UYGULAMA

Kasım 2018

1) $y''' + 2y'' - 5y' - 6y = 0$ g.ç.!

$r^3 + 2r^2 - 5r - 6 = 0$

$r = -1 \rightarrow -1 + 2 + 5 - 6 = 0$

$(r+1)(\ ?\ ) = 0$

$r^3 + 2r^2 - 5r - 6 \ \big|\ r+1$

$r^3 + r^2$

$r^2 - 5r - 6$

$r^2 + r$

$-6r - 6$

$-6r - 6$

0

$(r+1)(r^2 + r - 6) = 0$

$-2 \quad 3$

$r = -1, \ r = 2, \ r = -3$

$y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-3x}$

2) $y''' + 4y' = 0$ g.ç.!

$r^3 + 4r = 0 \rightarrow r(r^2 + 4) = 0 \rightarrow \begin{cases} r = 0 \\ r = \pm 2i \end{cases}$

$y = c_1 + c_2 \cos 2x + c_3 \sin 2x$

3) $y'' + 4y' + 4y = 0$ g.ç.!

$r^2 + 4r + 4 = 0 \rightarrow (r+2)^2 = 0 \rightarrow r = -2$ (2 katlı)

$y = (c_1 + c_2 x) e^{-2x}$

4) $y'' - 5y' = 0$ g.ç.!

5) $y'' + 9y = 0$ g.ç.!

6) $y'' - 2y' + 2y = 0$ g.ç.!

$\Delta = 4 - 4 \cdot 1 \cdot 2 = -4$

$r_{1,2} = \dfrac{2 \pm \sqrt{-4}}{2} = 1 \pm i$

$y = e^x (c_1 \cos x + c_2 \sin x)$

7) $e^{x+y}y' = e^{2x-y}$ g.ç.?

$e^x e^y dy = e^{2x} \cdot e^{-y} dx$

$e^{2y} dy = e^x dx$

$\frac{e^{2y}}{2} = e^x + \frac{c}{2}$

$e^{2y} = 2e^x + c$

$2y = \ln(2e^x + c)$ //

8) $(3x-2y+2)y' = 3x-2y$

$3x-2y = u$

$3 - 2\frac{dy}{dx} = \frac{du}{dx}$

$\frac{dy}{dx} = \frac{1}{2}\left(3 - \frac{du}{dx}\right)$

$(u+2)\frac{1}{2}\left(3-\frac{du}{dx}\right) = u$

$-(u+2)\frac{du}{dx} + 3u + 6 = 2u$

$\frac{u+2}{u+6} du = dx$

$x = \int \left(1 - \frac{4}{u+6}\right) du$

$x = u - 4\ln|u+6| + 2c$

$x = 3x-2y-4\ln|3x-2y+6| + 2c$

$y = x - 2\ln|3x-2y+6| + c$ //

9) $-y' = e^{-x}y^2 - y - e^x$, $y_1 = -e^x$ (özel çözüm)

$y = y_1 + \frac{1}{u} \rightarrow y = -e^x + \frac{1}{u} \rightarrow y' = -e^x - \frac{u'}{u^2}$

$e^x + \frac{u'}{u^2} = e^{-x}\left(e^{2x} - 2e^x \cdot \frac{1}{u} + \frac{1}{u^2}\right) + e^x - \frac{1}{u} - e^x$

$e^x + \frac{u'}{u^2} = e^x - \frac{2}{u} + \frac{e^{-x}}{u^2} - \frac{1}{u}$

$u' = -3u + e^{-x}$

$u' + 3u = e^{-x}$

$\lambda(x) = e^{\int 3dx} = e^{3x}$

$u = \frac{1}{e^{3x}}\left[\int e^{-x} e^{3x} dx + c\right]$

$u = \frac{1}{e^{3x}}\left[\frac{1}{2}e^{2x} + \frac{c}{2}\right]$

$u = \frac{e^{2x} + c}{2e^{3x}}$

$\Rightarrow y = -e^x + \frac{2e^{3x}}{e^{2x}+c}$ //

10) $\underbrace{(3xy+y^2+1)}_{P}dx + \underbrace{(x^2+xy)}_{Q}dy = 0$

$\frac{\partial P}{\partial y} = 3x+2y$

$\frac{\partial Q}{\partial x} = 2x+y$ $\Big\}$ $\neq$ Tam dif. denk. değil

$$\frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{Q} = \frac{3x+2y-2x-y}{x(x+y)} = \frac{x+y}{x(x+y)} = \frac{1}{x}$$

$$\lambda(x) = e^{\int \frac{1}{x}dx} = e^{\ln x} = x \Rightarrow \boxed{\lambda = x}$$ İntegral çarpan

