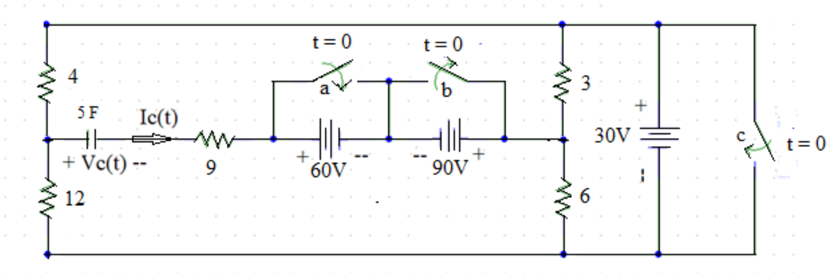
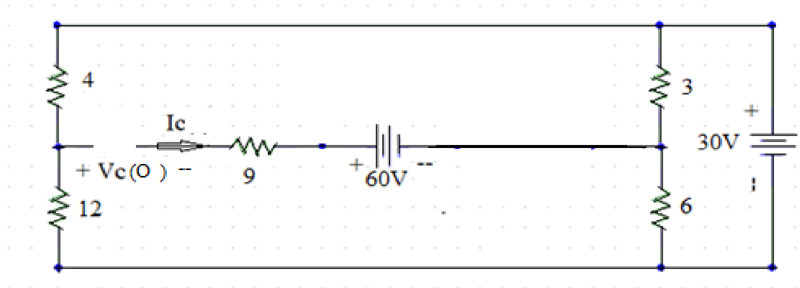


a ve c anahtarları uzun süre açık kaldıktan sonra t = 0 anında a ve c anahtarları kapatılıyor .

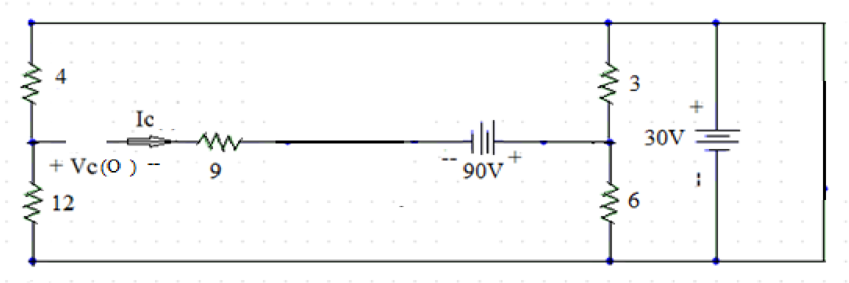
b anahtarı ise uzun süre kapalı kaldıktan sonra t = 0 anında b anahtarı açılıyor. Vc(t) ve Ic(t)' yi bulunuz.



$+ V_c(0^-) + 9x0 + 60 - (30/9)x3 + (30/16)x4 = 0$

$V_c(0^-) = - 57,5$ V $= V_c(0^+)$

$+ V_c(0^-) + 9x0 + 60 + (30/9)x6 - (30/16)x12 = 0$



$V_c(\infty) = 90$ V

$Reş=((4x12)/(4+12) + 9 + (3X6)/(3+6)) = 14$

τ = ReşxC = 14x5 = 70

$V_c(t) = A.e^{-(t/70)} + 90$

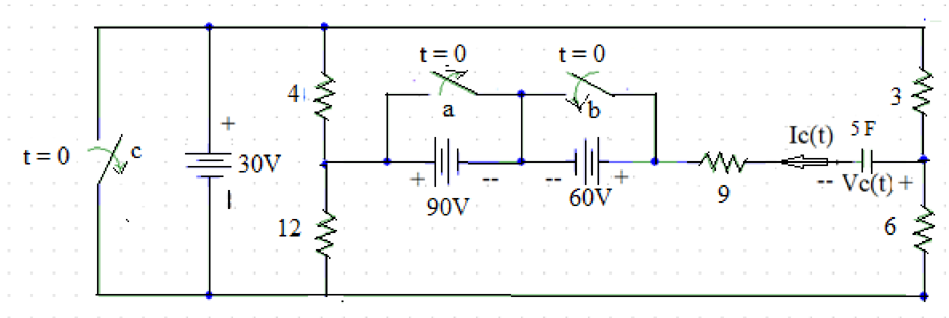
t = 0 için $V_c(0^+) = - 57,5$

$- 57,5 = A.1 + 90$

$A = - 147,5$

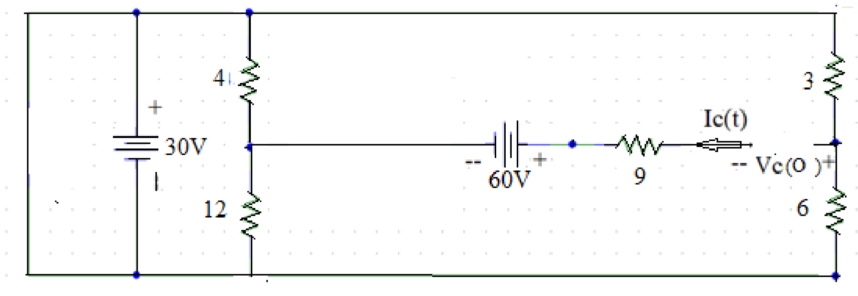
$V_c(t) = - 147,5 .e^{-(t/70)} + 90$

$I_c(t) = C \times (dV_c(t) / dt) = 5.(-147,5).(- 1/70).e^{-(t/70)} = 10,536.\ ).e^{-(t/70)}$

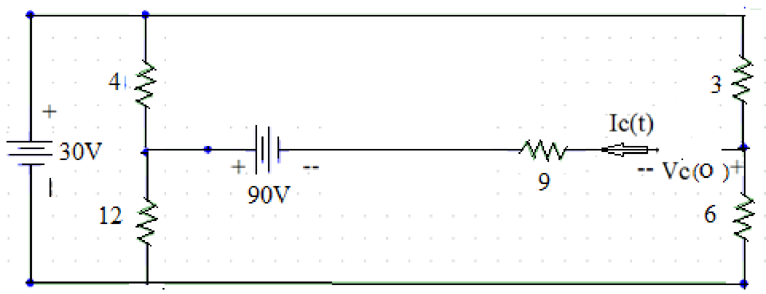


a ve c anahtarları uzun süre kapalı kaldıktan sonra t = 0 anında a ve c anahtarları açılıyor.

b anahtarı ise uzun süre açık kaldıktan sonra t = 0 anında b anahtarı kapatılıyor. Vc(t) ve Ic(t)' yi bulunuz.



$Vc(0^-)$ = - 60 V



+ Vc(∞) + 9x0 - 90 -4x(30/16) + 3x(30/9) = 0

Vc(∞) = 87,5 V

Yada + Vc(∞) - 9x0 - 90 + 12x(30/16) - 6 x(30/9) = 0

Vc(∞) = 87,5 V

Reş=((4x12)/(4+12) + 9 + (3X6)/(3+6)) = 14

τ = ReşxC = 14x5 = 70

Vc(t) = A.$e^{-(t/70)}$ + 87,5

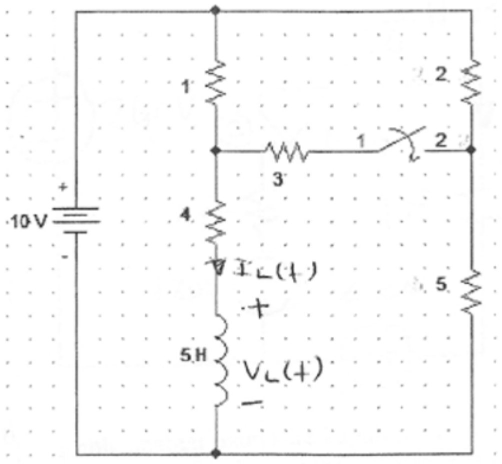
t = 0 için $Vc(0^+)$ = - 60

- 60 = A.1 + 87,5

A = - 147,5

Vc(t) = - 147,5 .$e^{-(t/70)}$ + 87,5

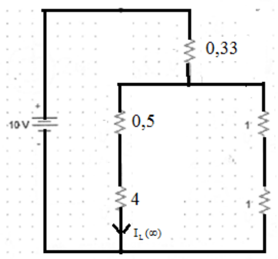
Ic(t) = C x (dVc(t) / dt) = 5.(147,5).(- 1/70).$e^{-(t/70)}$ = 10,53.).$e^{-t/}$



