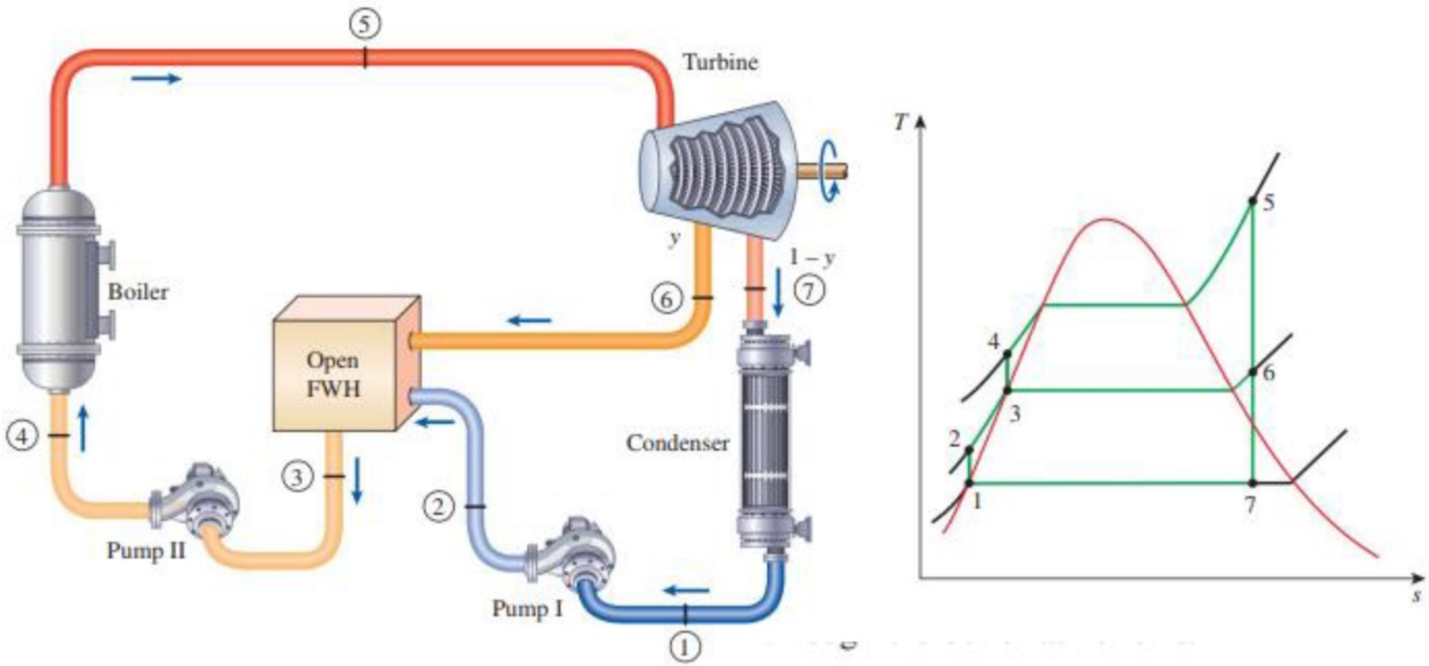


FIGURE 10–15
The ideal regenerative Rankine cycle with an open feedwater heater.

$$q_{in} = h_5 - h_4 \tag{10–14}$$

$$q_{out} = (1 - y)(h_7 - h_1) \tag{10–15}$$

$$w_{turb,out} = (h_5 - h_6) + (1 - y)(h_6 - h_7) \tag{10–16}$$

$$w_{pump,in} = (1 - y)w_{pump\ I,in} + w_{pump\ II,in} \tag{10–17}$$

where

$$y = \dot{m}_6/\dot{m}_5 \quad \text{(fraction of steam extracted)}$$

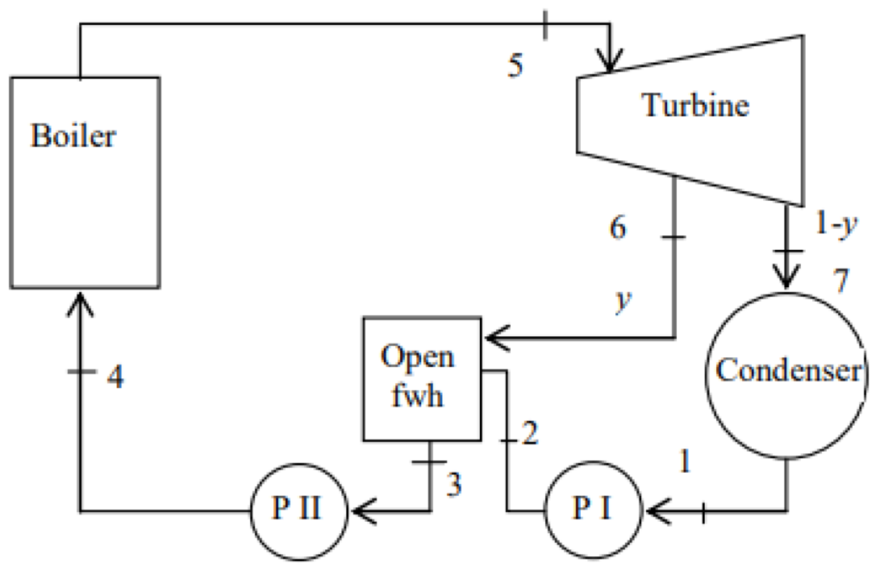
$$w_{pump\ I,in} = v_1(P_2 - P_1)$$

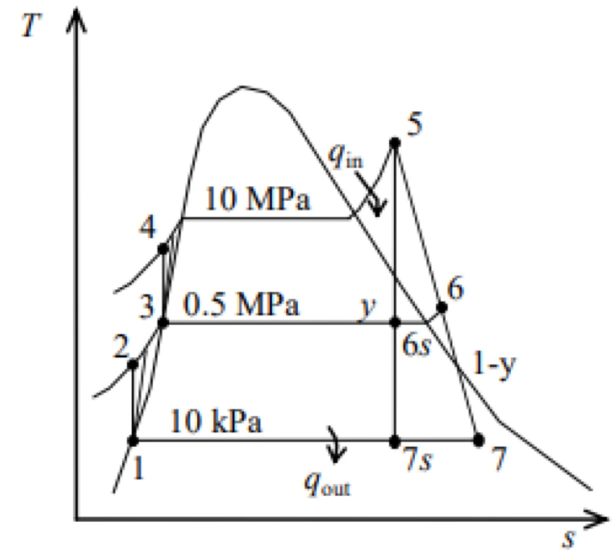
$$w_{pump\ II,in} = v_3(P_4 - P_3)$$

An 150-MW steam power plant operating on a regenerative Rankine cycle with an open feedwater heater is considered. The mass flow rate of steam through the boiler, the thermal efficiency of the cycle, and the irreversibility associated with the regeneration process are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis





