MAT1072 MATEMATİK 2

ÇIKMIŞ SINAV SORULARI - 1 ÇÖZÜMLERİ

1) $$\lim_{n\to\infty}\left(\frac{n^{5/2}}{2n^2+1}\cdot \sin\frac{1}{\sqrt{n}}\right)=\lim_{n\to\infty}\left(\frac{n^2.\sqrt{n}}{2n^2+1}\cdot \sin\frac{1}{\sqrt{n}}\right)$$

$$=\lim_{n\to\infty}\left(\frac{n^2}{2n^2+1}\cdot\frac{\sin\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}\right)$$

$$=\frac{1}{2}\cdot 1=\frac{1}{2}$$

2) $$S_n=\sum_{k=1}^{n}\frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k^2+k}}=\sum_{k=1}^{n}\frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k}.\sqrt{k+1}}$$

$$=\sum_{k=1}^{n}\left(\frac{1}{\sqrt{k}}-\frac{1}{\sqrt{k+1}}\right)$$

$$=\left(1-\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right)+\dots+\left(\frac{1}{\sqrt{n-1}}-\frac{1}{\sqrt{n}}\right)+\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$$

$$=1-\frac{1}{\sqrt{n+1}}$$

$$\sum_{n=1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n^2+n}}=\lim_{n\to\infty}S_n=\lim_{n\to\infty}\left(1-\frac{1}{\sqrt{n+1}}\right)=1$$

3) $a_n=n.\ln\left(1-\frac{1}{n}\right)$; $\forall n\geq 2$ için $\left(1-\frac{1}{n}\right)^n>0$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\ln\left(1-\frac{1}{n}\right)^n=\ln\left(\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n\right)$$

$$=\ln e^{-1}=-1\neq 0$$

olduğundan n. terim testine göre verilen seri ıraksaktır.

4) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{2}\left(1+\frac{1}{n}\right) = \frac{1}{2} < 1$

oldugundan, Oran testine göre verilen seri yakınsaktır.

5) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)!\,(x-1)^{n+1}}{n!\,(x-1)^n}\right| = \lim_{n\to\infty} (n+1).\,|x-1| = \infty > 1 \quad (x \neq 1)$

$x \neq 1$ için verilen seri ıraksaktır.

Sadece $x=1$ için yakınsaktır.

6) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(x+3)^{n+1}}{4^{n+2}} \cdot \frac{4^{n+1}}{(x+3)^n}\right| = \lim_{n\to\infty} \frac{|x+3|}{4} = \frac{|x+3|}{4} < 1$

$-4 < x+3 < 4$

$-7 < x < 1$

$x=1$ için $\sum_{n=0}^{\infty} \frac{4^n}{4^{n+1}} = \sum_{n=0}^{\infty} \frac{1}{4}$ ıraksak

$x=-7$ için $\sum_{n=0}^{\infty} \frac{(-1)^n}{4}$ ıraksak

Yakınsaklık aralığı $(-7, 1)$

$\sum_{n=0}^{\infty} \frac{1}{4} \cdot \left(\frac{x+3}{4}\right)^n \qquad a = \frac{1}{4}, \; r = \frac{x+3}{4}$

$$= \frac{a}{1-r} = \frac{\frac{1}{4}}{1-\frac{x+3}{4}} = \frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{4-x-3} = \frac{1}{1-x} \;//$$

7) $\sum_{k=n}^{\infty} 3^{n-k} = \underset{k=n}{1} + \underset{k=n+1}{\frac{1}{3}} + \underset{k=n+2}{\frac{1}{3^2}} + \underset{k=n+3}{\frac{1}{3^3}} + \cdots = \frac{1}{1-\frac{1}{3}} = \frac{3}{2} \;//$

$a = 1$

$r = \frac{1}{3}$

8) i) $\forall n \geq 1$ için $3n \leq 3n + 3(n-1) = 6n-3$ olduğundan,

$$3n \leq 6n-3$$
$$3n+2 \leq 6n-1$$
$$\frac{6n-1}{3n+2} \geq 1$$
$$\ln\left(\frac{6n-1}{3n+2}\right) \geq \ln 1 = 0 \;\;//$$

ii) $\forall n \geq 1$ için $6n < 6n+5 = 2(3n+2)+1$ olduğundan,

$$6n-1 < 2(3n+2)$$
$$\frac{6n-1}{3n+2} < 2$$
$$\ln \frac{6n-1}{3n+2} < \ln 2 \;\;//$$

9) $\sum_{k=3}^{\infty} \left| (-1)^k \frac{\ln k+1}{\sqrt{k}} \right| = \sum_{k=3}^{\infty} \frac{\ln k+1}{\sqrt{k}}$ (mutlak yakınsak mı?)

$\forall k \geq 3$ için

$$1 < \ln k$$
$$2 < \ln k + 1$$
$$\frac{2}{\sqrt{k}} < \frac{1+\ln k}{\sqrt{k}}$$

$\sum_{k=3}^{\infty} \frac{2}{\sqrt{k}}$ serisi p-testine göre ıraksaktır.

Mukayese testine göre de $\sum_{k=3}^{\infty} \frac{2}{\sqrt{k}}$ ıraksak olduğundan $\sum_{k=3}^{\infty} \frac{\ln k+1}{\sqrt{k}}$ ıraksaktır.
Yani mutlak yakınsak değildir.
Alterne seri testine göre;

i) $a_k = \frac{\ln k+1}{\sqrt{k}} > 0 \quad (k \geq 3)$

ii) $a_{k+1} \leq a_k$? (Azalan mı?) $\quad a_k = f(k)$ olsun. $f(x) = \frac{\ln x + 1}{\sqrt{x}} \quad (x \geq 3)$

$$f'(x) = \frac{1-\ln x}{2x^{3/2}} < 0 \quad (\ln x > 1)$$

Azalandır.

Yani $\{a_k\}$ dizisi azalandır.

iii) $\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{\ln k+1}{\sqrt{k}} \overset{L'H}{=} \lim_{k\to\infty} \frac{1/k}{1/2\sqrt{k}} = \lim_{k\to\infty} \frac{2}{\sqrt{k}} = 0$

şartları sağlandığından,
$\sum_{k=3}^{\infty} (-1)^k \frac{\ln k+1}{\sqrt{k}}$ serisi Şartlı Yakınsaktır.