Anahtar uzun süre açık kaldıktan sonra t = 0 anında kapatılıyor.
t ≥ 0 için $I_L(t)$'yi ve $V_L(t)$' yi bulunuz..

$$I_L(0^-) = \frac{10}{\frac{(5+2).(4+1)}{(5+2+4+1)}} \cdot \frac{5+2}{(5+2+4+1)} = 2A$$

Üçgen yıldız dönüşümü yapılırsa İkinci durum devresi



$$I_L(\infty) = \frac{10}{0{,}33 + \frac{(0{,}5+4).(1+5)}{(0{,}5+4+1+5)}} \cdot \frac{(1+5)}{(0{,}5+4+1+5)} = 1{,}97A$$

τ = L/$R_{eş}$ =5/4,81 =1,038

$$I_L(t) = A.e^{-\frac{t}{\tau}} + I_L(\infty) = A.e^{-\frac{t}{1{,}038}} + 1{,}97\,A$$

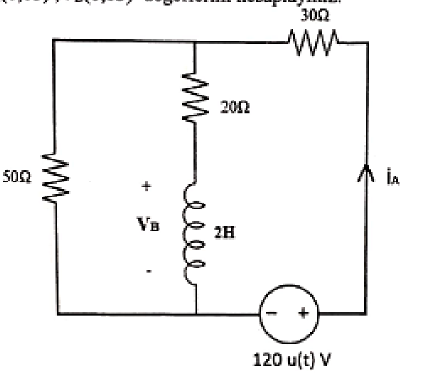
t = 0 için $I_L(0^+)$ =2 A

$2 = A.1 + 1{,}97$ A=0,03

$$I_L(t) = 0{,}03.e^{-0{,}963t} + 1{,}97A$$

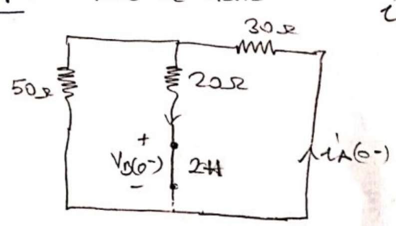
$$V_L(t) = L.\frac{d_{iL}}{dt} = 5x\left((0{,}03x(-0{,}963)\right).e^{-0{,}963t} = -0{,}144.e^{-0{,}963t}V$$

Aşağıda verilen devre için, a) $i_A(0^-)$, $V_B(0^-)$ b) $i_A(0^+)$, $V_B(0^+)$ c) $i_A(\infty)$, $V_B(\infty)$ d) $t \geq 0$ için $i_A(t)=?$, $V_B(t)=?$ e) $i_A(0{,}03)$, $V_B(0{,}03)$ değerlerini hesaplayınız.



35p Cevap 1

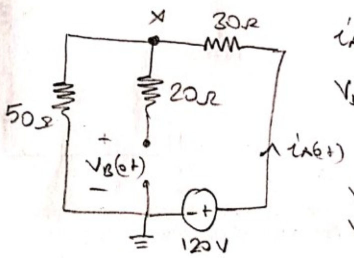
a) t=0⁻ da devre



$i_A(0^-) = \frac{V_{kaynak}}{R_{eş}} = \frac{0}{R_{eş}} = 0\,A$ 5p

$V_B(0^-) = 0\,V$ (kısa devre)

b) t=0⁺ da devre,



$i_A(0^+) = \frac{120}{50+30} = 1{,}5\,A$

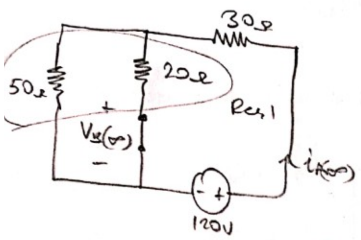
$V_B(0^+) = V_X$

$V_B(0^+) = i_A(0^+) \cdot 50$ 5p

$V_B(0^+) = 1{,}5 \cdot 50$

$V_B(0^+) = 75\,V$

c) t=∞ için devre,



$R_{eş1} = \frac{50 \cdot 20}{50+20} = \frac{100}{7}$

$R_{eş} = \frac{100}{7} + 30$

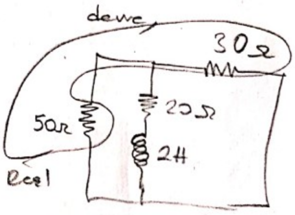
$R_{eş} = \frac{310}{7} = 44{,}286\,\Omega$

$i_A(\infty) = \frac{120}{R_{eş}} =$

$i_A(\infty) = \frac{120}{44{,}286} = 2{,}71\,A$ 5p

$V_B(\infty) = 0\,V$ (Kısa Devre)

d) Öz çözüm için kaynaksız devre,



$R_{eş} = R_{eş1} + 20\,\Omega$

$R_{eş} = \frac{50 \cdot 30}{(50+30)} + 20 = 38{,}75\,\Omega$

$\tau = \frac{L}{R} = \frac{2}{38{,}75} = 0{,}0516\,s$ 5p

$i_A(t) = i_{A\,zor}(t) + i_{A\,öz}(t)$

$i_A(t) = 2{,}71 + A\exp\left(\frac{-t}{\tau}\right)$

$i_A(t) = 2{,}71 + A\exp(-19{,}38\,t)$

$i_A(0) = 2{,}71 + A\exp(0) = 1{,}5$

$\boxed{A = -1{,}21}$ 5p

$i_A(t) = 2{,}71 - 1{,}21\exp(-19{,}38\,t)$

$V_B(t) = V_{B\,zor}(t) + V_{B\,öz}(t)$

$V_B(t) = 0 + B\exp\left(\frac{-t}{\tau}\right)$

$V_B(t) = B\exp(-19{,}38\,t)$

$V_B(0) = B\exp(0) = 75$ $\boxed{B = 75}$

$V_B(t) = 75\exp(-19{,}38\,t)$ 5p

e)

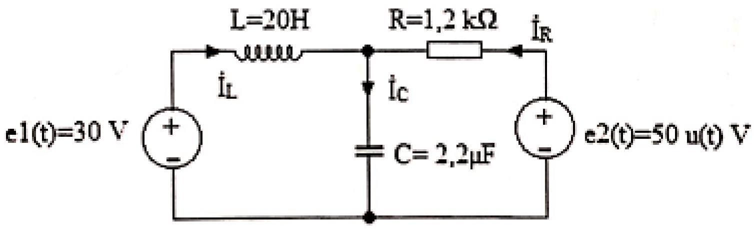
$i_A(0{,}03) = 2{,}71 - 1{,}21\exp(-19{,}38 \cdot 0{,}03)$

$i_A(0{,}03) = 2{,}035\,A$

$V_B(0{,}03) = 41{,}334\,V$

5p

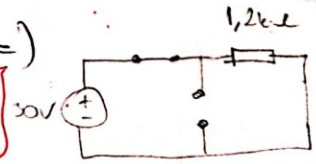
2) (Zorunlu) Aşağıda verilen devrede **a)** $\dot{I}_L(0^-)$, $V_C(0^-)$ ve $\dot{I}_R(0^+)$ değerlerini bulunuz. **b)** $t \geq 0$ için kondansatörün uç gerilimi, '$V_C(t)$' ifadesini bulunuz.