(*a*) From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f\,@\,10\,\text{kPa}} = 191.81 \text{ kJ/kg}$$
$$v_1 = v_{f\,@\,10\,\text{kPa}} = 0.00101 \text{ m}^3\text{/kg}$$

$$\begin{aligned} w_{\text{pI,in}} &= v_1(P_2 - P_1)/\eta_p \\ &= \left(0.00101 \text{ m}^3\text{/kg}\right)\left(500 - 10 \text{ kPa}\right)\left(\frac{1 \text{ kJ}}{1 \text{ kPa}\cdot\text{m}^3}\right)/(0.95) \\ &= 0.52 \text{ kJ/kg} \end{aligned}$$

$$h_2 = h_1 + w_{\text{pI,in}} = 191.81 + 0.52 = 192.33 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 0.5 \text{ MPa} \\ \text{sat.liquid} \end{array}\right\} \begin{array}{l} h_3 = h_{f @ 0.5 \text{ MPa}} = 640.09 \text{ kJ/kg} \\ v_3 = v_{f @ 0.5 \text{ MPa}} = 0.001093 \text{ m}^3\text{/kg} \end{array}$$

$$\begin{aligned} w_{\text{pII,in}} &= v_3(P_4 - P_3)/\eta_p \\ &= (0.001093 \text{ m}^3\text{/kg})(10{,}000 - 500 \text{ kPa})\left(\frac{1 \text{ kJ}}{1 \text{ kPa}\cdot\text{m}^3}\right)/(0.95) \\ &= 10.93 \text{ kJ/kg} \end{aligned}$$

$$h_4 = h_3 + w_{\text{pII,in}} = 640.09 + 10.93 = 651.02 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_5 = 10 \text{ MPa} \\ T_5 = 500°\text{C} \end{array}\right\} \begin{array}{l} h_5 = 3375.1 \text{ kJ/kg} \\ s_5 = 6.5995 \text{ kJ/kg}\cdot\text{K} \end{array}$$

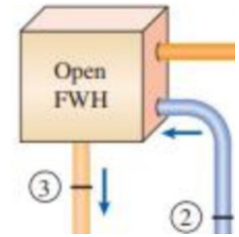
$$\left.\begin{array}{l} P_{6s} = 0.5 \text{ MPa} \\ s_{6s} = s_5 \end{array}\right\} \begin{array}{l} x_{6s} = \dfrac{s_{6s} - s_f}{s_{fg}} = \dfrac{6.5995 - 1.8604}{4.9603} = 0.9554 \\ h_{6s} = h_f + x_{6s} h_{fg} = 640.09 + (0.9554)(2108.0) \\ \quad = 2654.1 \text{ kJ/kg} \end{array}$$

$$\eta_T = \frac{h_5 - h_6}{h_5 - h_{6s}} \longrightarrow \begin{aligned} h_6 &= h_5 - \eta_T (h_5 - h_{6s}) \\ &= 3375.1 - (0.80)(3375.1 - 2654.1) \\ &= 2798.3 \text{ kJ/kg} \end{aligned}$$

$$\left.\begin{array}{l} P_{7s} = 10 \text{ kPa} \\ s_{7s} = s_5 \end{array}\right\} \begin{array}{l} x_{7s} = \dfrac{s_{7s} - s_f}{s_{fg}} = \dfrac{6.5995 - 0.6492}{7.4996} = 0.7934 \\ h_{7s} = h_f + x_{7s} h_{fg} = 191.81 + (0.7934)(2392.1) \\ \quad = 2089.7 \text{ kJ/kg} \end{array}$$

$$\eta_T = \frac{h_5 - h_7}{h_5 - h_{7s}} \longrightarrow \begin{aligned} h_7 &= h_5 - \eta_T (h_5 - h_{7s}) \\ &= 3375.1 - (0.80)(3375.1 - 2089.7) \\ &= 2346.8 \text{ kJ/kg} \end{aligned}$$

The fraction of steam extracted is determined from the steady-flow energy balance equation applied to the feedwater heaters. Noting that $\dot{Q} \cong \dot{W} \cong \Delta ke \cong \Delta pe \cong 0$,



$$\dot{E}_{in} - \dot{E}_{out} = \Delta\dot{E}_{system}^{\nearrow 0\,(steady)} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e \longrightarrow \dot{m}_6 h_6 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \longrightarrow y h_6 + (1-y) h_2 = 1(h_3)$$

where y is the fraction of steam extracted from the turbine ($= \dot{m}_6 / \dot{m}_3$). Solving for y,

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{640.09 - 192.33}{2798.3 - 192.33} = 0.1718$$

Then,

$$q_{in} = h_5 - h_4 = 3375.1 - 651.02 = 2724.1 \text{ kJ/kg}$$

$$q_{out} = (1-y)(h_7 - h_1) = (1 - 0.1718)(2346.8 - 191.81) = 1784.7 \text{ kJ/kg}$$

$$w_{net} = q_{in} - q_{out} = 2724.1 - 1784.7 = 939.4 \text{ kJ/kg}$$

and

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{150{,}000 \text{ kJ/s}}{939.4 \text{ kJ/kg}} = \mathbf{159.7\ kg/s}$$

(*b*) The thermal efficiency is determined from

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1784.7 \text{ kJ/kg}}{2724.1 \text{ kJ/kg}} = \mathbf{34.5\%}$$

Also,

$$\left.\begin{array}{l} P_6 = 0.5 \text{ MPa} \\ h_6 = 2798.3 \text{ kJ/kg} \end{array}\right\} s_6 = 6.9453 \text{ kJ/kg} \cdot \text{K}$$

$$s_3 = s_{f\,@\,0.5\text{ MPa}} = 1.8604 \text{ kJ/kg} \cdot \text{K}$$

$$s_2 = s_1 = s_{f\,@\,10\text{ kPa}} = 0.6492 \text{ kJ/kg} \cdot \text{K}$$

Then the irreversibility (or exergy destruction) associated with this regeneration process is

$$i_{\text{regen}} = T_0 s_{\text{gen}} = T_0 \left(\sum m_e s_e - \sum m_i s_i + \cancelto{0}{\frac{q_{\text{surr}}}{T_L}} \right) = T_0 \left[s_3 - y s_6 - (1-y) s_2 \right]$$