10) $|x|<1$ için $\dfrac{1}{1-x}=\sum\limits_{n=0}^{\infty} x^n$

$x\to -x^2$ yazalım : $\dfrac{1}{1-(-x^2)}=\sum\limits_{n=0}^{\infty}(-x^2)^n \qquad (|-x^2|<1)$

$$\frac{1}{1+x^2}=\sum_{n=0}^{\infty}(-1)^n . x^{2n}$$

Her tarafı x ile çarpalım : $\dfrac{x}{1+x^2}=\sum\limits_{n=0}^{\infty}(-1)^n . x^{2n+1}$

Her tarafın integralini alalım: $\displaystyle\int_0^x \frac{t}{1+t^2}\,dt=\sum_{n=0}^{\infty}(-1)^n\int_0^x t^{2n+1}\,dt$

$$\frac{1}{2}\ln(1+x^2)=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+2}}{2n+2}$$

$$x.\ln(1+x^2)=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+3}}{\cancel{2}.(n+1)}.\cancel{2}$$

$$g(x)=x\ln(1+x^2)=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+3}}{n+1} \quad // \qquad (|x|<1)$$

$x=\pm 1$ için $\pm\sum\limits_{n=0}^{\infty}\dfrac{(-1)^n}{n+1}$ alterne (Harmonik) seri olduğundan şartlı yakınsaktır.

Dolayısıyla yakınsaklık aralığı $[-1,1]$ dır.

11) $f(x)=\dfrac{1}{1-x}=\sum\limits_{k=0}^{\infty}x^k \; ; \; (|x|<1)$

$$f'(x)=\frac{1}{(1-x)^2}=\sum_{k=1}^{\infty}k.x^{k-1}, \qquad (|x|<1)$$

$$f''(x)=\frac{2}{(1-x)^3}=\sum_{k=2}^{\infty}k.(k-1)\,x^{k-2}, \quad (|x|<1)$$

$k=n+2$ alınırsa ;

$$\sum_{n=0}^{\infty}(n+2)(n+1)\,x^n=\frac{2}{(1-x)^3}$$

$$x=\frac{1}{2} \Rightarrow \sum_{n=0}^{\infty}\frac{(n+2)(n+1)}{2^n}=\frac{2}{\left(1-\frac{1}{2}\right)^3}=16 \; //$$

12) $\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{2^n \sqrt[3]{n^2+5}}$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x-3)^{n+1}}{2^{n+1}\sqrt[3]{(n+1)^2+5}} \cdot \frac{(-1)^n\, 2^n \sqrt[3]{n^2+5}}{(x-3)^n} \right| = \frac{1}{2}|x-3| < 1$$

$|x-3| < 2 \Rightarrow -2 < x-3 < 2 \Rightarrow \underline{1 < x < 5}$

$\underline{x=1}$ için ; $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2+5}}$ $\left\{ \begin{array}{l} \sum_{n=1}^{\infty} \frac{1}{n^{2/3}} \;;\; P = \frac{2}{3} < 1 \;\; \underline{\text{ıraksak}} \\ \lim_{n\to\infty} \frac{\frac{1}{\sqrt[3]{n^2+5}}}{\frac{1}{\sqrt[3]{n^2}}} = 1 \neq 0,\infty \quad \text{aynı karakterdeler.} \end{array} \right.$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2+5}}$ ıraksak

$\underline{x=5}$ için; $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2+5}}$ mutlak yakınsak değil.

Alterne seri testine göre;

$*\; a_n = \frac{1}{\sqrt[3]{n^2+5}} > 0$, $**\; a_{n+1} < a_n$, $***\; \lim_{n\to\infty} a_n = 0$

şartları sağlandığından seri şartlı yakınsaktır.

i) $1 < x < 5$ aralığında Mutlak yakınsak

ii) $x=5$ de Şartlı yakınsak

iii) $R \setminus (1,5]$ de Iraksak

13) i) $\sum_{n=1}^{\infty} \frac{n^2 . \sin\frac{1}{n}}{\sqrt{n^2+n+1}}$, $\sum_{n=1}^{\infty} \frac{1}{n}$ harmonik serisini seçelim.

$$\lim_{n\to\infty} \underbrace{\frac{n^2.}{\sqrt{n^2+n+1}}}_{\infty} \cdot \underbrace{\frac{\sin\frac{1}{n}}{\frac{1}{n}}}_{1} = \infty \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n}$$ ıraksak olduğundan verilen seri ıraksaktır.

ii) $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)}$

$f(x) = \frac{1}{x(1+\ln x)}$, $[1,\infty)$ da pozitif, sürekli ve azalan bir fonk.

$$\int_1^{\infty} \frac{dx}{x(1+\ln x)} = \lim_{R\to\infty} \int_1^R \frac{dx}{x(1+\ln x)} = \lim_{R\to\infty} \int_1^{1+\ln R} \frac{du}{u} = \lim_{R\to\infty} \ln u \Big|_1^{1+\ln R}$$

$1+\ln x = u$, $x=1 \Rightarrow u=1$

$\frac{dx}{x} = du$, $x=R \Rightarrow u = 1+\ln R$

$$= \lim_{R\to\infty} \left[\ln(1+\ln R) - \ln 1\right] = \infty \Rightarrow \text{ıraksak}$$

integral testine göre verilen seri ıraksaktır.

14) $$\sum_{n=1}^{\infty} \frac{2+(-1)^n}{2^n} = \sum_{n=1}^{\infty} \frac{1}{2^{n-1}} + \sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} + \sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^{n-1}$$

$\Downarrow$ $a=1, r=\frac{1}{2}$ $\quad$ $\Downarrow$ $a=-\frac{1}{2}$, $r=-\frac{1}{2}$

$$= \frac{1}{1-\frac{1}{2}} + \frac{-\frac{1}{2}}{1+\frac{1}{2}} = 2 - \frac{1}{3} = \frac{5}{3}$$ //

15) $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad , |x|<1$

$x \to -x^2$ yazalım.

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \quad , \quad |x|<1$$

integral alalım.

$$\arctan x + c = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad , |x|<1$$

$x=0 \Rightarrow c=0$ olur.