Cevap Anahtarı ~~[illegible]~~ Devre Analizi ve Sistemleri 2. Arasınav 2020-2021

Cevap 2

a) $t<0$ için devre ($t=0^-$ de devre)



$i_L(0^-) = \frac{30}{1.200} = 0,025\ A \Rightarrow 25\ mA$ 5

$V_C(0^-) = 30V$ kaynak gerilimi 5

$i_R(0^+) = \frac{50-30}{1.200} = 0,01666 \Rightarrow 16,67$ 5

$i_L(0^+) + i_R(0^+) = i_C(0^+) = 41,7\ mA$

b) $\alpha = \frac{1}{2RC} = \frac{1}{2.1200.10^{-6}.2,2} = 189,4\ rad/s$

$\alpha > \omega_0$ aşırı sönümlü

$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{20.2,2.10^{-6}}} = 150,76\ rad/s$

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$
$= -189,4 + \sqrt{189,4^2 - 150...}$
$= -74,75$

$V_C(0^+) = V_C(\infty) + A_1 \exp(-74,75t) + A_2 \exp(s_2 t)$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$= -304,04$ 5

$V_C(0^+) = 30 + A_1 \exp(-74,75t) + A_2 \exp(-304,04t) = 30 \Rightarrow A_1 + A_2 = 0$

$i_C(0^+) = i_L(0^+) + i_R(0^+) = 25 + 16,67 = 41,67\ mA$ 5

$i_C(t) = C.\frac{dv(t)}{dt} = 2,2.10^{-6}(-74,75A_1 - 304,04A_2) = 41,67$

$-74,75A_1 - 304,04A_2 = 18,94.10^3$

$304,04 /$

$A_1 + A_2 = 0$

$229,29 A_1 = 18,94.10^3$

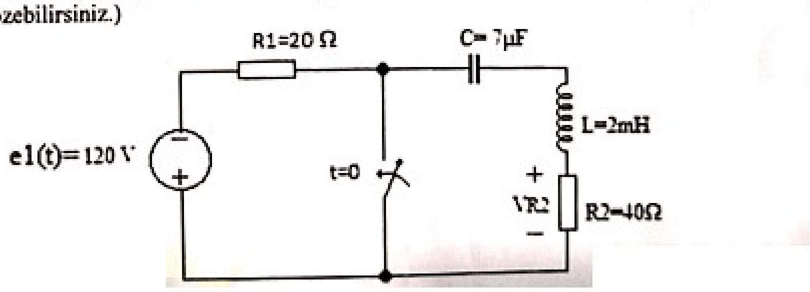
$A_1 = 82,6$

$A_2 = -82,6$

$V_C(t) = 30 + 82,6 \exp(-74,75t) - 82,6 \exp(-304,04t)$

10

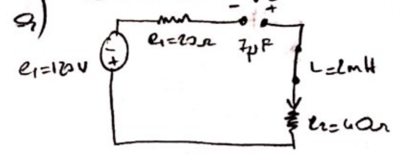
3) (Seçimlik) Aşağıda verilen devrede t=0 anında anahtar kapatılmaktadır. **a)** $I_L(0^-)$, $V_C(0^-)$ ve $V_{R2}(0^+)$ değerlerini bulunuz. **b)** $t \geq 0$ için R_2 direncinin uç gerilimi, $V_{R2}(t)$ ifadesini bulunuz. (s veya t domeninde çözebilirsiniz.)



Soru 3

t = ∞ için devre

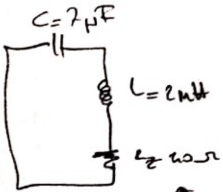
a) t = 0⁻ da devre



$V_C(0^-) = 120\,V$

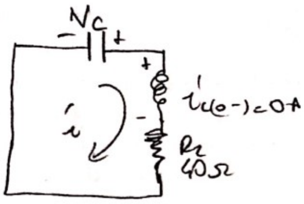
$i_L(0^-) = 0\,A$ 5

5



$i(\infty) = 0\,A$ 0

t = 0⁺ da devre,



$i(0^+) = i_L(0^+) = 0A$ 0

$V_{R2}(0^+) = i(0^+) \cdot R_2 = 0 \cdot 40 = 0\,V$ 0 5

$V_L(0^+) = \cancel{6}\,120$ 0

b) $\alpha = \dfrac{R}{2L} = \dfrac{40}{2 \cdot 2 \cdot 10^{-3}} = 10^4 = 10000\ rad/s$

$\alpha > w_0$ aşırı sönümlü

$w_0 = \sqrt{\dfrac{1}{LC}} = \sqrt{\dfrac{1}{2 \cdot 10^{-3} \cdot 7 \cdot 10^{-6}}} = 8451\ rad/s$

$s_1 = -\alpha + \sqrt{\alpha^2 - w_0^2}$

$= -10000 + \sqrt{10000^2 - 8451^2} = -4654$

$i(t) = i(\infty) + A_1 \exp(-4654t) + A_2 \exp(-15346t)$

$s_2 = -15346$ 5

$i(0) = A_1 \exp(0) + A_2 \exp(0) = 0 \Rightarrow A_1 + A_2 = 0$

$V_L(0^+) = L \cdot \left.\dfrac{di(t)}{dt}\right|_{t=0} = L(-4654 A_1 - 15346 A_2) = 120$

4654 / $A_1 + A_2 = 0$

$-4654 A_1 - 15346 A_2 = 60000$

$-10692\,A_2 = 60000$

$i(t) = 5{,}61 \exp(-4654t) - 5{,}61 \exp(-15346t)$

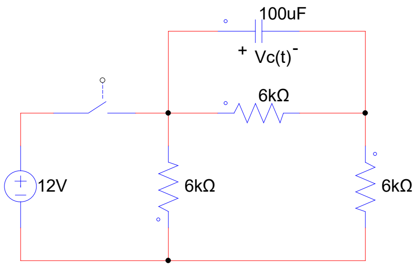
$A_2 = -5{,}61$

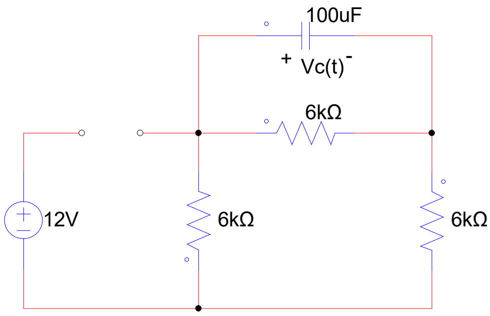
$V_{R2}(t) = i(t) \cdot 40 = 224{,}46 \exp(-4654t) - 224{,}46 \exp(-15346t)$

$A_1 = 5{,}61$

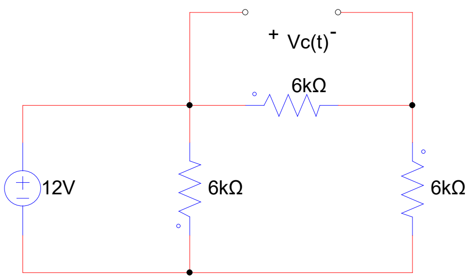
10

t= 0 anında anahtar kapatılıyor. t>0 için $V_C(t)$ = ?

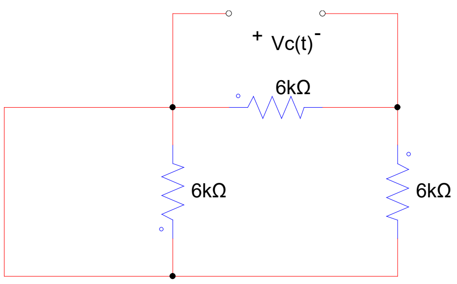




$V_{C(0^-)} = 0 = V_{C(0^+)}$; $I_{C(0^-)} = 0 = I_{C(0^+)}$



$V_{C(\infty)} = 6.10^3 x 1.10^{-3} = 6V$; $I_{C(\infty)} = 0A$

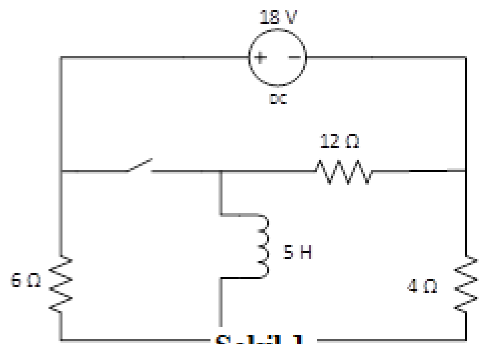


$\tau = R.C$; $R_{eş} = \frac{6000x6000}{6000+6000} = 3000 = 3k\Omega$

$\tau = 3.10^3 x 100.10^{-6} = 3.10^{-1} = \frac{3}{10}$

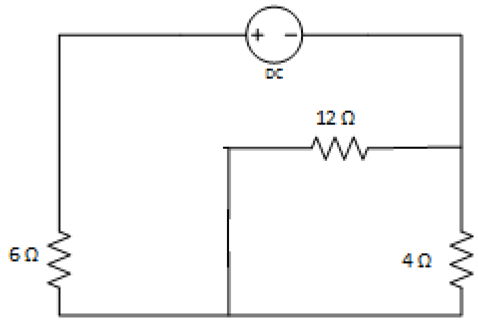
$V_C(t) = A.e^{-t/\tau} + V_{C(\infty)}$

t=0 için $V_{C(0^+)}$ =0 V 0 = A.1 + 6 ; A = -6 **$V_C(t) = -6.e^{-10t/3} + 6$ V**