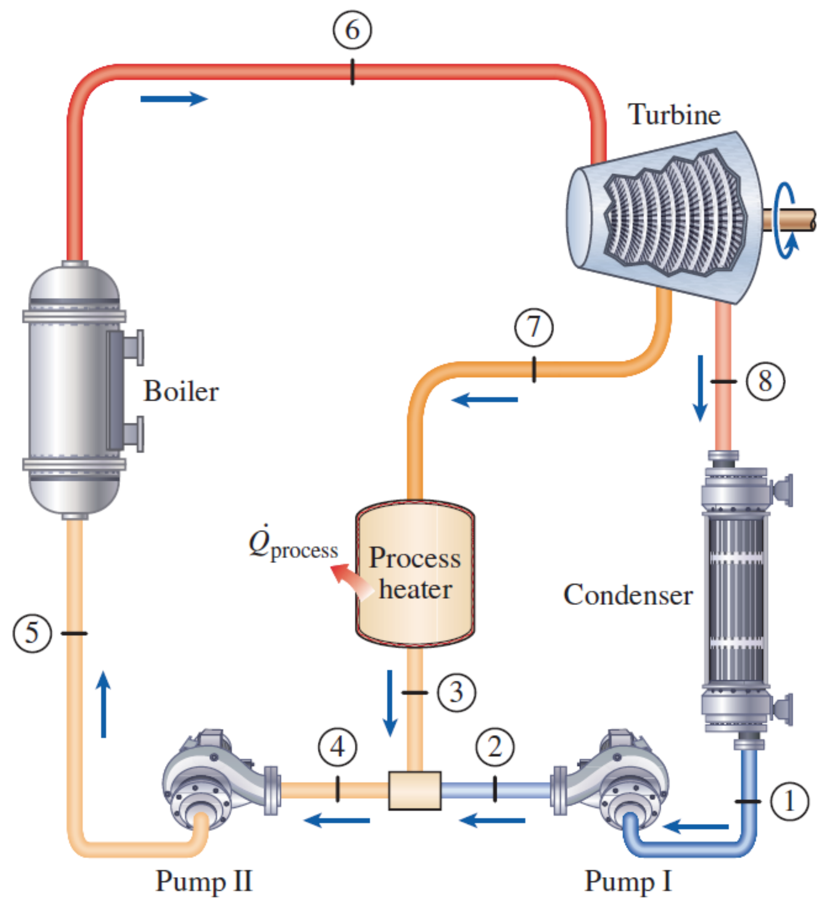
$$= (303 \text{ K})[1.8604 - (0.1718)(6.9453) - (1 - 0.1718)(0.6492)]$$

$$= \mathbf{39.25 \text{ kJ/kg}}$$

Cogeneration

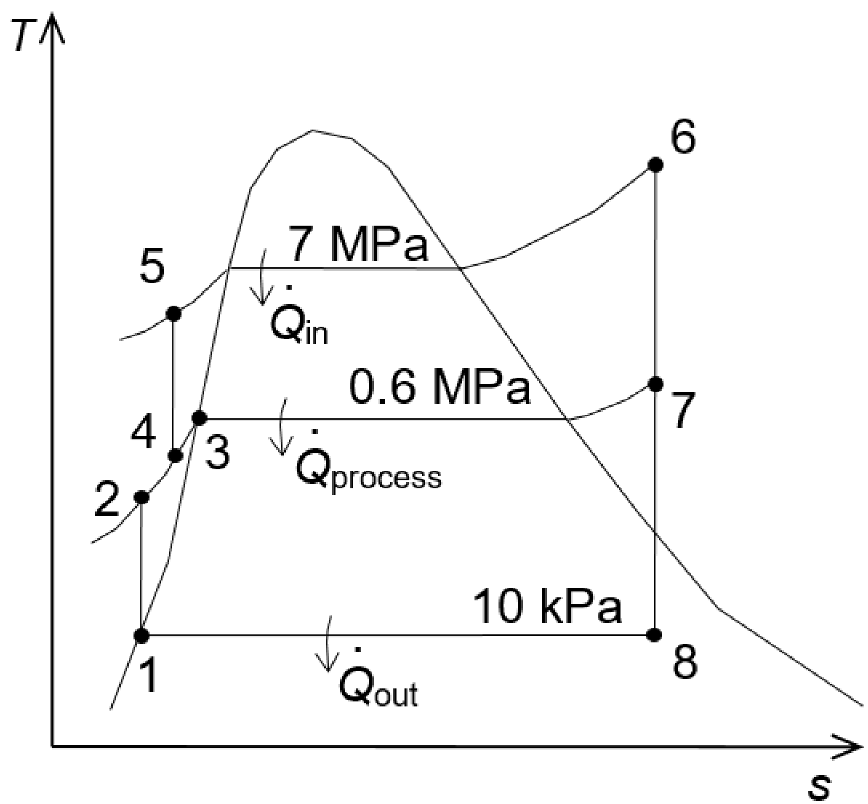
An ideal cogeneration steam plant is to generate power and 8600 kJ/s of process heat. Steam enters the turbine from the boiler at 7 MPa and 500°C. One-fourth of the steam is extracted from the turbine at 600-kPa pressure for process heating. The remainder of the steam continues to expand and exhausts to the condenser at 10 kPa.

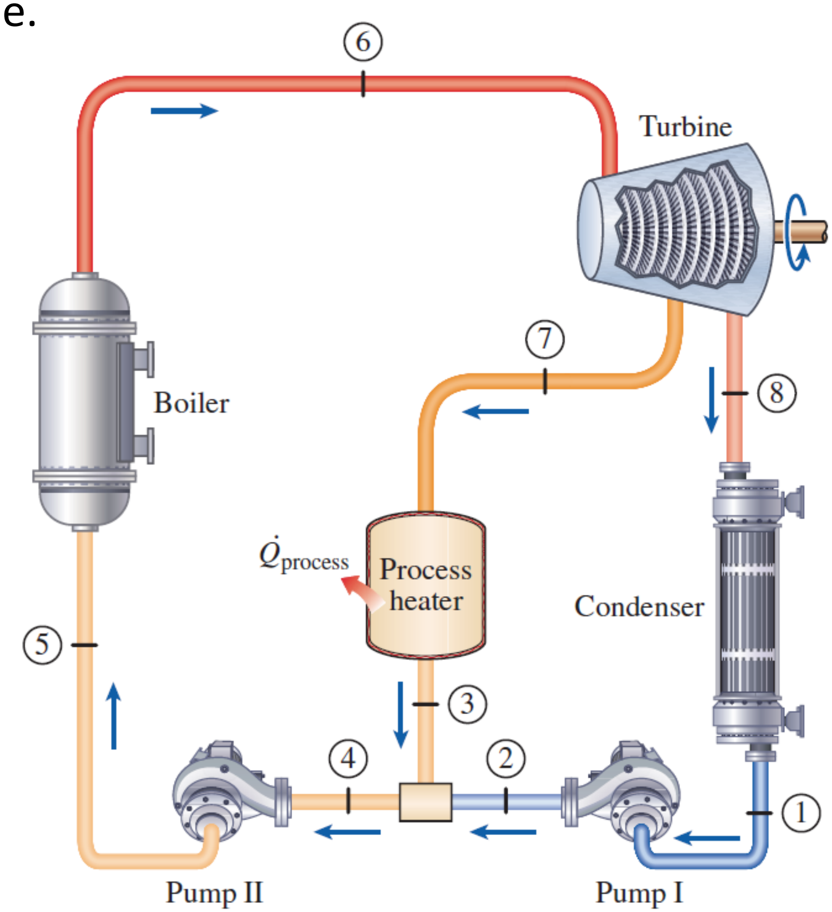
The steam extracted for the process heater is condensed in the heater and mixed with the feedwater at 600 kPa. The mixture is pumped to the boiler pressure of 7 MPa. Show the cycle on a T-s diagram with respect to saturation lines, and determine (a) the mass flow rate of steam that must be supplied by the boiler, (b) the net power produced by the plant, and (c) the utilization factor.