Öyle bir $u(x,y)$ fonk.nu vardır ki

$$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = x(3xy+y^2+1)dx + x(x^2+xy)dy$$

(1°) $\frac{\partial u}{\partial x} = 3x^2y+y^2x+x \rightarrow u = \int (3x^2y+y^2x+x)dx + r(y)$

$$u = \frac{3}{3}x^3y + \frac{x^2}{2}y^2 + \frac{x^2}{2} + r(y) \quad (*)$$

$$\frac{\partial u}{\partial y} = x^3 + x^2y + r'(y)$$

(2°) $\frac{\partial u}{\partial y} = x^3+x^2y$

$x^3 + x^2y + r'(y) = x^3 + x^2y \Rightarrow r'(y) = 0 \rightarrow \boxed{r(y) = c_1}$

$r(y)$ $(*)$ da yazılırsa $u = x^3y + \frac{1}{2}x^2y^2 + \frac{1}{2}x^2 + c_1$

G.Ç. : $u = c_2 \rightarrow x^3y + \frac{1}{2}x^2y^2 + \frac{1}{2}x^2 = K \quad (K = c_2 - c_1)$

11) $y' \sin x + \sin^2 x - y\cos x = 0$

$y' - \frac{\cos x}{\sin x} y = -\sin x$ Lineer dif. denk.

$\lambda(x) = e^{\int -\frac{\cos x}{\sin x} dx} = e^{-\ln(\sin x)} = \frac{1}{\sin x}$

$y = \sin x \left[\int -\sin x . \frac{1}{\sin x} dx + c \right]$

$y = \sin x (-x + c)$ //

12) $(x^2 + yx + y^2) dx - x^2 dy = 0$

$(1 + \frac{y}{x} + \frac{y^2}{x^2}) - \frac{dy}{dx} = 0$ Homojen dif. denk.

$\frac{y}{x} = u \rightarrow y = ux \rightarrow \frac{dy}{dx} = u'x + u$

$(1 + u + u^2) = u'x + u \rightarrow 1 + u^2 = x \frac{du}{dx}$

$\frac{dx}{x} = \frac{1}{1+u^2} du \rightarrow \ln x = \arctan u + c$

$\Rightarrow \ln x = \arctan \frac{y}{x} + c$ //

13) $3(y+1)dx - (x+y+3)dy = 0$

$3y+3=0 \rightarrow y=-1$

$x+y+3=0 \rightarrow x=-2$

$x = x_1 - 2 \rightarrow dx = dx_1$

$y = y_1 - 1 \rightarrow dy = dy_1$

$(3(y_1-1)+3)dx_1 - (x_1-2+y_1-1+3)dy_1$

$3y_1 dx_1 - (x_1+y_1)dy_1 = 0$

$3\frac{y_1}{x_1} - (1+\frac{y_1}{x_1})\frac{dy_1}{dx_1} = 0$ Homojen dif. denk.

$\frac{y_1}{x_1} = u \rightarrow y_1 = ux_1 \rightarrow \frac{dy_1}{dx_1} = \frac{du}{dx_1}x_1 + u$

$3u - (1+u)(\frac{du}{dx_1}x_1 + u) = 0$

$x_1(1+u)\frac{du}{dx_1} = 3u - u - u^2$

$\frac{dx_1}{x_1} = \frac{u+1}{-u^2+2u}du$

$\ln x_1 = -\int \frac{u+1}{u^2-2u}du$

$\frac{u+1}{u(u-2)} = \frac{A}{u} + \frac{B}{u-2}$

$A = -\frac{1}{2}, \quad B = \frac{3}{2}$

$\ln x_1 = +\frac{1}{2}\ln u - \frac{3}{2}\ln(u-2) + \frac{1}{2}\ln c$

$x_1^2 = \frac{u \cdot c}{(u-2)^3} \rightarrow \left(\frac{y+1}{x+2} - 2\right)^3 (x+2)^2 = \frac{y+1}{x+2} \cdot c$ //