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad , \quad |x|<1$$

$x \to \sqrt{x}$ yazalım.

$$\arctan\sqrt{x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^n \sqrt{x}}{2n+1} \quad , \quad 0 \leq |\sqrt{x}| \leq 1$$

$\sqrt{x}$ ile çarpalım.

$$\sqrt{x} \cdot \arctan\sqrt{x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{2n+1} \quad , \quad 0 \leq x \leq 1$$

16) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\sin n}{n!}$

$-1 \leq \sin n \leq 1$

$0 \leq 1+\sin n \leq 2$

$0 \leq \dfrac{1+\sin n}{n!} \leq \dfrac{2}{n!}$

$n! = n(n-1)(n-2)\ldots 3.2.1 \geq 2.2.2\ldots 2.1 = 2^{n-1}$

$\dfrac{1}{n!} \leq \dfrac{1}{2^{n-1}} \Rightarrow \dfrac{2}{n!} \leq \dfrac{2}{2^{n-1}}$

$$0 \leq \frac{1+\sin n}{n!} \leq \frac{2}{2^{n-1}}$$

$\displaystyle\sum_{n=1}^{\infty} 2\cdot\left(\frac{1}{2}\right)^{n-1}$ geometrik seri $r=\frac{1}{2}<1$ olduğundan yakınsaktır.

Dolayısıyla $\displaystyle\sum_{n=1}^{\infty} \frac{1+\sin n}{n!}$ serisi de yakınsaktır.

17) $\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = \sum_{n=1}^{\infty} \frac{n+1-1}{(n+1)!} = \sum_{n=1}^{\infty} \left[\frac{n+1}{(n+1)!} - \frac{1}{(n+1)!}\right] = \sum_{n=1}^{\infty} \left[\frac{1}{n!} - \frac{1}{(n+1)!}\right]$

$S_k = \sum_{n=1}^{k} \frac{n}{(n+1)!} = \left(\frac{1}{1!} - \frac{1}{2!}\right) + \left(\frac{1}{2!} - \frac{1}{3!}\right) + \left(\frac{1}{3!} - \frac{1}{4!}\right) + \dots + \left(\frac{1}{(k-1)!} - \frac{1}{k!}\right) + \left(\frac{1}{k!} - \frac{1}{(k+1)!}\right)$

$S_k = 1 - \frac{1}{(k+1)!} \Rightarrow S = \lim_{k\to\infty} S_k = \lim_{k\to\infty} \left(1 - \frac{1}{(k+1)!}\right) = 1$ //

18) $f(x) = \ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$

$f'(x) = \frac{1}{1+x} + \frac{1}{1-x} = \sum_{n=0}^{\infty} (-x)^n + \sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} (-1)^n x^n + \sum_{n=0}^{\infty} x^n$

$= \sum_{n=0}^{\infty} \left[(-1)^n + 1\right] . x^n$ $\left[(-1)^n + 1 = \begin{cases} 2, & n \text{ çift} \\ 0, & n \text{ tek} \end{cases} \right]$

$f'(x) = \sum_{k=0}^{\infty} 2 . x^{2k}$

integralini alalım.

$f(x) = \int_0^x \left(\sum_{k=0}^{\infty} 2 . t^{2k} \right) dt = 2 \sum_{k=0}^{\infty} \int_0^x t^{2k} dt = 2 . \sum_{k=0}^{\infty} \frac{x^{2k+1}}{2k+1}$

$a_n = \frac{2 \; x^{2n+1}}{2n+1}$ → Genel terim

19) $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)! \, 9^n} = \sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{3^{2n} (2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{\pi}{3}\right)^{2n}}{(2n)!}$

$\Rightarrow \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

$x = \frac{\pi}{3}$ alınırsa,

$\sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{\pi}{3}\right)^{2n}}{(2n)!} = \cos \frac{\pi}{3} = \frac{1}{2}$ //

20) $\displaystyle\sum_{n=1}^{\infty} \ln\left(1+\frac{2}{n(n+3)}\right)$

$$\ln\left[\frac{n^2+3n+2}{n(n+3)}\right] = \ln[(n+1)(n+2)] - \ln[n(n+3)]$$
$$= \ln(n+1) + \ln(n+2) - \ln n - \ln(n+3)$$

$$\begin{aligned} S_n = & \cancel{\ln 2} + \ln 3 - \ln 1 - \cancel{\ln 4} \\ & + \cancel{\ln 3} + \cancel{\ln 4} - \cancel{\ln 2} - \cancel{\ln 5} \\ & + \ln 4 + \cancel{\ln 5} - \cancel{\ln 3} - \ln 6 \\ & \vdots \\ & + \ln(n+1) + \ln(\cancel{n+2}) - \cancel{\ln n} - \ln(n+3) \end{aligned}$$

$$S_n = \ln 3 + \ln\left(\frac{n+1}{n+3}\right) \qquad \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[\ln 3 + \ln\left(\frac{n+1}{n+3}\right)\right] = \ln 3 \;//$$

21) $\{a_n\}$ dizisi yakınsak ise $\lim\limits_{n\to\infty} a_n = A$ $(A \in \mathbb{R})$ olmalı.

$\{a_{2n+1}\}$, $\{a_n\}$ dizisinin alt dizisidir ve $\{a_n\}$ ile aynı sayıya yakınsar. Yani; $\lim\limits_{n\to\infty} a_{2n+1} = A$ dır.

$$\lim_{n\to\infty}\left(2a_n + 3a_{2n+1}\right) = \lim_{n\to\infty}\frac{5n+1}{2n+3} = \frac{5}{2}$$
$$2A + 3A = \frac{5}{2} \Rightarrow A = \frac{1}{2} \Rightarrow \lim_{n\to\infty} a_n = \frac{1}{2} \;//$$

22) $a_n = \ln e^n - \ln(e^n+1) = \ln\left(\dfrac{e^n}{e^n+1}\right) = \ln\left(1 - \dfrac{1}{e^n+1}\right)$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \ln\left(1 - \frac{1}{e^n+1}\right) = \ln\left(\lim_{n\to\infty}\left(1 - \frac{1}{e^n+1}\right)\right)$$
$$= \ln 1 = 0 \;//$$

23)

