2) $y'' = y'^3 + y'$

(x is absent)

$y' = p, \quad y'' = p\dfrac{dp}{dy}$

$p\dfrac{dp}{dy} = p^3 + p \rightarrow \dfrac{dp}{p^2+1} = dy \rightarrow \arctan p = y + c_1$

$\Rightarrow p = \tan(y+c_1) \rightarrow \dfrac{dy}{\tan(y+c_1)} = dx$

$\ln|\sin(y+c_1)| + \ln c_2 = x \rightarrow e^x = c_2 \sin(y+c_1)$

veya $y + c_1 = \arcsin \dfrac{e^x}{c_2}$

2.40c (y is absent) $y' = p, \ y'' = \dfrac{dp}{dx}$

$\dfrac{dp}{dx} = p^3 + p \rightarrow dx = \dfrac{dp}{p(p^2+1)} = \left(\dfrac{1}{p} - \dfrac{p}{p^2+1}\right)dp$

$\dfrac{1}{p(p^2+1)} = \dfrac{A}{p} + \dfrac{Bp+C}{p^2+1}$

$1 = Ap^2 + A + Bp^2 + Cp$

$A + B = 0 \rightarrow B = -1$

$C = 0$

$A = 1$

$x = \ln p - \frac{1}{2}\ln(p^2+1) - \frac{1}{2}\ln c_1$

$2x = \ln \dfrac{p^2}{(p^2+1)c_1} \rightarrow \dfrac{p^2}{(p^2+1)c_1} = e^{2x}$

$\cancel{1} + \dfrac{1}{p^2} = \dfrac{1}{c_1 e^{2x}} - 1 \rightarrow p^2 = \dfrac{c_1 e^{2x}}{1 - c_1 e^{2x}}$

$p = \pm \dfrac{\sqrt{c_1}\, e^x}{\sqrt{1 - c_1 e^{2x}}} \rightarrow y = \pm \displaystyle\int \dfrac{\sqrt{c_1}\, e^x}{\sqrt{c_1}\sqrt{\frac{1}{c_1} - (e^x)^2}}\, dx$

$\Rightarrow y = \pm \arcsin c_1 e^x + c_2$

örnek $yy'' = 2y'^2 - 2y'$ (x is absent)

$y' = p \; ; \; y'' = p\dfrac{dp}{dy}$

$yp\dfrac{dp}{dy} = 2p^2 - 2p \rightarrow \dfrac{dp}{p-1} = 2\dfrac{dy}{y} \rightarrow \ln(p-1) = 2\ln y + \ln c_1$

$p - 1 = y^2 . c_1 \rightarrow \dfrac{dy}{dx} = c_1 y^2 + 1 \rightarrow dx = \int \dfrac{dy}{c_1 y^2 + 1}$

$\Rightarrow x = \dfrac{1}{c_1} . \sqrt{c_1} \arctan \sqrt{c_1}\, y + c_2$ //

örnek $2xy'' = (y')^2 + 2y'$ (y is absent)

$y' = p \; ; \; y'' = \dfrac{dp}{dx}$

$2x\dfrac{dp}{dx} = p^2 + 2p \rightarrow \dfrac{2\,dp}{p(p+2)} = \dfrac{dx}{x}$

$\ln x = \int \left(\dfrac{1}{p} - \dfrac{1}{p+2} \right) dp \Rightarrow \ln x + \ln c_1 = \ln p - \ln(p+2)$

$\Rightarrow \dfrac{p}{p+2} = x c_1 \rightarrow p = (p+2) x c_1 \rightarrow (1 - c_1 x) p = 2 x c_1$

$\Rightarrow p = \dfrac{2 x c_1}{1 - c_1 x} \rightarrow \dfrac{dy}{dx} = \dfrac{2 x c_1}{1 - c_1 x} \rightarrow y = \int \dfrac{2 x c_1}{1 - c_1 x} dx$

$\Rightarrow y = -2 \int \left(1 + \dfrac{1}{c_1 x - 1} \right) dx \Rightarrow y = -2x - \dfrac{2}{c_1} \ln |c_1 x - 1| + c_2$

Örnek $y'' = 2y^3 + 8y$ $x = \frac{\pi}{4}$ için $y = 2$, $y' = 8$

olarak verilen başlangıç koşullarına uygun çözümü

bulunuz. $y = 2\tan\left(2x - \frac{\pi}{4}\right)$

x in bulunmadığı denk.

$y' = p$, $y'' = p\frac{dp}{dy}$

$p\frac{dp}{dy} = 2y^3 + 8y \rightarrow p\,dp = (2y^3 + 8y)\,dy$

$\frac{p^2}{2} = \frac{y^4}{2} + 4y^2 + \frac{c_1}{2} \rightarrow p^2 = y^4 + 8y^2 + c_1$

$c_1 = ?$ $8^2 = 2^4 + 8.2^2 + c_1 \rightarrow c_1 = 64 - 16 - 32 = 16$

$\Rightarrow p^2 = y^4 + 8y^2 + 16 \rightarrow p^2 = (y^2+4)^2 \rightarrow p = \mp (y^2+4)$

$\Rightarrow p = y^2 + 4 \rightarrow \frac{dy}{dx} = y^2 + 4 \rightarrow dx = \frac{dy}{y^2+4}$

$\frac{c_2}{2} + x = \frac{1}{2}\arctan\frac{y}{2} \rightarrow \frac{y}{2} = \tan(2x + c_2)$; $c_2 = ?$

$\frac{2}{2} = \tan\left(2.\frac{\pi}{4} + c_2\right) \rightarrow \tan\left(\frac{\pi}{2} + c_2\right) = 1 \rightarrow \frac{\pi}{2} + c_2 = \arctan 1$

$\frac{\pi}{2} + c_2 = \frac{\pi}{4} \rightarrow c_2 = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}$ //

$\Rightarrow y = 2\tan\left(2x - \frac{\pi}{4}\right)$ //

SS

Örnek $\quad x\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

(y nin bulunmadığı dek.)

$\frac{dy}{dx}=p \rightarrow \frac{d^2y}{dx^2}=\frac{dp}{dx}$

$x\frac{dp}{dx}=\sqrt{1+p^2} \rightarrow \frac{dp}{\sqrt{1+p^2}}=\frac{dx}{x} \rightarrow \ln\left(p+\sqrt{1+p^2}\right)=\ln x+\ln c_1$

$p+\sqrt{1+p^2}=c_1x \rightarrow \sqrt{1+p^2}=c_1x-p \rightarrow 1+p^2=c_1^2x^2-2c_1xp+p^2$

$2c_1xp=c_1^2x^2-1 \rightarrow \underset{\uparrow \atop \frac{dy}{dx}}{p}=\frac{c_1x}{2}-\frac{1}{2c_1x}$

$y=\frac{c_1}{4}x^2-\frac{1}{2c_1}\ln x+c_2$