VEYA

$\frac{(y+1-2(x+2))^3}{(x+2)^3} \cdot (x+2)^2 = \frac{y+1}{x+2} \cdot c$

$(y-2x-3)^3 = c(y+1)$ //

$u = \frac{y_1}{x_1}$

$u = \frac{y+1}{x+2}$

14) $x^2y + (y-x^3)y' = 0$, $\lambda = \lambda(y)$ şeklinde integral çarpanı bularak çözünüz.

$$\underbrace{x^2y}_{P}\,dx + \underbrace{(y-x^3)}_{Q}\,dy = 0$$

$\frac{\partial P}{\partial y} = x^2$ ⎫
⎬ $\neq$ Tam dif. denk değil
$\frac{\partial Q}{\partial x} = -3x^2$ ⎭

$$\frac{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}{P} = \frac{-3x^2 - x^2}{x^2y} = -\frac{4}{y}$$

$$\lambda(y) = e^{\int -\frac{4}{y}dy} = e^{-4\ln y} = \frac{1}{y^4} \Rightarrow \lambda(y) = \frac{1}{y^4}$$

$$\frac{1}{y^4}(x^2y)\,dx + \frac{1}{y^4}(y-x^3)\,dy = 0$$

$$\frac{x^2}{y^3}\,dx + \left(\frac{1}{y^3} - \frac{x^3}{y^4}\right)dy = 0$$ Tam dif. denk.

$$u = \int \frac{x^2}{y^3}dx \rightarrow u = \frac{x^3}{3y^3} + r(y)$$

$$\frac{\partial u}{\partial y} = -\frac{x^3}{y^4} + r'(y) = \frac{1}{y^3} - \frac{x^3}{y^4}$$

$$r'(y) = \frac{1}{y^3}$$

$$\int dr = \int \frac{1}{y^3}dy$$

$$r = -\frac{1}{2y^2} + c_1$$

G.Ç. $u = c_2$

$$\frac{x^3}{3y^3} - \frac{1}{2y^2} = K \quad (K = c_2 - c_1)$$

15) $y^2dx + x(x-y)dy = 0$ dif. denk.nin $x=1$ için $y=1$ koşuluna uyan özel çözümünü bulunuz.

$$y^2 + (x^2 - xy)\frac{dy}{dx} = 0$$

$$\frac{y^2}{x^2} + \left(1 - \frac{y}{x}\right)\frac{dy}{dx} = 0$$

Homojen

$$\frac{y}{x} = u \rightarrow y = ux$$

$$\frac{dy}{dx} = \frac{du}{dx}x + u$$

$$u^2 + (1-u)\left(\frac{du}{dx}x + u\right) = 0$$

$$(1-u)x\frac{du}{dx} + u^2 + u - u^2 = 0$$

$$\frac{dx}{x} = \frac{u-1}{u}du \rightarrow \ln x = u - \ln u + c$$

$$\Rightarrow \ln x = \frac{y}{x} - \ln\frac{y}{x} + c \quad // \text{ g.ç.}$$

$x=1,\ y=1$: $\ln\frac{1}{1} = \frac{1}{1} - \ln\frac{1}{1} + c \Rightarrow \boxed{c=-1}$

Özel çözüm: $\underline{\ln y = \frac{y}{x} + 1}$ VEYA $y = e^{\frac{y}{x}-1}$

16) $y' = \frac{y}{x} + e^{y/x}$ Homojen

(~~$e^{-\frac{y}{x}} = -\ln x + c$~~) :)

17) $y' = \dfrac{9x-2y-7}{2x-y-3}$

$(2x-y-3)dy = (9x-2y-7)dx$

$(9x-2y-7)dx - (2x-y-3)dy = 0$

$\underbrace{(9x-2y-7)}_{P}dx + \underbrace{(-2x+y+3)}_{Q}dy = 0$

$\frac{\partial y}{\partial x}$ $\qquad$ $\frac{\partial y}{\partial y}$

$\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x} \Rightarrow$ Tam dif.