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{(\sqrt{n^2+n}-n)\cdot \cancel{a_n}}{\cancel{a_n}}=\lim_{n\to\infty}(\sqrt{n^2+n}-n)\cdot\frac{(\sqrt{n^2+n}+n)}{(\sqrt{n^2+n}+n)}$$

$$=\lim_{n\to\infty}\frac{\cancel{n^2}+n-\cancel{n^2}}{\sqrt{n^2+n}+n}=\lim_{n\to\infty}\frac{\cancel{n}}{\cancel{n}\left(\sqrt{1+\underbrace{\tfrac{1}{n}}_{0}}+1\right)}=\frac{1}{2}<1 \text{ yakınsak}$$

24)

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\frac{|x+1|^{n+1}}{2^{n+2}}\cdot\frac{2^{n+1}}{|x+1|^n}=\frac{|x+1|}{2}<1$$

$$-2<x+1<2 \Rightarrow -3<x<1$$

$\underline{x=1}$ için, $\sum_{n=0}^{\infty}\frac{2^n}{2^{n+1}}=\sum_{n=0}^{\infty}\frac{1}{2}$ → ıraksak

$\underline{x=-3}$ için, $\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}$, $\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}=\sum_{n=0}^{\infty}\frac{1}{2}\cdot\frac{1}{2^n}$ → Geometrik seri

$r=\frac{1}{2}<1$ yakınsak

dolayısıyla Mutlak yakınsak.

Yakınsaklık aralığı $[-3,1)$

$$\sum_{n=0}^{\infty}\left(\frac{x+1}{2}\right)^n\cdot\frac{1}{2} \to \text{Geometrik seri}$$

$$a=\frac{1}{2}\quad,\quad r=\frac{x+1}{2}$$

$$=\frac{\frac{1}{2}}{1-\frac{x+1}{2}}=\frac{2}{2(2-x-1)}=\frac{1}{1-x}\ //$$

25) $\sum_{n=1}^{\infty} (\sqrt{e})^{1-4n} = \sum_{n=1}^{\infty} e^{1/2} \cdot e^{-2n} = \sum_{n=1}^{\infty} e^{1/2} \cdot \left(\frac{1}{e^2}\right)^n$

$= \sum_{n=1}^{\infty} e^{1/2} \cdot \frac{1}{e^2} \left(\frac{1}{e^2}\right)^{n-1}$

$a = e^{-3/2}$, $r = \frac{1}{e^2}$

$\sum_{n=1}^{\infty} (\sqrt{e})^{1-4n} = \frac{e^{-3/2}}{1-\frac{1}{e^2}} = \frac{\sqrt{e}}{e^2-1}$ //

26) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right) = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} + \cdots$

$S_n = 1 - \cancel{\frac{1}{\sqrt{2}}} + \cancel{\frac{1}{\sqrt{2}}} - \cancel{\frac{1}{\sqrt{3}}} + \cancel{\frac{1}{\sqrt{3}}} - \cdots + \cancel{\frac{1}{\sqrt{n}}} - \frac{1}{\sqrt{n+1}} = 1 - \frac{1}{\sqrt{n+1}}$

$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right) = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(1 - \underbrace{\frac{1}{\sqrt{n+1}}}_{0}\right) = 1$ //

27) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

$\frac{2n+1}{n^2(n+1)^2} = \frac{2n+1+n^2-n^2}{n^2(n+1)^2} = \frac{\cancel{(n+1)^2}}{\cancel{(n+1)^2} \cdot n^2} - \frac{\cancel{n^2}}{\cancel{n^2}(n+1)^2}$

$= \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$S_n = 1 - \cancel{\frac{1}{2^2}} + \cancel{\frac{1}{2^2}} - \cancel{\frac{1}{3^2}} + \cancel{\frac{1}{3^2}} - \cancel{\frac{1}{4^2}} + \cdots + \cancel{\frac{1}{n^2}} - \frac{1}{(n+1)^2} = 1 - \frac{1}{(n+1)^2}$

$\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left[1 - \underbrace{\frac{1}{(n+1)^2}}_{0}\right] = 1$ //

28) $\sum_{n=1}^{\infty} \frac{\sin(3^{-n})}{1+3^n}$

$\sin(3^{-n}) \leq 1$

$\frac{\sin(3^{-n})}{1+3^n} \leq \frac{1}{1+3^n} < \frac{1}{3^n}$

$\sum_{n=1}^{\infty} \frac{1}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3} \cdot \left(\frac{1}{3}\right)^{n-1} \rightarrow$ Geometrik seri $r = \frac{1}{3} < 1 \Rightarrow$ Yakınsak

Dolayısıyla $\sum_{n=1}^{\infty} \frac{\sin(3^{-n})}{1+3^n}$ serisi yakınsaktır.

29)

$$\lim_{n\to\infty} \frac{a_{n+1}-1}{a_n-1} = \lim_{n\to\infty} \frac{\sqrt{3+a_n}-1-1}{a_n-1} = \lim_{n\to\infty} \frac{\sqrt{3+a_n}-2}{a_n-1}$$

$$= \lim_{n\to\infty} \frac{(\sqrt{3+a_n}-2)(\sqrt{3+a_n}+2)}{(a_n-1)(\sqrt{3+a_n}+2)} = \lim_{n\to\infty} \frac{(\cancel{a_n-1})}{(\cancel{a_n-1})(\sqrt{3+\underset{1}{\underset{\sim}{a_n}}}+2)} = \frac{1}{4} \,//$$

30) i) $$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2}} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(1-\frac{1}{n+1}\right)^{n+1} \cdot \left(1-\frac{1}{n+1}\right)^{-1}$$

$$= e^{-1} \cdot 1 = \frac{1}{e} < 1 \Rightarrow \text{Yakınsak}$$

ii) $b_n = \frac{1}{n}$ olmak üzere $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n}$ harmonik seri ıraksak

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = 1 \neq 0, \infty$$

Limit karşılaştırma testinden her iki seri aynı karakterdedir.

$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ ıraksaktır.