Şekil 1

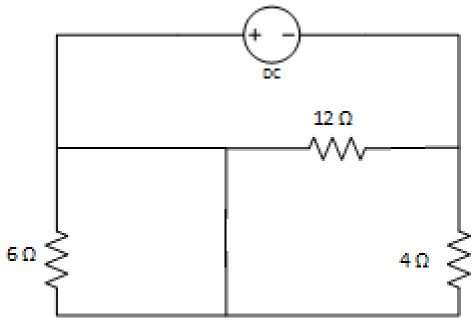
Şekildeki devrede anahtar t=0 anında kapatılıyor. Buna göre **t>0** için $\boldsymbol{I_L}$**(t)** ve $\boldsymbol{V_L}$**(t)**'yi bulunuz.



Çözüm; Öncelikle devremizin $\boldsymbol{t = 0^-}$ anı için olan durumunu incelemeliyiz. Ve $\boldsymbol{V_L(0^-)}$ **ve** $\boldsymbol{i_L(0^-)}$ değerlerini bulmalıyız. $\boldsymbol{t = 0^-}$ için devrenin hali;

$$\boldsymbol{i_k(0^-) = \frac{V}{Reş} = \frac{18}{9} = 2A}$$

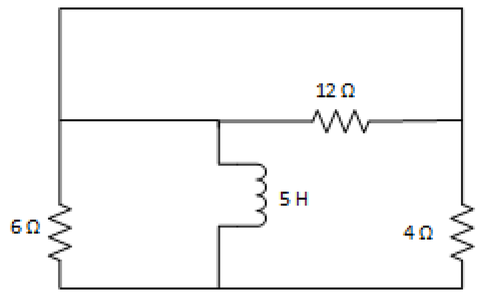
$$i_L(0^-) = -2.\frac{4}{4+12} = -0{,}5A = i_L(0^+) \qquad \boldsymbol{V_L 0^-) = 0}$$



Bundan sonra $\boldsymbol{t} = \infty$ için devremizi incelemeliyiz.

$$\boldsymbol{i_L(\infty) = \frac{18}{3} . \frac{12}{4+12} = 4,5A} \qquad \boldsymbol{V_L(\infty) = 0}$$

Daha sonra τ'yı bulmalıyız.



$$\boldsymbol{R_{eş} = \frac{4x6}{4+6} = 2,4}$$

$$\boldsymbol{\tau = \frac{L}{R_{eş}} = \frac{5}{2,4} = 2,088}$$

$\boldsymbol{I_L}$**(t) = A.**$\boldsymbol{e^{-t/\tau}}$ + $\boldsymbol{I_L}$(∞)

$\boldsymbol{I_L(\mathbf{t}) = \mathbf{A}.e^{-\frac{t}{2{,}088}} + 4{,}5}$

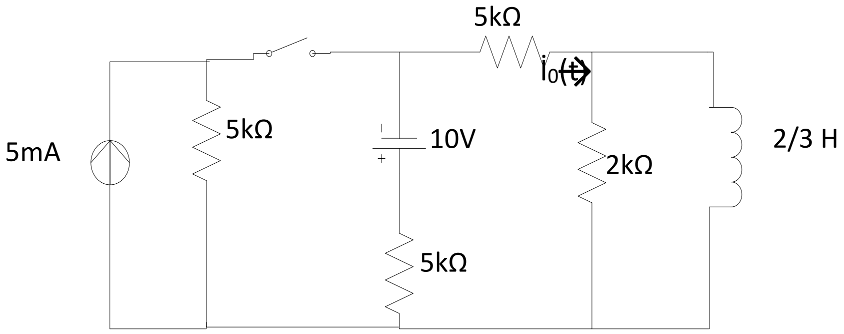
A'yı bulmak için t=0 veririz $\boldsymbol{I_L(0^+)} =$ **-0,5 A** bulunmuştu

-0,5= A.1 + 4,5 'dan A = -5 bulunur

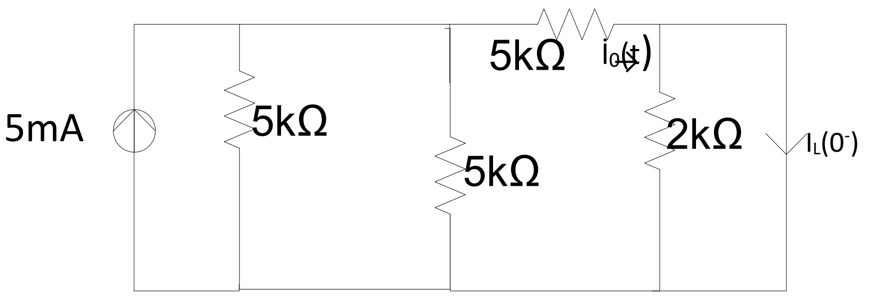
$\boldsymbol{I_L(\mathbf{t}) = -5.e^{-\frac{t}{2{,}088}} + 4{,}5}$

$\boldsymbol{V_L(t)} = \mathbf{L}\ .\ \boldsymbol{\frac{dI(t)}{dt}}$

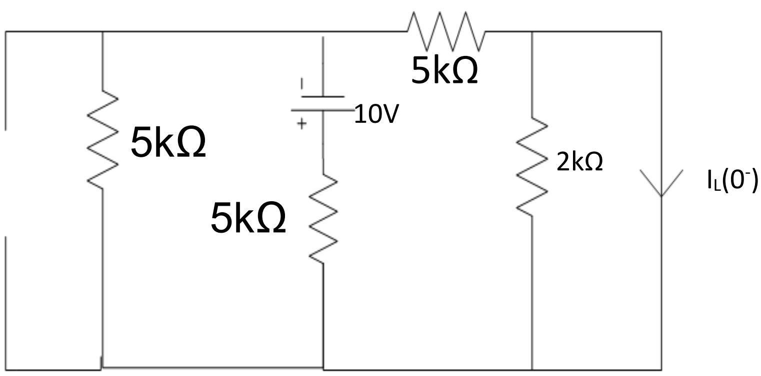
$.\boldsymbol{V_L(\mathbf{t}) = 5\,(-5).\frac{1}{2{,}088}\,e^{-\frac{t}{2{,}088}}}$



Anahtar uzun süre kapalı kaldıktan sonra t= anında açılıyor. Buna göre **t≥0** için $\boldsymbol{I_L}$**(t)** ve $\boldsymbol{V_L}$**(t)**'yi bulunuz.