$\downarrow \qquad \downarrow$

$-2 \qquad -2$

① $\dfrac{\partial u}{\partial x} = 9x-2y-7 \rightarrow u = \int (9x-2y-7)dx$

$u = \dfrac{9}{2}x^2 - 2xy - 7x + r(y)$ (*)

$\dfrac{\partial u}{\partial y} = -2x + r'(y)$

② $\dfrac{\partial u}{\partial y} = -2x+y+3$

$\downarrow$

$-2x + r'(y) = -2x + y + 3 \rightarrow \dfrac{dr}{dy} = y+3$

$dr = (y+3)dy$

$\boxed{r = \dfrac{y^2}{2} + 3y + c_1}$ (*) da yazılır;

$u = \dfrac{9}{2}x^2 - 2xy - 7x + \dfrac{y^2}{2} + 3y + c_1$

G.Ç.: $\dfrac{9}{2}x^2 - 2xy - 7x + \dfrac{y^2}{2} + 3y = K \quad (K = c_2 - c_1)$

$u = c_2 \rightarrow$

18) $dy + (y\cot x - 5e^{\cos x})dx = 0$ dif. denk.ni tam dif. denk haline getirerek $x = \frac{\pi}{2}$ için $y = 0$ koşuluna uyan çözümü bulunuz. :)

$-5e^{\cos x}\frac{dy}{dx}$:(

$\frac{dy}{dx} + \cot x \cdot y = 5e^{\cos x}$ Lineer dif denk.

Sabitin değişimi yöntemi ile çözelim: :)

$\frac{dy}{dx} + \cot x \cdot y = 0$ (Denk.in homojen kısmı alınır)

$\frac{dy}{y} = -\cot x\,dx \to \ln y - \ln c = -\ln(\sin x) \to \frac{y}{c} = \frac{1}{\sin x}$

$\Rightarrow y = \frac{c}{\sin x}$; $c = c(x)$ o/d.nu varsayalım

$\Rightarrow y = \frac{c(x)}{\sin x} \to y' = \frac{c'\sin x - c\cos x}{\sin^2 x}$

$\frac{dy}{dx} + \cot x \cdot y = 5e^{\cos x}$

$\frac{c'\sin x}{\sin^2 x} - c\frac{\cos x}{\sin^2 x} + \cot x \cdot \frac{c}{\sin x} = 5e^{\cos x}$

$c' = 5\sin x e^{\cos x} \to \int dc = \int 5\sin x e^{\cos x}$

$\Rightarrow \boxed{c = -5e^{\cos x} + k} \Rightarrow y = \frac{-5e^{\cos x} + k}{\sin x}$ g.ç. :)

$x = \frac{\pi}{2}, y = 0 \Rightarrow 0 = \frac{-5e^{\cos\frac{\pi}{2}} + k}{\sin\frac{\pi}{2}} \Rightarrow \boxed{k = 5}$

$\Rightarrow$ Özel çözüm: $\sin x \cdot y + 5e^{\cos x} = 5$ //

19) $y' = \frac{y}{x+\sqrt{xy}}$

$y' = \frac{\frac{y}{x}}{1+\frac{1}{x}\sqrt{xy}} \rightarrow y' = \frac{\frac{y}{x}}{1+\sqrt{\frac{y}{x}}}$ Homojen dif.denk.