A cogeneration plant is to generate power and process heat. Part of the steam extracted from the turbine at a relatively high pressure is used for process heating. The mass flow rate of steam that must be supplied by the boiler, the net power produced, and the utilization factor of the plant are to be determined.

1 Steady operating conditions exist.
2 Kinetic and potential energy changes are negligible.





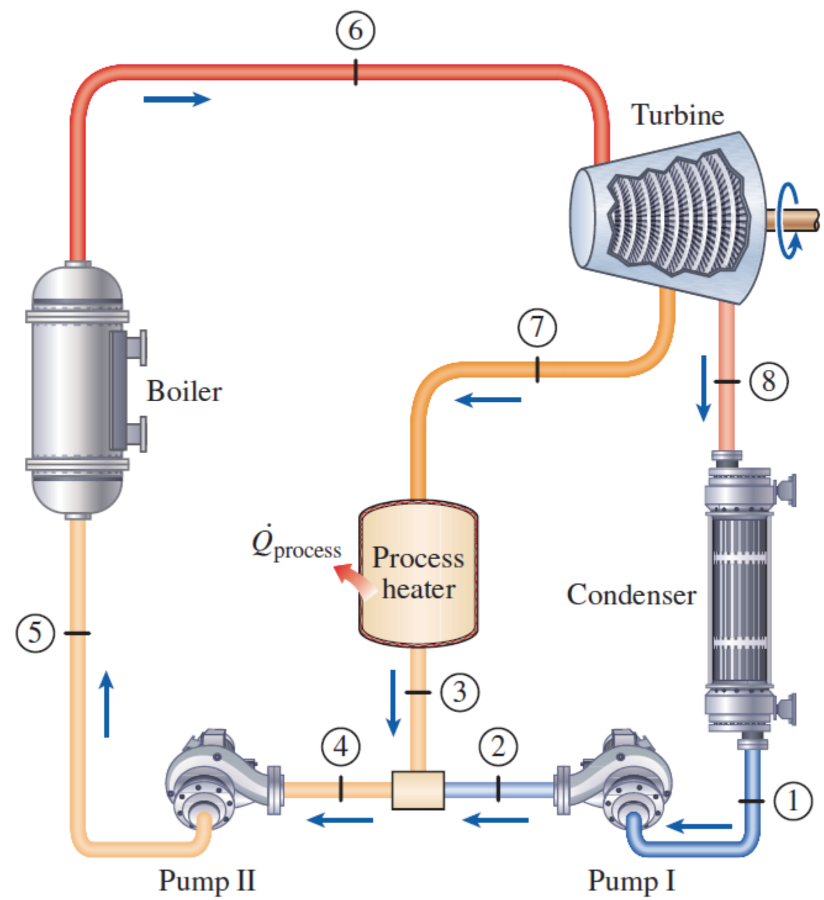
Analysis From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f\ @\ 10\ \text{kPa}} = 191.81\ \text{kJ/kg}$$
$$v_1 = v_{f\ @\ 10\ \text{kPa}} = 0.00101\ \text{m}^3/\text{kg}$$

$$\begin{aligned} w_{\text{pI, in}} &= v_1(P_2 - P_1) \\ &= (0.00101\ \text{m}^3/\text{kg})(600 - 10\ \text{kPa})\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right) \\ &= 0.596\ \ \text{kJ/kg} \end{aligned}$$

$$h_2 = h_1 + w_{\text{pI, in}} = 191.81 + 0.596 = 192.40\ \text{kJ/kg}$$

$$h_3 = h_{f\ @\ 0.6\ \text{MPa}} = 670.38\ \text{kJ/kg}$$



Mixing chamber:

$$\dot{m}_3 h_3 + \dot{m}_2 h_2 = \dot{m}_4 h_4$$

$$(0.25)(670.38 \text{ kJ/kg}) + (0.75)(192.40 \text{ kJ/kg})) = (1)h_4 \longrightarrow h_4 = 311.90 \text{ kJ/kg}$$

$$v_4 \cong v_{f\ @\ h_f = 311.90 \text{ kJ/kg}} = 0.001026 \text{ m}^3/\text{kg}$$

$$\begin{aligned} w_{p\text{II, in}} &= v_4 (P_5 - P_4) \\ &= (0.001026 \text{ m}^3/\text{kg})(7000 - 600 \text{ kPa})\left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right) \\ &= 6.563 \text{ kJ/kg} \end{aligned}$$

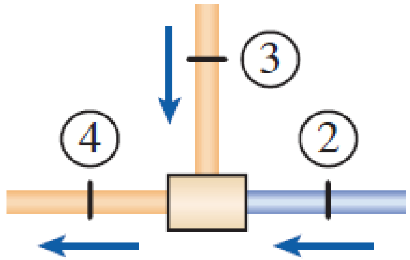
$$h_5 = h_4 + w_{p\text{II, in}} = 311.90 + 6.563 = 318.47 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_6 = 7 \text{ MPa} \\ T_6 = 500°\text{C} \end{array}\right\} \begin{array}{l} h_6 = 3411.4 \text{ kJ/kg} \\ s_6 = 6.8000 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\left.\begin{array}{l} P_7 = 0.6 \text{ MPa} \\ s_7 = s_6 \end{array}\right\} h_7 = 2773.9 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_8 = 10 \text{ kPa} \\ s_8 = s_6 \end{array}\right\} h_8 = 2153.6 \text{ kJ/kg}$$