31) $$\sum_{n=0}^{\infty} \frac{\pi^{-n}}{\cos(n\pi)} = 1 - \frac{1}{\pi} + \frac{1}{\pi^2} - \frac{1}{\pi^3} + \dots + (-1)^n \frac{1}{\pi^n} + \dots = \sum_{n=0}^{\infty} \left(-\frac{1}{\pi}\right)^n$$

Geometrik seridir. $a = 1$, $r = -\frac{1}{\pi}$ $|r| < 1$ olduğundan

serinin toplamı $S = \frac{a}{1-r} = \frac{1}{1-\left(-\frac{1}{\pi}\right)} = \frac{\pi}{1+\pi} \,//$

32) $$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{(3-2x)^{k+1}}{(k+1)^2\ln(k+1)}\cdot\frac{k^2\ln k}{(3-2x)^k}\right|$$

$$= |3-2x| \cdot \lim_{k\to\infty}\left[\underbrace{\left(\frac{k}{k+1}\right)^2}_{1}\cdot\underbrace{\frac{\ln k}{\ln(k+1)}}_{1}\right] = |3-2x| < 1$$

$$-1 < 3-2x < 1 \Rightarrow \boxed{1 < x < 2}$$

$x=1$ için;

$$\sum_{k=2}^{\infty}\frac{1}{k^2\ln k}$$

$b_k = \frac{1}{k^2}$ olmak üzere $\sum_{k=2}^{\infty}\frac{1}{k^2}$ $p=2>1$ yakınsak

$$\lim_{k\to\infty}\frac{a_k}{b_k} = \lim_{k\to\infty}\frac{\frac{1}{k^2\ln k}}{\frac{1}{k^2}} = \lim_{k\to\infty}\frac{1}{\ln k} = 0 \quad \text{ve}$$

$\sum_{k=2}^{\infty}\frac{1}{k^2}$ serisi yakınsak olduğundan $\sum_{k=2}^{\infty}\frac{1}{k^2\ln k}$ serisi de yakınsak

$x=2$ için;

$\sum_{k=2}^{\infty}(-1)^k\frac{1}{k^2\ln k}$ alterne serisi mutlak yakınsaktır.

Dolayısıyla yakınsaklığı aralığı :

$$\boxed{1 \le x \le 2}$$

33) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n\ln n}$

$\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ serisini ele alalım.

$f(x) = \frac{1}{x\ln x}$ olsun. $[2,\infty)$ aralığında sürekli, pozitif ve azalandır.

$\left(f'(x) = -\frac{(\ln x+1)}{(x\ln x)^2} < 0 \Rightarrow f(x) \text{, azalan} \right)$

Yani integral testi kullanılabilir.

$$\int_2^{\infty} \frac{dx}{x\ln x} = \lim_{R\to\infty} \int_2^{R} \frac{dx}{x\ln x} = \lim_{R\to\infty} \int_{\ln 2}^{\ln R} \frac{du}{u} = \lim_{R\to\infty} (\ln u)\Big|_{\ln 2}^{\ln R}$$

$\ln x = u$ | $x=2 \Rightarrow u=\ln 2$

$\frac{dx}{x} = du$ | $x=R \Rightarrow u=\ln R$

$$= \lim_{R\to\infty} \left[\ln(\ln R) - \ln(\ln 2) \right] = \infty \rightarrow \text{ıraksak}$$

O halde integral testine göre $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ serisi ıraksaktır.

Yani mutlak yakınsak değildir.

$\sum_{n=2}^{\infty} (-1)^n \frac{1}{n\ln n}$ alterne serisini ele alalım.

i) $a_n = \frac{1}{n\ln n} > 0$

ii) $a_{n+1} < a_n$ (Azalan olduğu yukarıda gösterildi.)

iii) $\lim_{n\to\infty} \frac{1}{n\ln n} = 0$

şartları sağlandığından $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n\ln n}$ serisi alterne seri testine göre yakınsaktır. Yani şartlı yakınsaktır.

34) $L(x)=\int_0^x \cos(t^2)\,dt$

$$\cos x=\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$$

$x\to t^2$ $$\cos(t^2)=\sum_{n=0}^{\infty}(-1)^n \frac{t^{4n}}{(2n)!}$$

$$L(x)=\int_0^x \cos(t^2)\,dt=\int_0^x\left[\sum_{n=0}^{\infty}(-1)^n\frac{t^{4n}}{(2n)!}\right]dt=\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+1}}{(2n)!}\cdot\frac{1}{(4n+1)}$$

$$L(x)=x-\frac{x^5}{2.5}+\cdots$$

$$L(0{,}1)\simeq 0{,}1-\frac{(0{,}1)^5}{10}=0{,}1-0{,}000001=0{,}099999 \;//$$

35) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{n^4+\ln n}$

Mutlak yakınsak mı?

$\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{n^4+\ln n}$ ile $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$ serisini karşılaştıralım.

$\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$ $(p=2>1)$ yakınsak bir seridir.

$$\lim_{n\to\infty}\frac{n^2}{n^4+\ln n}\cdot n^2=\lim_{n\to\infty}\frac{\cancel{n^4}}{\cancel{n^4}\left(1+\underbrace{\frac{\ln n}{n^4}}_{0}\right)}=1\neq 0,\infty$$

Limit testine göre iki seri aynı karakterdedir.

Dolayısıyla $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$ yakınsak olduğundan $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{n^4+\ln n}$ yakınsaktır.

Sonuç olarak $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{n^4+\ln n}$ serisi mutlak yakınsaktır.

36) $\displaystyle\sum_{k=0}^{\infty}\int_k^{k+1}\frac{1}{1+e^x}\,dx$

$$I=\int\frac{1}{1+e^x}dx=\int\frac{du}{u(u-1)}\qquad \frac{1-u+u}{u(u-1)}=\frac{1-u}{u(u-1)}+\frac{u}{u(u-1)}=\frac{1}{u-1}-\frac{1}{u}$$