$\frac{y}{x} = u \rightarrow y = ux \rightarrow y' = u'x + u$

Denklemde yazılırsa, $u'x + u = \frac{u}{1+\sqrt{u}}$

$(1+\sqrt{u}) \times \frac{du}{dx} + u + u\sqrt{u} = u \rightarrow \int \frac{dx}{x} = -\int \frac{1+\sqrt{u}}{u\sqrt{u}}\, du$

$\Rightarrow \ln x = -\int (u^{-\frac{3}{2}} + \frac{1}{u})\, du$

$\ln c + \ln x = 2\frac{1}{\sqrt{u}} - \ln u \rightarrow \ln(c \times u) = \frac{2}{\sqrt{u}}$

$\ln(c \cdot x \cdot \frac{y}{x}) = \frac{2}{\sqrt{\frac{y}{x}}} \rightarrow cy = e^{2\sqrt{\frac{x}{y}}}$ //

20) $y\sqrt{1-\ln^2 y}\sin x\, dx + (1+\cos^2 x)\, dy = 0$

$\frac{\sin x}{1+\cos^2 x}\, dx + \frac{1}{y\sqrt{1-\ln^2 y}}\, dy = 0$

$\cos x = u$
$-\sin x\, dx = du$

$\ln y = t$
$\frac{1}{y} dy = dt$

$\int -\frac{du}{1+u^2} + \int \frac{dt}{\sqrt{1-t^2}} = 0$

$-\arctan u + \arcsin t = c$

$-\arctan(\cos x) + \arcsin(\ln y) = c$ //

21) $2y\,dy + 4x^3\sqrt{4-y^4}\,dx = 0$

$$\frac{2y}{\sqrt{4-y^4}}dy + 4x^3dx = 0$$

$$\frac{2y}{\sqrt{2^2-(y^2)^2}}dy + x^4 = C$$

$$\arcsin\frac{y^2}{2} + x^4 = C \;//$$

22) $y = -xy'^2 + \ln y'$ Lagrange dif.denk.

$y' = p \rightarrow y = -xp^2 + \ln p \rightarrow \frac{dy}{dx} = -p^2 - 2xpp' + \frac{p'}{p}$ ($\frac{dy}{dx} \downarrow p$)

$$\Rightarrow p^2 + p = \frac{dp}{dx}\left(-2xp + \frac{1}{p}\right)$$

$$(p^2+p)\frac{dx}{dp} = -2xp + \frac{1}{p}$$

$$\frac{dx}{dp} + \frac{2p}{p^2+p}x = \frac{1}{p(p^2+p)}$$ Lineer dif.denk.

$$\lambda(p) = e^{\int \frac{2p}{p(p+1)}dp} = e^{2\ln(p+1)} = (p+1)^2$$

$$x = \frac{1}{(p+1)^2}\left[\int \frac{1}{p^2(p+1)}(p+1)^2dp + k\right]$$

$$x = \frac{1}{(p+1)^2}\left[\ln p - \frac{1}{p} + k\right]$$

$$\begin{cases} x = \frac{1}{(p+1)^2}\left(\ln p - \frac{1}{p} + k\right) \\ y = -\frac{p^2}{(p+1)^2}\left(\ln p - \frac{1}{p} + k\right) + \ln p \end{cases}$$

Genel çözümün parametrik denklemleri

23) $\frac{y'}{x} + \frac{2}{x^2}y = -x\sec^2 x\, y^2$

$y' + \frac{2}{x}y = -x^2\sec^2 x\, y^2$ Bernoulli dif. denk.

$y' y^{-2} + \frac{2}{x} y^{-1} = -x^2\sec^2 x$

↳ $z = y^{-1} \rightarrow z' = -y^{-2}y' \rightarrow y'y^{-2} = -z'$

Düzenlersek

Dif. denk. de yazılırsa

$-z' + \frac{2}{x}z = -x^2\sec^2 x$

$z' - \frac{2}{x}z = x^2\sec^2 x$ Lineer dif denk.

$\lambda(x) = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = \frac{1}{x^2}$

$z = x^2\left[\int +x^2\sec^2 x \frac{1}{x^2}dx + k\right]$

$z = x^2\left[+\int \sec^2 x\, dx + k\right] \Rightarrow z = x^2\left[+\tan x + k\right]$

$z = y^{-1}$ idi $\Rightarrow \frac{1}{y} = z \Rightarrow \frac{1}{y} = x^2(+\tan x + k)$ //

24) $y'\sec x + y^2 = y$ (Cevap:) $y = \frac{1}{1+ce^{-\sin x}}$

$y' - \frac{1}{\sec x}y = -\frac{1}{\sec x}y^2$

$y^{-2}y' - \frac{1}{\sec x}y^{-1} = -\frac{1}{\sec x}$

↳ $z = y^{-1} \rightarrow z' = -y^{-2}y'$

$+z' + \frac{1}{\sec x}z = +\frac{1}{\sec x}$

$\lambda(x) = e^{\int \frac{1}{\sec x}dx} = e^{\int \cos x\, dx} = e^{\sin x}$

---- devamı (sizde) :)

25) $y + xy' = y'^2$

$y = -xy' + p^2$ Lagrange dif. denk.