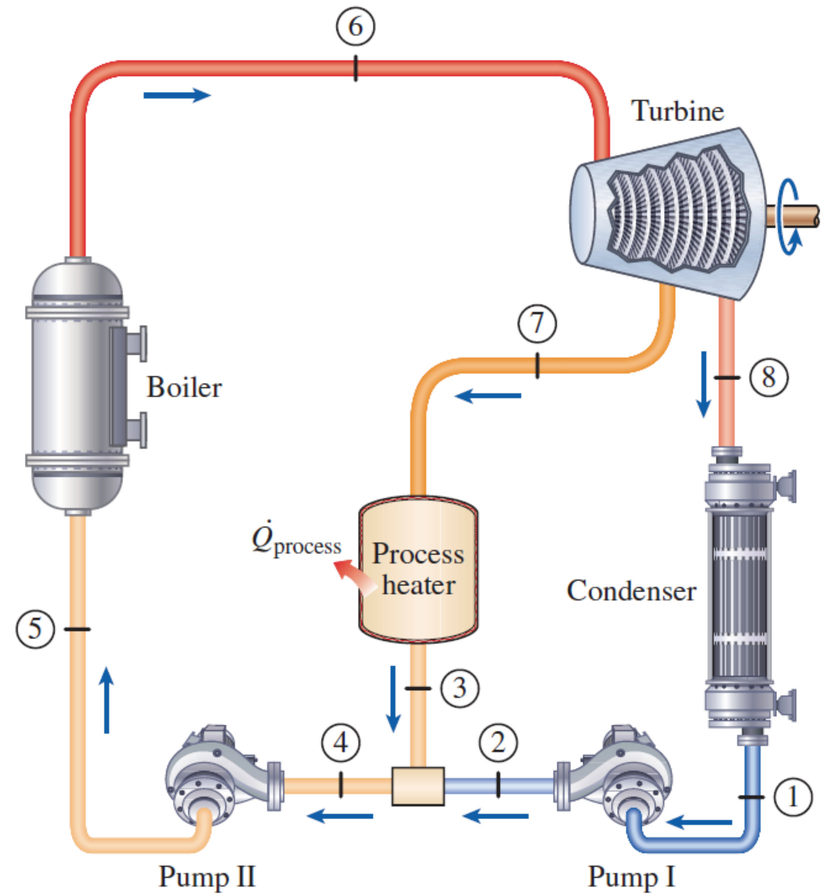
$$\begin{aligned} \dot{Q}_{\text{process}} &= \dot{m}_7 (h_7 - h_3) \\ 8600 \text{ kJ/s} &= \dot{m}_7 (2773.9 - 670.38) \text{ kJ/kg} \\ \dot{m}_7 &= 4.088 \text{ kg/s} \end{aligned}$$



$$\dot{Q}_{\text{process}} = \dot{m}_7 \left(h_7 - h_3\right)$$
$$8600 \text{ kJ/s} = \dot{m}_7 \left(2773.9 - 670.38\right) \text{ kJ/kg}$$
$$\dot{m}_7 = 4.088 \text{ kg/s}$$

This is one-fourth of the mass flowing through the boiler. Thus, the mass flow rate of steam that must be supplied by the boiler becomes

$$\dot{m}_6 = 4\dot{m}_7 = 4(4.088 \text{ kg/s}) = \mathbf{16.35\ kg/s}$$



(*b*) Cycle analysis:

$$\begin{aligned}\dot{W}_{T,out} &= \dot{m}_7(h_6-h_7)+\dot{m}_8(h_6-h_8)\\ &= (4.088\text{ kg/s})(3411.4-2773.9)\text{ kJ/kg}+(16.35\text{-}4.088\text{ kg/s})(3411.4-2153.6)\text{ kJ/kg}\\ &= 18{,}033\text{ kW}\end{aligned}$$

$$\begin{aligned}\dot{W}_{p,\,in} &= \dot{m}_1 w_{pI,\,in}+\dot{m}_4 w_{pII,\,in}\\ &= (16.35\text{-}4.088\text{ kg/s})(0.596\text{ kJ/kg})+(16.35\text{ kg/s})(6.563\text{ kJ/kg})=114.6\text{ kW}\end{aligned}$$

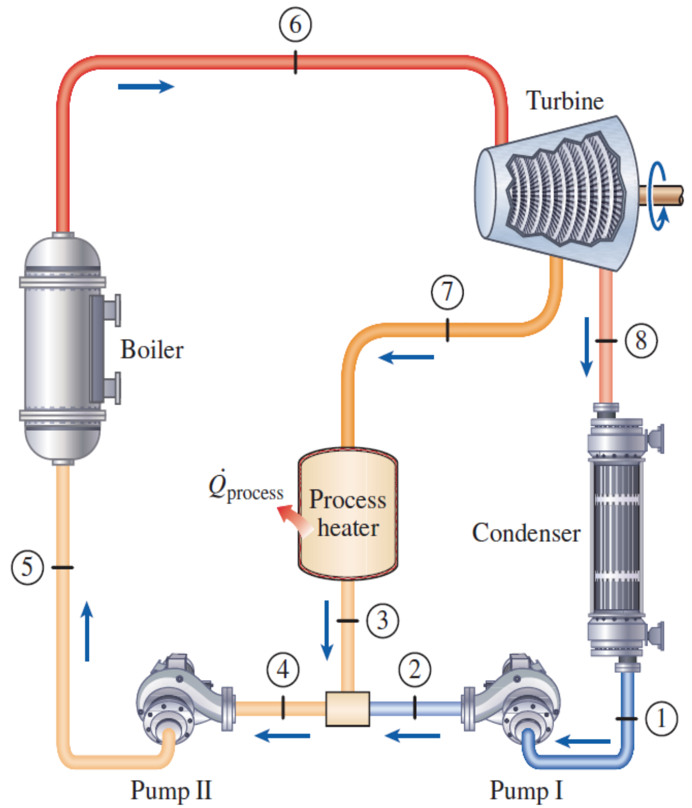
$$\dot{W}_{net}=\dot{W}_{T,out}-\dot{W}_{p,\,in}=18{,}033-115=\mathbf{17{,}919\ kW}$$

(*c*) Then,

$$\dot{Q}_{in}=\dot{m}_5(h_6-h_5)=(16.35\text{ kg/s})(3411.4-318.46)=50{,}581\text{ kW}$$

and

$$\varepsilon_u=\frac{\dot{W}_{net}+\dot{Q}_{process}}{\dot{Q}_{in}}=\frac{17{,}919+8600}{50{,}581}=0.524=\mathbf{52.4\%}$$



The gas-turbine portion of a combined gas-steam power plant has a pressure ratio of 16. Air enters the compressor at 300 K at a rate of 14 kg/s and is heated to 1500 K in the combustion chamber. The combustion gases leaving the gas turbine are used to heat the steam to 400 ºC at 10 MPa in a heat exchanger. The combustion gases leave the heat exchanger at 420 K. The steam leaving the turbine is condensed at 15 kPa. Assume all the compression and expansion processes to be isentropic. ($T_0 = 293\ K, T_\mathrm{b} = 300\ K\ and\ T_s = 2200\ K$). For air, assume constant specific heats at room temperature (c_p=1.005 kJ/kg·K and k=1.4).