$1+e^x=u$

$e^x dx=du$

$$=\int\left(\frac{1}{u-1}-\frac{1}{u}\right)du=\ln|u-1|-\ln|u|+C$$
$$=\ln(e^x)-\ln(e^x+1)+C$$
$$=x-\ln(1+e^x)+C$$

$$\int_k^{k+1}\frac{1}{1+e^x}dx=x-\ln[1+e^x]\Big|_k^{k+1}$$
$$=\cancel{k}+1-\ln(1+e^{k+1})-\cancel{k}+\ln(e^k+1)$$
$$=1+\ln(e^k+1)-\ln(e^{k+1}+1)$$

$$\sum_{k=0}^{\infty}\int_k^{k+1}\frac{1}{1+e^x}dx=\sum_{k=0}^{\infty}\left[1+\ln(e^k+1)-\ln(e^{k+1}+1)\right]$$

$$S_n=1+\ln 2-\cancel{\ln(e+1)}+1+\cancel{\ln(e+1)}-\ln(e^2+1)+\dots+1+\cancel{\ln(e^n+1)}-\ln(e^{n+1}+1)$$
$$=n.1+\ln 2-\ln(e^{n+1}+1)=\ln(e^n)+\ln 2-\ln(e^{n+1}+1)$$
$$=\ln 2+\ln\left(\frac{e^n}{e^{n+1}+1}\right)=\ln 2-\ln\left(\frac{e^{n+1}+1}{e^n}\right)=\ln 2-\ln\left(e+\frac{1}{e^n}\right)$$

$$\sum_{k=0}^{\infty}\int_k^{k+1}\frac{1}{1+e^x}dx=\lim_{n\to\infty}S_n=\lim_{n\to\infty}\left[\ln 2-\ln\left(e+\frac{1}{e^n}\right)\right]=\ln 2-1 \;//$$

37) $e^u=1+u+\frac{u^2}{2}+\frac{u^3}{6}+\dots$

$$e^{2t}-1=1+2t+2t^2+\frac{4t^3}{3}+\dots-1$$

$$\lim_{x\to 0}\int_x^{2x}\frac{e^{2t}-1}{t^2}dt=\lim_{x\to 0}\int_x^{2x}\left(\frac{2}{t}+2+\frac{4}{3}t+\dots\right)dt$$
$$=\lim_{x\to 0}\left[2\ln|t|+2t+\frac{2}{3}t^2+\dots\right]\Big|_x^{2x}$$
$$=\lim_{x\to 0}\left[2\ln 2+\cancel{2\ln|x|}+4x+\dots-\cancel{2\ln|x|}-2x-\dots\right]$$
$$=2\ln 2 \;//$$

38) $x(t)=t^3$, $y(t)=(1-t^2)^{3/2}$, $-1\leq t\leq 1$

$x'(t)=3t^2$

$y'(t)=-3t(1-t^2)^{1/2}$

$$S=\int_{-1}^{1}\sqrt{[x'(t)]^2+[y'(t)]^2}\,dt=\int_{-1}^{1}\sqrt{9t^4+9t^2(1-t^2)}\,dt$$

$$=\int_{-1}^{1}3|t|\,dt=6\int_{0}^{1}t\,dt=3t^2\Big|_0^1=6\text{ br.}$$

39) $t^2\sin x+x^3=e^t$, $y=t\sin t-2t$

$t=0\Rightarrow x=1$, $y=0$

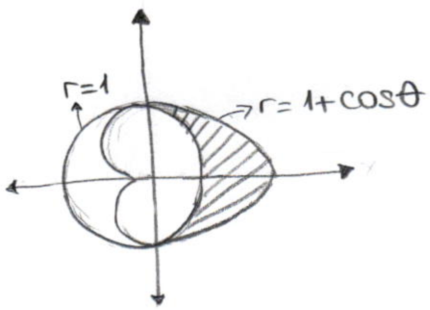
$$2t\sin x+t^2\cdot\cos x\cdot\frac{dx}{dt}+3x^2\frac{dx}{dt}=e^t$$

$$\left.\begin{matrix}t=0\\x=1\end{matrix}\right]\Rightarrow\quad 0+0+3\frac{dx}{dt}=1\Rightarrow\left.\frac{dx}{dt}\right|_{t=0}=\frac{1}{3}$$

$$y=t\sin t-2t\Rightarrow\frac{dy}{dt}=\sin t+t.\cos t-2\Rightarrow\left.\frac{dy}{dt}\right|_{t=0}=-2$$

$$\left.\frac{dy}{dx}\right|_{t=0}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-2}{\frac{1}{3}}=-6$$

40)



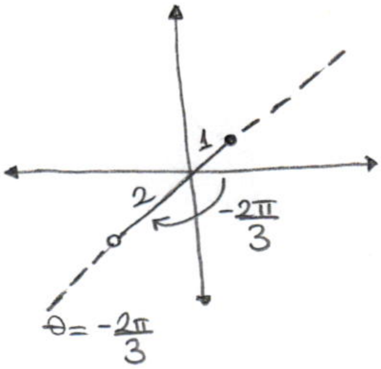
$1=1+\cos\theta\Rightarrow\cos\theta=0\Rightarrow\theta=\frac{\pi}{2},\frac{3\pi}{2}$

$$\frac{A}{2}=\int_0^{\pi/2}\frac{1}{2}\left[(1+\cos\theta)^2-1^2\right]d\theta$$

$$=\frac{1}{2}\int_0^{\pi/2}\left(2\cos\theta+\frac{1+\cos2\theta}{2}\right)d\theta$$

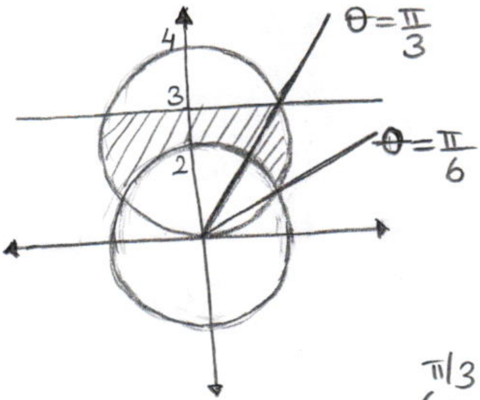
$$=\frac{1}{2}\left[2\sin\theta+\frac{\theta}{2}+\frac{1}{4}\sin2\theta\right]\Big|_0^{\pi/2}=1+\frac{\pi}{8}$$

$$\Rightarrow A=2+\frac{\pi}{4}\text{ br}^2$$

41) 