$y' = p \rightarrow y = -xp + p^2$

$\hookrightarrow \frac{dy}{dx} = -p - x\frac{dp}{dx} + 2p\frac{dp}{dx}$

$\downarrow p$

$2p = \frac{dp}{dx}(-x + 2p) \rightarrow 2p\frac{dx}{dp} = -x + 2p$

$\Rightarrow \frac{dx}{dp} + \frac{1}{2p}x = 1$ Lineer dif. denk

$\lambda(p) = e^{\int \frac{1}{2}\cdot\frac{1}{p}dp} = e^{\frac{1}{2}\ln p} = \sqrt{p}$

$x = \frac{1}{\sqrt{p}}\left[\int 1\cdot\sqrt{p}\,dp + k\right] \rightarrow x = \frac{1}{\sqrt{p}}\left(\frac{2}{3}p^{\frac{3}{2}} + k\right)$

$$\begin{cases} x = \frac{1}{\sqrt{p}}\left(\frac{2}{3}p^{\frac{3}{2}} + k\right) \\ y = -\frac{p}{\sqrt{p}}\left(\frac{2}{3}p^{3/2} + k\right) + p^2 \end{cases}$$

Genel çözümün parametrik denklemleri

Soru $y^2y' - \sqrt{x^4 - x^4y^6} = 0$

$y^2\frac{dy}{dx} - x^2\sqrt{1-y^6} = 0$

$\frac{y^2}{\sqrt{1-(y^3)^2}}dy - x^2dx = 0$

$\frac{1}{3}\arcsin y^3 - \frac{x^3}{3} = \frac{c}{3}$ // veya

$\Rightarrow \arcsin y^3 - x^3 = c$ // :)

26) $x\,dx + (y\,dx - x\,dy)\sin^2\frac{y}{x} = 0$

$1 + \left(\frac{y}{x} - \frac{dy}{dx}\right)\sin^2\frac{y}{x} = 0$ Homojen dif. denk.

$\frac{y}{x} = u \to y = ux \to y' = u'x + u$

$1 + (u - u'x - u)\sin^2 u = 0$

$\frac{du}{dx} \times \sin^2 u = 1 \to \frac{dx}{x} = \sin^2 u\,du$

$\ln x = \int \frac{1-\cos 2u}{2}\,du \to \ln x = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

$\Rightarrow \ln x = \frac{1}{2}\frac{y}{x} - \frac{1}{4}\sin\frac{2y}{x} + C$ //

27) $(4x + 3y^2)\,dx + 2xy\,dy = 0$, $\lambda = x^n$ şeklinde bir integral çarpanı bularak çözünüz.

$\underbrace{(4x^{n+1} + 3y^2x^n)}_{P}\,dx + \underbrace{2x^{n+1}y}_{Q}\,dy = 0$

$\frac{\partial P}{\partial y} = 6yx^n$ $\left.\begin{matrix}\\ \\ \end{matrix}\right]$ $= 0$ olması için

$\frac{\partial Q}{\partial x} = 2(n+1)x^n y$ $\left.\begin{matrix}\\ \\ \end{matrix}\right]$

$6yx^n = 2(n+1)x^n y$

$6 = 2(n+1)$

$n+1 = 3$

$n = 2$ olmalı

$\Rightarrow$ İntegral çarpanı : $\lambda = x^2$

$\Rightarrow (4x^3 + 3x^2y^2)\,dx + 2x^3y\,dy = 0$ Tam dif. denk. olur.

devamı size :)

28) $y = c_1 x + c_2 x \ln x$ eğri ailesinin dif denk.i?