$$h_{f_{@\ P=15\ kPa}} = 225.94\ kJ/kg$$

$$v_{f_{@\ P=15\ kPa}} = 0.001014\ m^3/kg$$

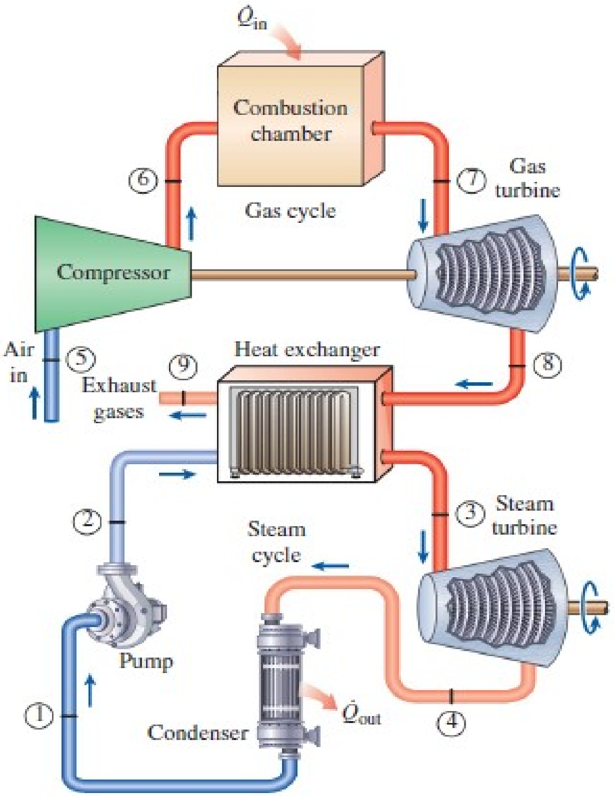
$$h_{@\ P=10\ MPa\ and\ T=400°C} = 3097.0\ kJ/kg$$

$$s_{@\ P=10\ MPa\ and\ T=400°C} = 6.2141\ kJ/kg \cdot K$$

$$s_{f_{@\ P=15\ kPa}} = 0.7549\ kJ/kg \cdot K$$

$$s_{fg_{@\ P=15\ \mathrm{kPa}}} = 7.2522\ kJ/kg \cdot K$$

$$h_{fg_{@\ P=15\ \mathrm{kPa}}} = 2372.3\ kJ/kg$$



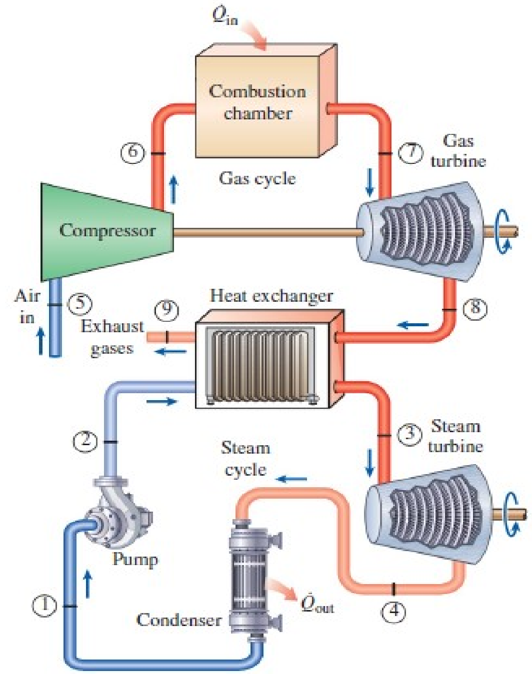
a) Draw the cycle T-s diagram.
b) Write all mass, energy, entropy and exergy balance equations for each device.
c) Determine the mass flow rate of the steam,
d) Determine the net power output,
e) Find the entropy generation in the combustion chamber
f) Find the energy and exergy efficiencies of the combined cycle
g) Find the exergy destruction in the conderser.

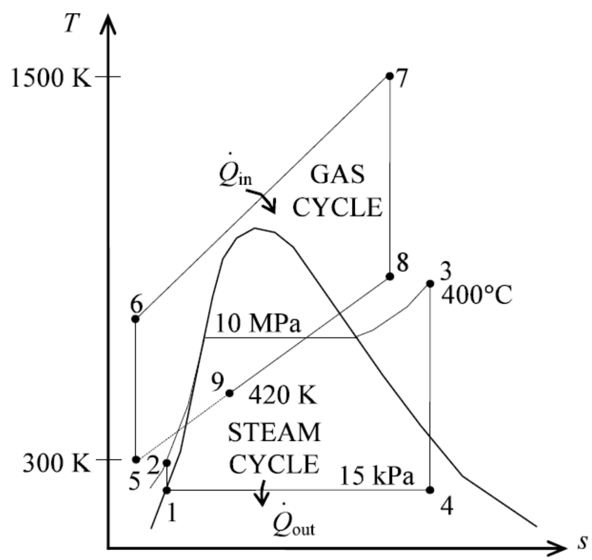
1-2 Isentropic compression in the pump

$$mBE: \quad \dot{m}_1 = \dot{m}_2 = \dot{m}_s$$
$$EBE: \dot{m}_s h_1 + \dot{W}_{pump} = \dot{m}_s h_2$$
$$h_1 + w_{pump} = h_2$$
$$w_{pump} = v(P_2 - P_1)$$
$$EnBE: \dot{m}_s s_1 + \dot{S}_{gen} = \dot{m}_s s_2$$
$$s_1 + s_{gen} = s_2$$
$$ExBE: \dot{m}_s ex_1 + \dot{W}_c = \dot{m}_s ex_2 + \dot{Ex}_D$$
$$ex_1 + w_C = ex_2 + ex_D$$

2-3 Constant-pressure heat addition in the heat exchanger

$$mBE: \quad \dot{m}_2 = \dot{m}_3 = \dot{m}_s$$
$$EBE: \dot{m}_s h_2 + \dot{Q}_{in} = \dot{m}_s h_3$$
$$h_2 + q_{in} = h_3$$
$$EnBE: \dot{m}_s s_2 + \frac{Q_{in}}{T_s} + \dot{S}_{gen} = \dot{m}_s s_3$$
$$s_2 + \frac{q_{in}}{T_s} + s_{gen} = s_3$$
$$ExBE: \dot{m}_s ex_2 + \dot{Ex}^{Q_{in}} = \dot{m}_s ex_3 + \dot{Ex}_D$$
$$ex_2 + ex^{Q_{in}} = ex_3 + ex_D$$





3-4 Isentropic expansion in the steam turbine

$$mBE: \dot{m}_3 = \dot{m}_4 = \dot{m}_s$$
$$EBE: \dot{m}_s h_3 = \dot{m}_s h_4 + \dot{W}_T$$
$$h_3 = h_4 + w_T$$
$$EnBE: \dot{m}_s s_3 + \dot{S}_{gen} = \dot{m}_s s_4$$
$$s_3 + s_{gen} = s_4$$
$$ExBE: \dot{m}_s ex_3 = \dot{m}_s ex_4 + \dot{W}_T + \dot{Ex}_D$$
$$ex_3 = ex_4 + w_T + ex_D$$