$\theta=-\frac{2\pi}{3}$, $-1\le r<2$

42) $r=2$, $r=4\sin\theta$, $r\sin\theta=3$



$4\sin\theta=\frac{3}{\sin\theta}$

$\sin^2\theta=\frac{3}{4}$

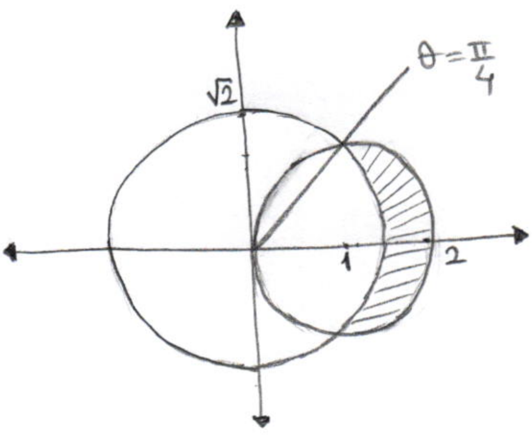
$\sin\theta=\pm\frac{\sqrt{3}}{2} \Rightarrow \theta=\frac{\pi}{3}$

$4\sin\theta=2$

$\sin\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{6}$

$$\frac{A}{2}=\frac{1}{2}\int_{\frac{\pi}{6}}^{\pi/3}\left[(4\sin\theta)^2-2^2\right]d\theta+\frac{1}{2}\int_{\frac{\pi}{3}}^{\pi/2}\left[\left(\frac{3}{\sin\theta}\right)^2-2^2\right]d\theta$$

43) $r=2\cos\theta$, $r=\sqrt{2}$



$2\cos\theta=\sqrt{2}$

$\Rightarrow \theta=\frac{\pi}{4}$

$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/4}(4\cos^2\theta-2)\,d\theta=\frac{1}{2}\int_0^{\pi/4}(\cancel{2}+2\cos2\theta-\cancel{2})\,d\theta$$

$$=\frac{1}{2}\sin2\theta\Big|_0^{\pi/4}$$

$A=1\ br^2$ //

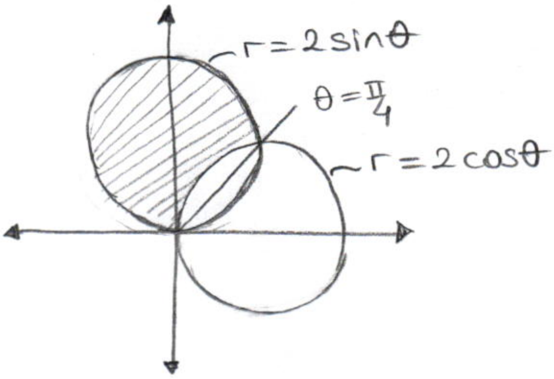
44) $$L=\int_{-\pi}^{\pi}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta=\int_{-\pi}^{\pi}\sqrt{e^{2a\theta}+a^2e^{2a\theta}}\,d\theta=\sqrt{1+a^2}\int_{-\pi}^{\pi}e^{a\theta}\,d\theta$$

$$=\frac{\sqrt{1+a^2}}{a}\,e^{a\theta}\Big|_{-\pi}^{\pi}=\frac{\sqrt{1+a^2}}{a}\left(e^{a\pi}-e^{-a\pi}\right)\text{br.}$$ //

45) $x = r\cos\theta$
$y = r\sin\theta$

$$x^2+y^2=2x$$
$$r^2\cos^2\theta + r^2\sin^2\theta = 2r\cos\theta$$
$$r^2 = 2r\cos\theta$$
$$\underline{r = 2\cos\theta}$$

$$x^2+y^2=2y$$
$$r^2\cos^2\theta + r^2\sin^2\theta = 2r\sin\theta$$
$$r^2 = 2r\sin\theta$$
$$\underline{r = 2\sin\theta}$$



46) $2\cos\theta = 2\sin\theta$

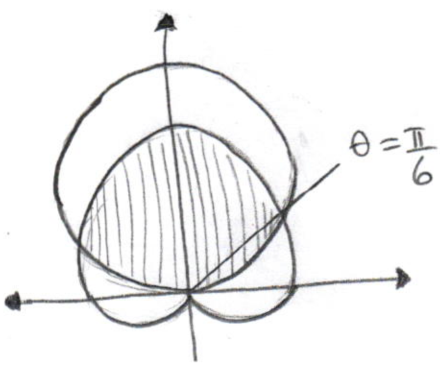
$$\theta = \frac{\pi}{4}$$

$$A = \frac{1}{2}\int_{\pi/4}^{\pi/2} (4\sin^2\theta - 4\cos^2\theta)\, d\theta + \frac{1}{2}\int_{\pi/2}^{\pi} 4\sin^2\theta\, d\theta$$

$$= 2\int_{\pi/4}^{\pi/2} (-\cos 2\theta)\, d\theta + \int_{\pi/2}^{\pi} (1-\cos 2\theta)\, d\theta$$

$$= -\sin 2\theta \Big|_{\pi/4}^{\pi/2} + (\theta - \sin 2\theta)\Big|_{\pi/2}^{\pi} = 1 + \pi - \frac{\pi}{2} = 1 + \frac{\pi}{2}\ br^2 \,/\!/$$

47)



$$3\sin\theta = 1+\sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/6} (3\sin\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} (1+\sin\theta)^2\, d\theta$$

$$A = \frac{9}{2}\int_0^{\pi/6} (1-\cos 2\theta)\, d\theta + \int_{\pi/6}^{\pi/2} \left(1 + 2\sin\theta + \frac{1-\cos 2\theta}{2}\right) d\theta$$

$$A = \frac{9}{2}\left[\theta - \frac{1}{2}\sin 2\theta \Big|_0^{\pi/6}\right] + \left[\theta - 2\cos\theta + \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \Big|_{\pi/6}^{\pi/2}\right]$$

$$A = \frac{9}{2}\left[\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right] + \left[\frac{3\pi}{4} - \frac{3\pi}{12} + \frac{1}{4}\frac{\sqrt{3}}{2} + \frac{2\sqrt{3}}{2}\right] = \frac{5\pi}{4}\ br^2 \,/\!/$$

48) İstenilen doğrunun yönlü vektörü $\vec{v}=(v_1,v_2,v_3)$ olsun.