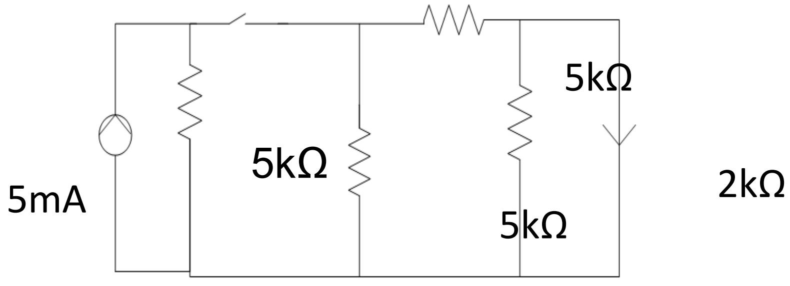


$$i_{L1}(0^-) = 5.\frac{\frac{5000x5000}{5000+5000}}{\frac{5000x5000}{5000+5000}+5000} = 1{,}666mA = i_{L1}(0^+)$$

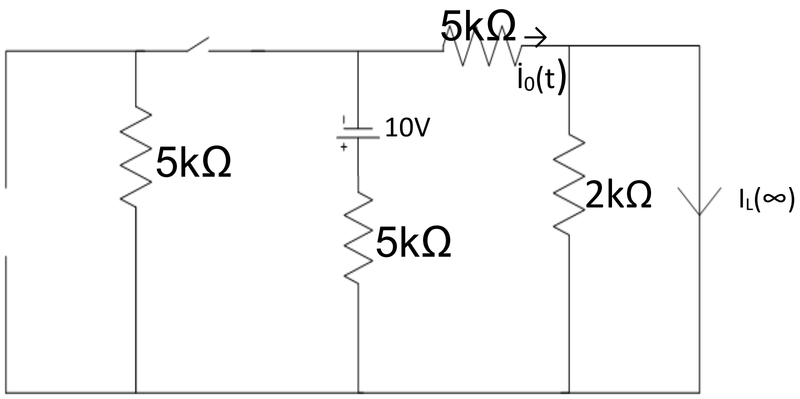


$$i_{L2}(0^-) = \frac{10}{\frac{5000x5000}{5000+5000}+5000} \cdot \frac{5000}{\frac{5000x5000}{5000+5000}} = -0,6661,666mA = i_{L2}(0^+)$$

$$i_L(0^+) = i_{L1}(0^+) + i_{L2}(0^+) = 1,666 - 0,666 = 1A$$

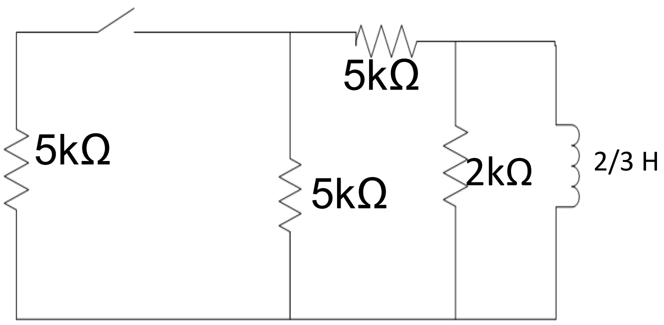


$I_{L1}(\infty)=0$



$R_{eş}=5+5 = 10$ kΩ $I_{L2}(\infty)=(10\ V/10\ k\Omega) = -1$ mA

$I_L(\infty) = 0-1 = -1$ mA



I

$R_{eş} = ((10\times2)\div(10+2))$

$= 5 / 3$ kΩ

$L/R_{eş} = ((2/3)\div(5/3))=0,4m$

$I_L(t) = A.e^{-2500t} + I_L(\infty)$ t= 0   1=A-1   A=2 m A

$I_L(t) = 2.10^{-3} .e^{-2500t} - 1.10^{-3}$ A

1. durumda; self kısa devre olarak bulunur.

$-\infty \cdot\cdot \rightarrow 0$ aralığında $I_L(0^+) = I_L(0^-) = ?$

$$\begin{vmatrix} 1+3 & 3 \\ 3 & 2+4+3 \end{vmatrix} \begin{vmatrix} I_L(0^-) \\ I_2(0^-) \end{vmatrix} = \begin{vmatrix} 60 \\ 24 \end{vmatrix}$$

$I_L(0^+) = 17{,}33A = I_L(0^-)$

2. durumda; self kısa devre olarak bulunur $0 \cdot\cdot \rightarrow +\infty$ aralığında $I_L(\infty) = ?$ $I_{L2}(\infty) = \frac{60}{1} = 60A$

Zaman sabiti $\tau = L/R_{eş}$ $R_{eş} = 1\ \Omega$ $\tau = L/R_{eş} = 9/1 = 9$ bulunur.

$I_L(t) = A\,e^{-t/\tau} + I_L(\infty)$ $I_L(t) = A.e^{-t/9} + 60$

t = 0 için $I_L(0^+) = 17{,}33A$ $17{,}33 = A + 60$ $A = -42{,}67$

$I_L(t) = -42{,}67\,e^{-t/9} + 60$ $V_L(t) = L\,\frac{di_L}{dt}$ $V_L(t) = 42{,}6.e^{-t/9}$ V

1. durumda; kondansatör açık devre olarak bulunur.

$-\infty \cdot\cdot \rightarrow 0$ aralığında $V_c(0^-) = V_c(0^+) = ?$ $V_c(0^-) = -12{,}44V = V_c(0^+)$

2. durumda; kondansatör açık devre olarak bulunur $0 \cdot\cdot \rightarrow +\infty$ aralığında $V_C(\infty) = ?$

$$-V_c(\infty) + 4x4 = 0 \quad V_c(\infty) = 16V$$

Zaman sabiti $\tau = R_{eş}xC$ $R_{eş} = \frac{2x4}{2+4} = \frac{4}{3}$ $\tau = R_{eş}xC = \frac{4}{3}$ x5 = $\frac{20}{3}$ bulunur

Tam çözüm $V_C(t) = A\,e^{-t/\tau} + V_C(\infty)$

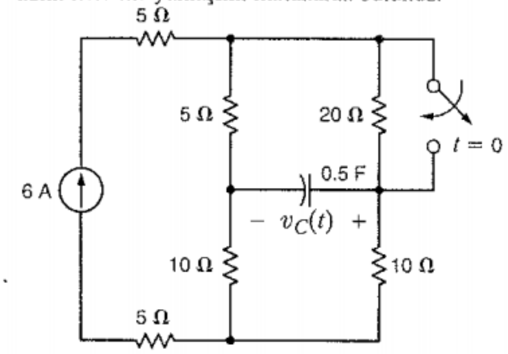
$V_C(t) = A.e^{-t/(4/3}+ 16$ A katsayısı başlangıç şartı olan $V_C(0^+)$'dan bulunur.

t = 0 için $V_C(0^+) = -12{,}44$ V $12{,}44 = A - 16$ A= 28,44

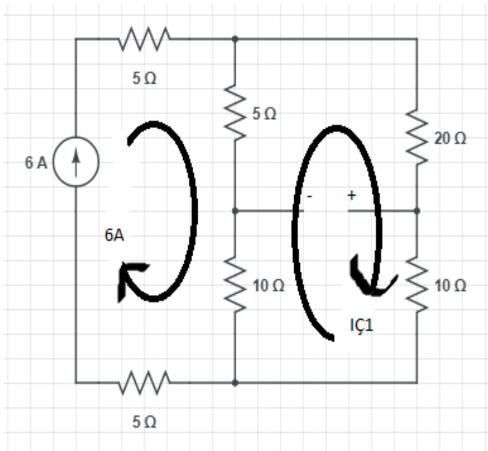
$v_C(t) = 28{,}44.\,.e^{-t/(4/3)} + 16$ V

$I_C(t) = C \times \frac{dv_C}{dt}$ $I_C(t) = -106{,}65.\,.e^{-t/(4/3)}$ A bulunur

7.59 Şekil 7.59'daki devrede $t > 0$ için $v_c(t)$ gerilimini adım adım ilerleme yaklaşımı kullanarak bulunuz.