4-1 Constant-pressure heat rejection in the condenser

$$mBE: \dot{m}_4 = \dot{m}_1$$
$$EBE: \dot{m}_s h_4 = \dot{m}_s h_1 + \dot{Q}_{out}$$
$$h_4 = h_1 + q_{out}$$
$$EnBE: \dot{m}_s s_4 + \dot{S}_{gen} = \dot{m}_s s_1 + \frac{\dot{Q}_{out}}{T_b}$$
$$s_4 + s_{gen} = s_1 + \frac{q_{out}}{T_b}$$
$$ExBE: \dot{m}_s ex_4 = \dot{m}_s ex_1 + \dot{Ex}^{Q_{out}} + \dot{Ex}_D$$
$$ex_4 = ex_1 + ex^{Q_{out}} + ex_D$$

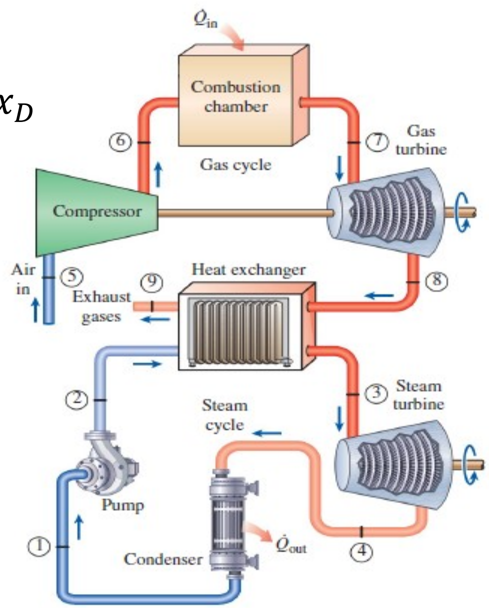
5-6 Isentropic compression in the compressor

$$mBE: \quad \dot{m}_5 = \dot{m}_6 = \dot{m}_{air}$$

$$EBE: \dot{m}_{air} h_5 + \dot{W}_c = \dot{m}_{air} h_6$$
$$h_5 + w_C = h_6$$

$$EnBE: \dot{m}_{air} s_5 + \dot{S}_{gen} = \dot{m}_{air} s_6$$
$$s_5 + s_{gen} = s_6$$

$$ExBE: \dot{m}_{air} ex_5 + \dot{W}_c = \dot{m}_{air} ex_6 + \dot{Ex}_D$$
$$ex_5 + w_C = ex_6 + ex_D$$



6-7 Constant-pressure heat addition in the combustion chamber

$$mBE: \quad \dot{m}_6 = \dot{m}_7 = \dot{m}_{air}$$

$$EBE: \dot{m}_{air} h_6 + \dot{Q}_{in} = \dot{m}_{air} h_7$$
$$h_6 + q_{in} = h_7$$

$$EnBE: \dot{m}_{air} s_6 + \frac{Q_{in}}{T_s} + \dot{S}_{gen} = \dot{m}_{air} s_7$$
$$s_6 + \frac{q_{in}}{T_s} + s_{gen} = s_7$$

$$ExBE: \dot{m}_{air} ex_6 + \dot{Ex}^{Q_{in}} = \dot{m}_{air} ex_7 + \dot{Ex}_D$$
$$ex_6 + ex^{Q_{in}} = ex_7 + ex_D$$

7-8 Isentropic expansion in the gas turbine

$$mBE: \dot{m}_7 = \dot{m}_8 = \dot{m}_{air}$$

$$EBE: \dot{m}_{air} h_7 = \dot{m}_{air} h_8 + \dot{W}_T$$
$$h_7 = h_8 + w_T$$

$$EnBE: \dot{m}_{air} s_7 + \dot{S}_{gen} = \dot{m}_{air} s_8$$
$$s_7 + s_{gen} = s_8$$

$$ExBE: \dot{m}_{air} ex_7 = \dot{m}_{air} ex_8 + \dot{W}_T + \dot{Ex}_D$$
$$ex_7 = ex_8 + w_T + ex_D$$

8-9 Constant-pressure heat rejection in the heat exchanger

$$mBE: \dot{m}_8 = \dot{m}_9 = \dot{m}_{air}$$

$$EBE: \dot{m}_{air} h_8 = \dot{m}_{air} h_9 + \dot{Q}_{out}$$
$$h_8 = h_9 + q_{out}$$

$$EnBE: \dot{m}_{air} s_8 + \dot{S}_{gen} = \dot{m}_{air} s_9 + \frac{\dot{Q}_{out}}{T_b}$$
$$s_8 + s_{gen} = s_9 + \frac{q_{out}}{T_b}$$

$$ExBE: \dot{m}_{air} ex_8 = \dot{m}_{air} ex_9 + \dot{Ex}^{Q_{out}} + \dot{Ex}_D$$
$$ex_8 = ex_9 + ex^{Q_{out}} + ex_D$$

A combined gas-steam power cycle is considered. The topping cycle is a gas-turbine cycle and the bottoming cycle is a simple ideal Rankine cycle. The mass flow rate of the steam, the net power output, and the thermal efficiency of the combined cycle are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4 (Table A-2).

Analysi (*c*) The analysis of gas cycle yields

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{(k-1)/k} = (300\text{ K})(16)^{0.4/1.4} = 662.5\text{ K}$$

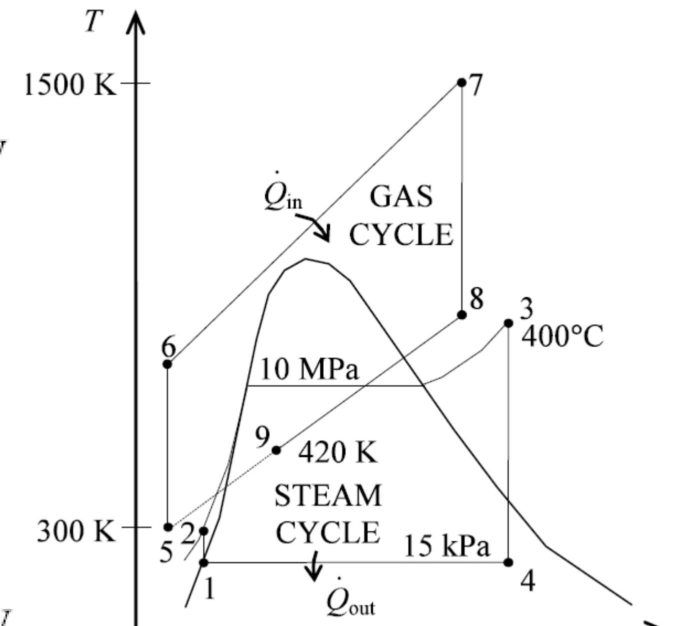
$$\dot{Q}_{\text{in}} = \dot{m}_{\text{air}}(h_7 - h_6) = \dot{m}_{\text{air}} c_p (T_7 - T_6)$$
$$= (14\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(1500 - 662.5)\text{ K} = 11{,}784\text{ kW}$$

$$\dot{W}_{C,\text{gas}} = \dot{m}_{\text{air}}(h_6 - h_5) = \dot{m}_{\text{air}} c_p (T_6 - T_5)$$
$$= (14\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(662.5 - 300)\text{ K} = 5100\text{ kW}$$

$$T_8 = T_7\left(\frac{P_8}{P_7}\right)^{(k-1)/k} = (1500\text{K})\left(\frac{1}{16}\right)^{0.4/1.4} = 679.3\text{ K}$$

$$\dot{W}_{T,\text{gas}} = \dot{m}_{\text{air}}(h_7 - h_8) = \dot{m}_{\text{air}} c_p (T_7 - T_8)$$
$$= (14\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(1500 - 679.3)\text{ K} = 11{,}547\text{ kW}$$

$$\dot{W}_{\text{net,gas}} = \dot{W}_{T,\text{gas}} - \dot{W}_{C,\text{gas}} = 11{,}547 - 5{,}100 = 6447\text{ kW}$$