Eğer doğru xy-düzlemine dik olacaksa;

$$\vec{v}\cdot\vec{i}=0 \quad \text{ve} \quad \vec{v}\cdot\vec{j}=0$$

olmalıdır. Öyleyse, $v_1=0$, $v_2=0$, $\vec{v}=(0,0,v_3)$ olur.

Buna göre doğrunun bir denklemi;

$$\vec{r}=\langle 1,1,-2\rangle + t\langle 0,0,v_3\rangle = \langle 1,1,-2+tv_3\rangle \;//\; \text{dir.}$$

49) $\vec{n}=(3,2,-1)$ $\quad P(1,2,-3)$

$$3(x-1)+2(y-2)-1(z+3)=0$$
$$3x+2y-z-10=0 \;//$$

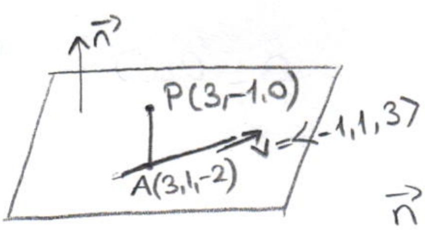
50) $\left.\begin{aligned}3x+y-2z+1&=0\\ x-3y+5z&=10\end{aligned}\right]$ $x=t$ için $\left.\begin{aligned}y-2z&=-1-3t\\ 3y-5z&=-10+t\end{aligned}\right] \Rightarrow \begin{aligned}y&=17t-15\\ z&=10t-7\end{aligned}$

$$\vec{r}(t)=\langle t,17t-15,10t-7\rangle=\langle 0,-15,-7\rangle+t\langle 1,17,10\rangle$$
$$\vec{R}(t)=\langle 1-3t,2-t,3+2t\rangle=\langle 1,2,3\rangle+t\langle -3,-1,2\rangle$$
$$\vec{v}\cdot\vec{V}=\langle 1,17,10\rangle\cdot\langle -3,-1,2\rangle=-3-17+20=0 \;//$$

O halde, arakesit doğrusu $\vec{r}(t)$, verilen doğru $\vec{R}(t)$ ye diktir.

51) $\left.\begin{aligned}x+y&=1\\ y+z&=2\end{aligned}\right]$ $x=t$ için $\begin{aligned}y&=1-t\\ z&=2-(1-t)=1+t\end{aligned}$

$$\cos\theta=\frac{\langle 1,-1,1\rangle\cdot\langle -1,1,2\rangle}{\sqrt{3}\cdot\sqrt{6}}=\frac{-1-1+2}{3\sqrt{2}}=0 \Rightarrow \theta=\frac{\pi}{2} \;//$$

52) 

$$\vec{n}=\vec{AP}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & -2 & 2\\ -1 & 1 & 3\end{vmatrix}=\langle -8,-2,-2\rangle$$

$$\vec{n}=\langle -8,-2,-2\rangle, \quad P(3,-1,0)$$
$$-8(x-3)-2(y+1)-2(z-0)=0$$
$$4x+y+z=11 \;//$$

53) a) $\left.\begin{array}{l} x+2y-z=1 \\ 2x+y+z=4 \end{array}\right]$ $x=t$ için $\left.\begin{array}{r} 2y-z=1-t \\ y+z=4-2t \end{array}\right]$ $\begin{array}{l} y=\frac{5}{3}-t \\ z=\frac{7}{3}-t \end{array}$

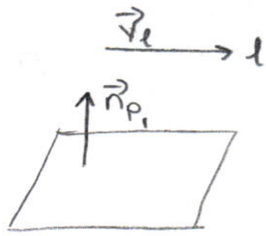
Arakesit doğrusu $\vec{r}(t) = \langle t, \frac{5}{3}-t, \frac{7}{3}-t \rangle$

$\vec{n} = \langle 1,-1,-1 \rangle$, $P(1,-2,1)$

$1.(x-1) - 1.(y+2) - 1.(z-1) = 0$

$x-y-z-2=0$ //

b)



P_1, l ye paralel ise $\vec{n}_{P_1} \perp \vec{v}_l$ olmalı yani;

$\vec{n}_{P_1} \cdot \vec{v}_l = 0$

$\vec{n}_{P_1} = \langle 1, 2, -1 \rangle$ $\vec{v}_l = \langle 1,-1,-1 \rangle$

$\vec{n}_{P_1} \cdot \vec{v}_l = \langle 1,2,-1 \rangle \cdot \langle 1,-1,-1 \rangle = 1-2+1 = 0$

$\Rightarrow \vec{n}_{P_1} \perp \vec{v}_l$

O halde P_1 düzlemi, l doğrusuna paraleldir.

54) a) $\left.\begin{array}{l} x+y=1 \\ 2x+y-2z=2 \end{array}\right]$ $x=t$ için $\begin{array}{l} y=1-t \\ z=\frac{t}{2}-\frac{1}{2} \end{array}$

$\left.\begin{array}{l} x=t \\ y=1-t \\ z=\frac{t}{2}-\frac{1}{2} \end{array}\right]$ kesişim doğrusunun parametrik denklemleri

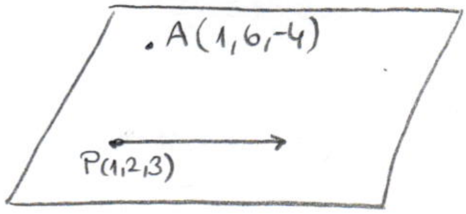
b) $\vec{n} = \langle 1,-1,\frac{1}{2} \rangle$, $P(3,1,-1)$

$1.(x-3) - 1.(y-1) + \frac{1}{2}(z+1) = 0$

$x-y+\frac{z}{2} - 3 + 1 + \frac{1}{2} = 0$

$2x-2y+z-3=0$ //

55)



$\overrightarrow{AP} = \langle 0,-4,7 \rangle$

$\vec{v} = \langle 2,-3,-1 \rangle$

$$\vec{n} = \overrightarrow{AP} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -4 & 7 \\ 2 & -3 & -1 \end{vmatrix} = \langle 25,14,8 \rangle$$

$\vec{n} = \langle 25,14,8 \rangle$, $A(1,6,-4)$

$$25\cdot(x-1) + 14\cdot(y-6) + 8\cdot(z+4) = 0$$

$$25x + 14y + 8z - 77 = 0$$

//