Şekil P7.59



45IÇ1-90=0

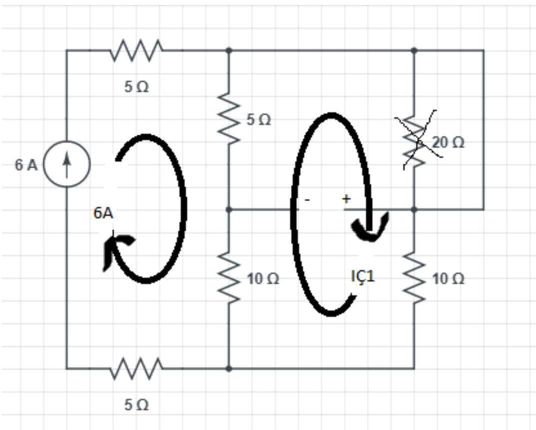
45IÇ1=90

IÇ1=2 amper

-Vc(0)+10.2+(-4).10=0

$V_c(0^-)=V_c(0^+)=-20$ volt

2.DURUM DEVRESİ



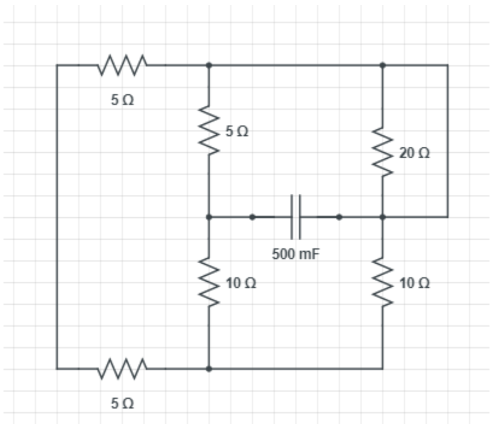
25IÇ1-90=0

25IÇ1=90

IÇ1=3.6 Amper

(-2,4).10-Vc(∞)+(3.6).10=0

Vc(∞)=12 volt



$R_{eş}=4$

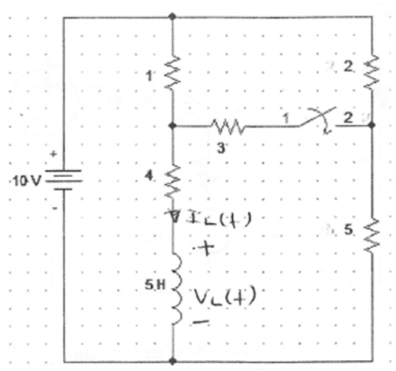
$$\tau = R_{eş} x C = 0.5x4 = 2$$

$$V_C(t) = A.e^{-\frac{t}{\tau}} + V_C(\infty)$$

t = 0 için $V_C(0^+)=-20$

-20=A+12 A=-32

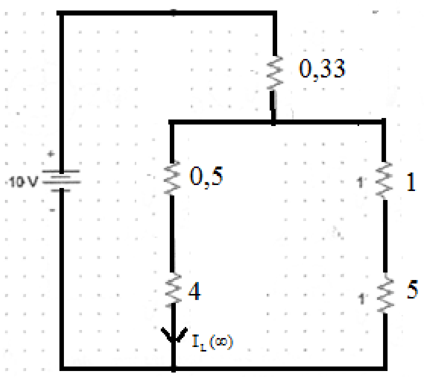
CEVAP: $V_C(t) = -32e^{-t/2}+12$



Anahtar uzun süre açık kaldıktan sonra t = 0 anında kapatılıyor.
t ≥ 0 için $I_L(t)$'yi ve $V_L(t)$' yi bulunuz..

$$I_L(0^-) = \frac{10}{\frac{(5+2).(4+1)}{(5+2+4+1)}} \cdot \frac{5+2}{(5+2+4+1)} = 2A$$

Üçgen yıldız dönüşümü yapılırsa



$$I_L(\infty) = \frac{10}{0{,}33 + \frac{(0{,}5+4).(1+5)}{(0{,}5+4+1+5)}} \cdot \frac{(1+5)}{(0{,}5+4+1+5)} = 1{,}97A$$

τ = L/$R_{eş}$ =5/4,81 =1,038

$$I_L(t) = A.e^{-\frac{t}{\tau}} + I_L(\infty) = A.e^{-\frac{t}{1{,}038}} + 1{,}97\,A$$

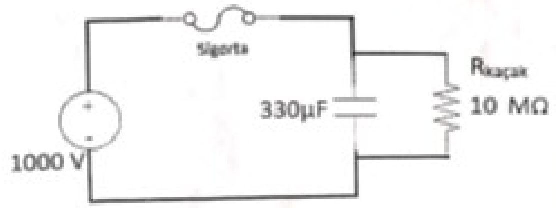
t = 0 için $I_L(0^+)$ =2 A

$2 = A.1 + 1{,}97$ A=0,03

$$I_L(t) = 0{,}03.e^{-0{,}963t} + 1{,}97A$$

$$V_L(t) = L.\frac{d_{iL}}{dt} = 5x\big((0{,}03x(-0{,}963)\big).e^{-0{,}963t} = -0{,}144.e^{-0{,}963t}V$$

3. Kaçak direnci 10MΩ olan 330µF'lık yüksek gerilim kondansatörünün bulunduğu devre aşağıda verilmişti 50 Volt'un üzerindeki gerilimlerin insan sağlığı açısından tehlikeli olduğu bilindiğine göre, a) Devrede sigorta attıktan(Kondansatörün gerilim kaynağıyla bağlantısı koptuktan) kaç saniye sonra kesin olara kondansatör uçlarına dokunulabilir? b) Sigorta attıktan 20 saniye sonra insanların kondansatör uçların dokunma ihtimali söz konusu ise insanların elektrik akımına kapılmaması için (Kaynak gerilimi aynı kalmak kaydıyla) ne tür bir önlem alınabilir? Hesaplayarak yanıtlayınız.



30

a) RC devresinin zaman sabiti

$$\tau = R.C = 10.10^{6}.330.10^{-6} \boxed{= 3300\, s}$$

$$V_c(0) = V_{kaynak} = 1200\, V$$

$$V_c(t) = V_0 \exp(-t/\tau) = 1200\, e^{\left(\frac{-t}{3300}\right)}$$

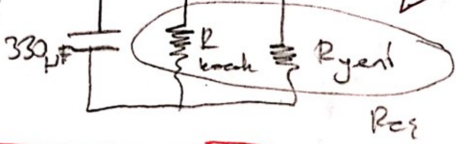
$$50 = 1200\, e^{\left(\frac{-t}{3300}\right)}$$

$$\ln\left(\frac{50}{1200}\right) = \left(\frac{-t}{3300}\right) \Rightarrow \boxed{t = 10487,6\, s \quad 10p}$$

5p

b) Devrede R kaçak direncine paralel bir direnç ekleyerek devrenin zaman sabiti (τ)'yu düşürebiliriz. Yeni oluşacak devrede zaman sabiti τ^* olsun

$$50 = 1200\, e^{\left(\frac{-20}{\tau^*}\right)}$$



$$\ln\left(\frac{50}{1200}\right) = \left(\frac{-20}{\tau^*}\right) \Rightarrow \boxed{\tau^* = 6,293\, s \quad 5p}$$

$$\tau^* = R_{eş}.C = 6,293 = R_{eş}.330.10^{-6} \Rightarrow R_{eş} = 19,07\, k\Omega$$

$$R_{eş} = \frac{R_{kaçak}.R_{yeni}}{(R_{kaçak}+R_{yeni})} = 19070 = \frac{10.10^6 R_{yeni}}{(10.10^6 + R_{yeni})}$$

$$19,07\, 10^{10} + 19070\, R_{yeni} = 10.10^6\, R_{yeni} \Rightarrow \boxed{R_{yeni} = 19,106 \quad 10p}$$

$$19,07 . 10^{10} = R_{yeni}(10.10^6 - 19070)$$

191,

3.62 A coil has a resistance of 5 Ω and an inductance of 1 H. At $t = 0$ it is connected to a 2 V battery. Find (a) the rate of rise of current at $t = 0$; (b) the rate of rise of current when $i = 0.2$ Amps and (c) the stored energy when $i = 0$ and $i = 0.3$ A.

Solution

$$\Upsilon = \frac{L}{R} = \frac{1}{5} = 0.2 \text{ sec.}$$

(a) $\dfrac{di}{dt} = \dfrac{E}{L} e^{-\frac{t}{\Upsilon}} = 2e^{-5t}$

at $t = 0$, $\dfrac{di}{dt} = 2$ A/sec.