From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f\,@\,15\,\text{kPa}} = 225.94\ \text{kJ/kg}$$
$$v_1 = v_{f\,@\,15\,\text{kPa}} = 0.001014\ \ \text{m}^3/\text{kg}$$

$$w_{\text{pI,in}} = v_1(P_2 - P_1) = \left(0.001014\ \ \text{m}^3/\text{kg}\right)\left(10{,}000 - 15\ \text{kPa}\right)\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right) = 10.12\ \text{kJ/kg}$$

$$h_2 = h_1 + w_{\text{pI,in}} = 225.94 + 10.13 = 236.06\ \text{kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 10\ \text{MPa} \\ T_3 = 400°\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3097.0\ \text{kJ/kg} \\ s_3 = 6.2141\ \text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_4 = 15\ \text{kPa} \\ s_4 = s_3 \end{array}\right\} \begin{array}{l} x_4 = \dfrac{s_4 - s_f}{s_{fg}} = \dfrac{6.2141 - 0.7549}{7.2522} = 0.7528 \\ h_4 = h_f + x_4 h_{fg} = 225.94 + (0.7528)(2372.3) = 2011.8\ \text{kJ/kg} \end{array}$$

Noting that $\dot{Q} \cong \dot{W} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0$ for the heat exchanger, the steady-flow energy balance equation yields

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta\dot{E}_{\text{system}}^{\nearrow 0\,\text{(steady)}} = 0 \longrightarrow \dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e \longrightarrow \dot{m}_s\left(h_3 - h_2\right) = \dot{m}_{\text{air}}\left(h_8 - h_9\right)$$

$$\dot{m}_s = \frac{h_8 - h_9}{h_3 - h_2}\dot{m}_{\text{air}} = \frac{c_p\left(T_8 - T_9\right)}{h_3 - h_2}\dot{m}_{\text{air}} = \frac{(1.005\text{ kJ/kg}\cdot\text{K})(679.3 - 420)\text{ K}}{(3097.0 - 236.06)\text{ kJ/kg}}(14\text{ kg/s}) = \mathbf{1.275\ \ kg/s}$$

(*d*)

$$\dot{W}_{\text{T,steam}} = \dot{m}_s\left(h_3 - h_4\right) = (1.275\text{ kg/s})(3097.0 - 2011.5)\text{ kJ/kg} = 1384\text{ kW}$$

$$\dot{W}_{\text{p,steam}} = \dot{m}_s w_p = (1.275\text{ kg/s})(10.12\text{ kJ/kg}) = 12.9\text{ kW}$$

$$\dot{W}_{\text{net,steam}} = \dot{W}_{\text{T,steam}} - \dot{W}_{\text{p,steam}} = 1384 - 12.9 = 1371\text{ kW}$$

and

$$\dot{W}_{\text{net}} = \dot{W}_{\text{net,steam}} + \dot{W}_{\text{net,gas}} = 1371 + 6448 = \mathbf{7819\,kW}$$

e) $\boldsymbol{EnBE\ for\ combustion\ chamber}$;

$$\dot{m}_{air}s_6 + \frac{\dot{Q}_{in}}{T_s} + \dot{S}_{gen} = \dot{m}_{air}s_7$$

$$s_6 + \frac{q_{in}}{T_s} + s_{gen} = s_7$$

$$s_7 - s_6 = c_{p,av}\left(\frac{T_7}{T_6}\right) - R\ ln\left(\frac{P_7}{P_6}\right) = 1.005 \times \left(\frac{1500}{662.5}\right) - 0 = 2.276\ kJ/kgK$$

$$\dot{S}_{gen,CC} = \frac{14kg}{s} \times \frac{2.276\ kJ}{kgK} - \frac{11{,}784kW}{2200K} = 26.508\ kW/K$$

f) $\boldsymbol{Energy\ Efficiency}$

$$\eta_{\mathrm{th}} = \frac{\dot{W}_{\mathrm{net}}}{\dot{Q}_{\mathrm{in}}} = \frac{7819\ \mathrm{kW}}{11{,}784\ \mathrm{kW}} = \mathbf{66.4\%}$$

$\boldsymbol{Exergy\ Efficiency}$

$$\dot{Ex}^{Q_{in}} = \left(1 - \left(\frac{T_0}{T_s}\right)\right)\dot{Q}_{in} = \left(1 - \left(\frac{293}{2200}\right)\right)11784\ kW = 10214\ kW$$

$$\eta_{ex} = \frac{\dot{W}_{net,GT} + \dot{W}_{net,ST}}{\dot{Ex}^{Q_{in}}} = \frac{6448\ kW + 1371\ kW}{10214\ kW} = 76.55\%$$

g) $\boldsymbol{Exergy\ destruction\ in\ the\ condenser}$

$$\dot{E}x_D = T_o.\dot{S}_{gen}$$

$$EnBE\text{:}\ \dot{m}_s s_4 + \dot{S}_{gen} = \dot{m}_s s_1 + \frac{\dot{Q}_{out}}{T_b}$$

To = 293 K , Tb = 300 K , s4=s3=6.2141 kj/kgK , s1=sf@15kPa = 0.7549 kj/kgK

$\dot{Q}_{out} = \dot{m}_s(h_4 - h_1)$ from EBE

$$\dot{Q}_{out} = 1.275\,\frac{kg}{s}\left(2011.8\,\frac{kJ}{kg} - 225.94\,\frac{kJ}{kg}\right) = 2277.3\ kW$$

$$\dot{S}_{gen,cond} = 1.275\,\frac{kg}{s}\left(0.7549\,\frac{kJ}{kgK} - 6.2141\,\frac{kJ}{kgK}\right) + \frac{2277.3\ kW}{300\ K} = 0.63\,\frac{kW}{K}$$

$$\dot{E}x_{D,cond} = 293\ K * 0.63\,\frac{kW}{K} = 184.6\ kW$$