$y' = c_1 + c_2 \ln x + c_2 x \frac{1}{x}$

$y'' = c_2 \cdot \frac{1}{x} \rightarrow c_2 = xy''$

$\rightarrow y - xy' = -c_2 x$

$y - xy' = -x^2 y'' \Rightarrow x^2 y'' - xy' + y = 0$

29) $y = (2+y')x + y'^2$

Lagrange dif. denk.

$y' = p \rightarrow y = x(2+p) + p^2$

$\downarrow y' = 2 + p + x\frac{dp}{dx} + 2p\frac{dp}{dx}$

$\downarrow p$

$\Rightarrow -2 = \frac{dp}{dx}(x+2p) \rightarrow -2\frac{dx}{dp} = x + 2p$

$\Rightarrow \frac{dx}{dp} + \frac{1}{2}x = -p$ Lineer dif denk.

$\int p e^{p/2} dp =$

30) $yy' = xy'^2 + a$

$y = xy' + \frac{a}{y'}$ clairaut dif. denk.

$y = p \rightarrow y = xp + \frac{a}{p} \rightarrow \frac{dy}{dx} = p + x\frac{dp}{dx} - \frac{a}{p^2}\frac{dp}{dx}$

$\downarrow p$

$p'\left(x - \frac{a}{p^2}\right) = 0 \rightarrow 1°)\ p' = 0 \rightarrow p = c \Rightarrow \begin{cases} y = xp + \frac{a}{p} \\ p = c \end{cases}$

$2°)\ x - \frac{a}{p^2} = 0$

$\Rightarrow y = xc + \frac{a}{c}$ ç.ç.

$\begin{cases} y = xp + \frac{a}{p} \\ x = \frac{a}{p^2} \rightarrow \frac{a}{p} = xp \end{cases} \rightarrow y = xp + xp$

$y = 2xp \rightarrow y^2 = 4x^2p^2$

$y^2 = 4ax$ Tekil çözüm

31) $x^2 \ln x\, y' + xy = 1$

$y' + \frac{1}{x\ln x} y = \frac{1}{x^2 \ln x}$ Lineer denk.

$\lambda(x) = e^{\int \frac{1}{x\ln x} dx} = e^{\int \frac{du}{u}} = e^{\ln u} = e^{\ln(\ln x)} = \ln x$

$y = \frac{1}{\ln x}\left[\int \frac{1}{x^2 \ln x} \ln x\, dx + c\right] = \frac{1}{\ln x}\left[-\frac{1}{x} + c\right]$

$\Rightarrow y \ln x = -\frac{1}{x} + c$ //

32) $\cos y + (1+e^{-x}) \sin y \cdot y' = 0$, $y(0) = \frac{\pi}{4}$

başlangıç değer problemini çözünüz.

$\frac{dx}{1+e^{-x}} + \frac{\sin y}{\cos y} dy = 0$

$1+e^{-x} = t$

$-e^{-x} dx = dt$

$dx = -\frac{dt}{t-1}$

$e^{-x} = t$

$-e^{-x} dx = dt$

$dx = -\frac{dt}{t}$

$\int \frac{dx}{1+e^{-x}} = \int \frac{dx}{1+\frac{1}{e^x}}$

$= \int \frac{e^x}{e^x+1} dx$

$= \ln(e^x+1)$

$-\int \frac{dt}{t(t-1)} = \int \left(\frac{1}{t} - \frac{1}{t-1}\right) dt$

$= \ln t - \ln(t-1) + c$

$= \ln \frac{t}{t-1} + c$

$-\int \frac{dt}{t(1+t)}$

$\Rightarrow \ln(e^x+1) - \ln(\cos y) = \ln c$

$\frac{e^x+1}{\cos y} = c$ g.ç.

$x=0,\ y=\frac{\pi}{4}$: $\frac{e^0+1}{\cos\frac{\pi}{4}} = c \rightarrow c = \frac{2}{\frac{\sqrt{2}}{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ //

$\Rightarrow$ Özel çözüm: $\frac{e^x+1}{\cos y} = 2\sqrt{2} \xrightarrow{\text{VEYA}} 2\sqrt{2}\cos y = e^x + 1$