(b) $i = \dfrac{E}{R}(1 - e^{-5t})$

$= 0.4(1 - e^{-5t})$

time t_1 when $i = 0.2$ A is

$$0.2 = 0.4\,(1 - e^{-5t_1})$$

or $$t_1 = 0.1386 \text{ sec.}$$

$$\frac{di}{dt} = 2e^{-5(0.1386)}$$

$$= 1 \text{ A/sec}$$

(c) At $i = 0$, stored energy = 0

when $i = 0.3$ A, stored energy

$$= \frac{1}{2}Li^2 = \frac{1}{2} \times 1 \times (0.3)^2 = 0.045 \text{ J.}$$

3.46 A coil having a resistance of 10 Ω and inductance of 15 H is connected across a d.c. voltage of 150 V. Calculate: (i) The value of current at 0.4 sec after switching on the supply. (ii) With the current having reached the final value the time it would take for the current to reach a value of 9 A after switching off the supply.

Solution

It is given that

$$V(d.c) = 150 \text{ V}$$
$$R = 10\ \Omega$$
$$L = 15 \text{ H}$$

(i) ∴ The value of the current

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{150}{10}\left(1 - e^{-\frac{10}{15}\times 0.4}\right)$$

$$= 15\left(1 - e^{-\frac{4}{15}}\right)$$

$$= 3.51 \text{ A}$$

(ii) Let us assume that at $t = t_1$, $i = 9$ A

∴ $$9 = 15 \times e^{-\frac{t_1}{1.5}} \qquad \text{(for decaying)}$$

$$e^{-\frac{t_1}{1.5}} = \frac{9}{15}$$

taking $\log_e$ in both sides,

$$-\frac{t_1}{1.5} = \log_e \frac{9}{15}$$

∴ $$t_1 = 0.7662 \text{ sec.}$$

....

3.47 For the network shown in Fig. 3.36

(a) Find the mathematical expression for the variation of the current in the inductor following the closure of the switch at $t = 0$ on to position 'a';

(b) The switch is closed on to position 'b' when $t = 100$ m/sec, calculate the new expression for the inductor current and also for the voltage across R;

(c) Plot the current waveforms for $t = 0$ to $t = 200$ m/sec.

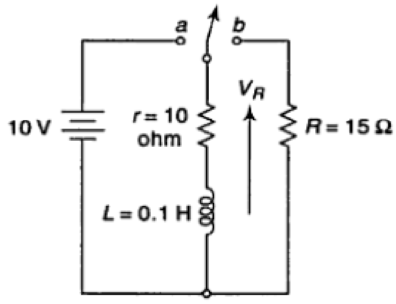


Fig. 3.36 *Network of Ex. 3.47*

Solution

(a) For the switch in position 'a', the time constant is

$$\Upsilon_a = \frac{L}{r} = \frac{0.1}{10} = 10 \text{ milli-sec(ms)}$$

$$\therefore \qquad i_a = \frac{V}{r}\left(1 - e^{-\frac{t}{\Upsilon_a}}\right) = \frac{10}{10}\left(1 - e^{-\frac{t}{10\times 10^{-3}}}\right)$$

$$= \left(1 - e^{-\frac{t}{10^{-2}}}\right) \text{A.}$$

(b) For the switch in position 'b' the time constant

$$\Upsilon_b = \frac{L}{R+r} = \frac{0.1}{15+10} = 4 \text{ ms}$$

$$\therefore \qquad i_b = \frac{V}{R} e^{-\frac{t}{\Upsilon_b}} \qquad \text{(for decaying)}$$

$$= \frac{10}{10} e^{-\frac{t}{4\times 10^{-3}}} = e^{-\frac{t}{4\times 10^{-3}}} \text{ A.}$$

The current continues to flow in the same direction as before, therefore the voltage drop across R is negative to the direction of the arrow shown in Fig. 3.36. $v_R = i_b \cdot R = -15 \times e^{-t/4 \times 10^{-3}}$ V.

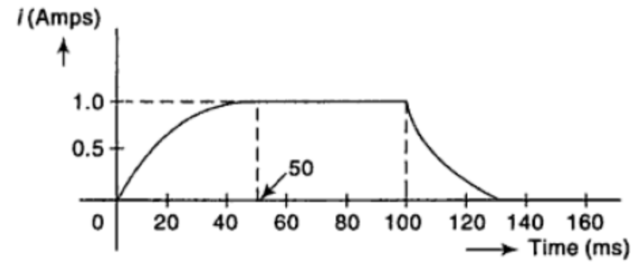


Fig. 3.37 *Current profile*

It will be noted that in the first switched period, five times the time constant is 50 m/sec. The transient has virtually finished at the end of this time and it would not have mattered whether the second switching took place then or later. During the second period the transient took only 25 m/sec.

(c) The profile current waveform has been plotted in Fig. 3.37.

3.48 For the network shown in Fig. 3.36 (Ex. No. 3.47) the switch is closed on the position 'a'. Next, it is closed on to position 'b' when τ = 10 ms. Again, find the expression of current and hence draw the current wave forms.

Solution

For the switch in position 'a', the time constant Υ is 10 m/sec as in Ex. No. 3.47, and the current expression as is before. However, the switch is moved to position 'b' while the transient is proceeding. When t = 10 m/sec.

$$i = \left(1 - e^{-\frac{t}{10\times10^{-3}}}\right) = \left(1.0 - e^{-\frac{10\times10^{-3}}{10\times10^{-3}}}\right)$$

$= (1 - e^{-1}) = 0.632$ A.

i.e., the second transient commences with an initial current in R of 0.632 A.

$\therefore$ The current decay is, $i_b = 0.632 \times \frac{t}{4\times10^{-3}}$ A. which is shown in Fig. 3.38.

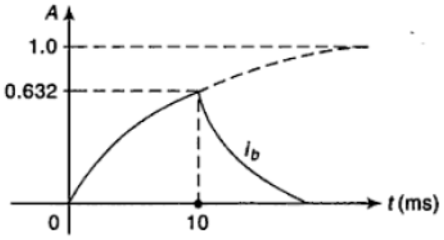


Fig. 3.38

3.49 A d.c. voltage of 150 V is applied to a coil whose resistance is 10 Ω and inductance is 15 H. Find: (i) the value of the current 0.3 sec after switching on the supply; (ii) with the current having reached the final value, how much time it would take for the current to reach a value of 6 A after switching off the supply.

Solution

(a) It is given that

V = 150 V, R = 10 Ω, L = 15 H

$\therefore$ The value of the current 0.3 sec after switching on is

$$i = \frac{150}{10}\left(1 - e^{-\frac{10}{15}\times0.3}\right) = 2.72 \text{ A.}$$

(b) After switching off the supply, the current will be decaying and is given by

$$i = \frac{V}{R} e^{-\frac{R}{L}t} \quad \therefore \quad 6 = \frac{150}{10} e^{-\frac{10}{15}\times t}$$

$\therefore$ t = 1 .375 sec.

3.50 A coil of resistance 24 Ω and having inductor 36 H is suddenly connected to a d.c. of 60 V supply. Determine

(a) the initial rate of change of current $\left(\dfrac{di}{dt}\right)$

(b) the time-constant

(c) the current after 3 sec.

(d) the enrgy stored in the magnetic field at t = 3 sec.

(e) the energy lost as heat energy at t = 3 sec.

Solution

It is given that: V = 60 V, R = 24 Ω, L = 36 H

(a) Initial rate of change of current:

$$i = \frac{V}{R}\left(1-e^{-\frac{R}{L}\cdot t}\right)$$

$$\therefore \quad \frac{di}{dt} = -\frac{V}{R}\cdot\left(-\frac{R}{L}\right)\cdot e^{-\frac{R}{L}\cdot t}$$

$$= \frac{V}{L}e^{-\frac{R}{L}\cdot t};$$

When $t = 0$,

$$\frac{di}{dt} = \frac{V}{L}\cdot e^{o} = \frac{V}{L} = \frac{60}{36} = 1.67 \text{ A/sec:}$$

(b) Time constant (ϒ):

$$\Upsilon = \frac{L}{R} = \frac{36}{24} = 1.5 \text{ sec.}$$

(c) Current; the current at t = 3 sec is

$$i = \frac{V}{R}\left(1-e^{-\frac{R}{L}\cdot t}\right) = \frac{60}{24}\left(1-e^{-\frac{24}{36}\times 3}\right) = 2.16 \text{ A.}$$

(d) Energy stored:

at t = 3 sec, the energy stored in the magnetic field is $\frac{1}{2}Li^2$

$$= \frac{1}{2}\times 36\times(2.16)^2 = 84 \text{ J.}$$

(e) Energy lost as heat energy:

at t = 3 sec, the energy lost as heat energy is

$$i^2\times R = (2.16)^2\times 24 = 112 \text{ J.}$$

3.51 If the vertical component of the earth's magnetic field be 4.0×10^{-5} Wb m^{-2}, then what will be the induced potential difference produced between the rails of a meter-gauge running north-south when a train is running on them with a speed of 36 km h^{-1}?

Solution

When a train is on the rails, it cuts the magnetic flux lines of the vertical component of the earth's magnetic field. Hence, a potential dfifference is induced between the ends of its axle.

Distance between the rails = 1 m; speed of train (v) = 36 km/hr = 10 m/sec. Magnetic field $B_V = 4.0\times10^{-5}$ Wb/m. ∴ The induced potential difference in $e = Bvl = (4.0\times10^{-5})\times10\times1 = 4.0\times10^{-9}$ V.

....

3.52 The current in the coil of a large electromagnet falls from 6 A to 2 A in 10 ms. The induced emf across the coil is 100 V. Find the self-inductance of the coil.

Solution

The self-induced emf is given by

$$e = -L\frac{di}{dt}$$

Here $\quad di = 2 - 6 = -4$ A

$\quad dt = 10$ ms $= 10^{-2}$ sec

and $\quad e = 100$ V

$$\therefore \quad L = -e\frac{dt}{di} = -100\times\frac{10^{-2}}{-4} = 0.25 \text{ H.}$$

....

3.53 The current (in ampere) in an inductor is given by $i = 5 + 16t$, where t is in seconds. The self-induced emf in it is 10 mV. Find (a) the self-inductance, and (b) the energy stored in the inductor and the power supplied to it at $t = 1$.

Solution

The induced emf in the inductor due to current change is

$$|e| = L\frac{di}{dt}$$

$$\therefore \quad L = \frac{|e|}{di/dt}$$

Hence $\quad i = 5 + 16t$, from this, we have

$$\frac{di}{dt} = 0 + 16 = 16 \text{ A sec}^{-1}, \text{ and } e = 10 \text{ mV} = 10 \times 10^{-3} \text{ V}$$

$$\therefore \quad L = \frac{10\times10^{-3}\,V}{15\,A\,\text{sec}^{-1}} = 0.666 \times 10^{-3} \text{ H} = 0.666 \text{ mH}$$

(b) The current at $t = 1$ sec is

$$i = 5 + 16t = 5 + 16 \times 1$$
$$= 21 \text{ A}$$

$\therefore$ Energy stored in the inductor is

$$\frac{1}{2}Li^2 = \frac{1}{2} \times (0.666 \times 10^{-3}) \times (21)^2$$
$$= 137.8 \times 10^{-3} = 137.8 \text{ mJ}$$

Power supplied to the inductor at $t = 1$ sec is

$$P = Ii = (10 \times 10^{-3} \text{ V}) \times 21 = 0.21 \text{ W}.$$

....

3.54 Calculate the self-inductance of an air-cored solenoid, 40 cm long, having an area of cross-section 20 cm^2 and 800 turns.

Hints: $$L = \frac{\mu_0 \cdot N^2 \cdot A}{l}$$

[here we assume $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}].

$$\therefore \quad L = \frac{4\pi\times10^{-7}\times800^2\times20\times10^{-4}}{40\times10^{-2}} = 4.022 \times 10^{-3} \text{ H}$$

....

3.55 A solenoid of inductance L and resistance R is connected to a battery. Prove that the time taken for the magnetic energy to reach 1/4 of its maximum value is $L/R \log_e(2)$.

Solution

The growth of current in an LR circuit is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}\cdot t}\right) \qquad \text{(i)}$$

where I_0 is the maximum current. The energy stored at time t is

$$u = \frac{1}{2}LI^2$$

We are required to find the time at which the energy stored is 1/4 the maximum value,

i.e., when $u = \frac{u_o}{4}$ where $u_o = \frac{1}{2}LI_0^2$.

i.e., $$\frac{1}{2}LI^2 = \frac{1}{4}\left(\frac{1}{2}LI_0^2\right) \quad \text{or} \quad I = \frac{I_0}{2}$$

$\therefore$ Using the equation 1, we have

$$\frac{I_0}{2} = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

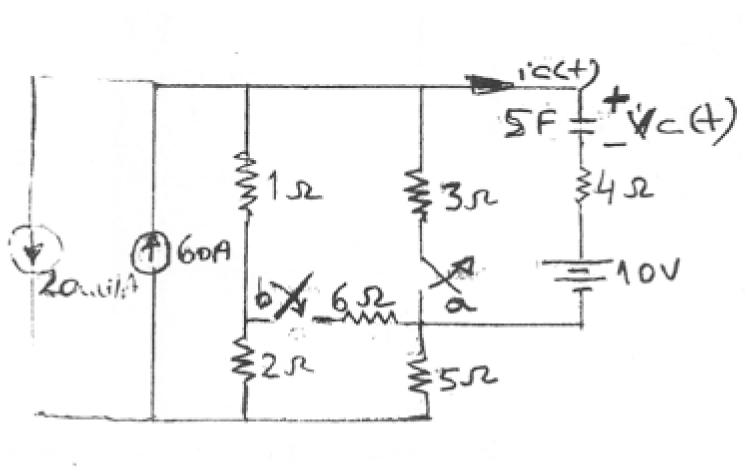
$$\frac{1}{2} = 1 - e^{-\frac{R}{L}\cdot t}$$

or $$e^{-\frac{R}{L}t} = \frac{1}{2}$$

or $$-\frac{R}{L}t = \log_e\left(\frac{1}{2}\right) - \log_e(2)$$

$$\therefore \quad t = \frac{L}{R}\log_e(2)$$

....



1) $t = 0$ anında a anahtarı açılıyor. b anahtarı ise kapatılıyor.
$t > 0$ için $V_c(t)$ ve $i_c(t)$'yi bulunuz.

3 ohm luk direncin üzerinden geçen akım=I_1 olsun

$I_1 = 60 \times \frac{3}{11} = 16.36A$

$-Vc(0+) + 3\Omega \times 16.36A - 10V = 0$

$V_c(0^+) = Vc(0^-) = 39.08V$

2.durumda devrede 2 tane akım kaynağı var bu yüzden superpozisyon yapacağız;

1.20A devrede 60A açık devre;

6Ωluk direç üzerinden geçen akım I_1 ve 1Ω luk direnç üzerinden geçen akım ise I_2 olsun

$I2 = 20 \times \frac{2}{13} = 3.07A$ ve $I1 = 20A$ olur

$-V_{c1}(\infty) - 1\Omega \times 20A - 3.07A \times 6\Omega - 10V = 0$

$V_{c1}(\infty) = -48.42V$

2.60A devrede 20A açık devre;

6Ωluk direç üzerinden geçen akım I_1 ve 1Ω luk direnç üzerinden geçen akım ise I_2 olsun

$I_2 = 60A$

$I_1=60\times\frac{2}{13}=9.23A$

$V_{c2}(\infty)+1\Omega\times60A+6\Omega\times9.23A-10V=0$

$V_{c2}(\infty)=105.38V$

$V_{ctop}(\infty)=105.38V-48.42V=56.96V$

Zaman sabiti $\tau = R_{eş}xC$ $R_{eş}$ => $R_{eş=}$ (2. durumda selften görülen eşdeğer direnç, gerilim kaynağı kısa devre, akım kaynağı açık devre yapılarak bulunur.)

$R_{eş}=1+\frac{2\times11}{2+11}+4=6.69\Omega$

$\tau = R_{eş}xC =5F\times6.69\Omega=33.46$

$Vc(t)=Ae^{-t/\tau}+Vc(\infty)$

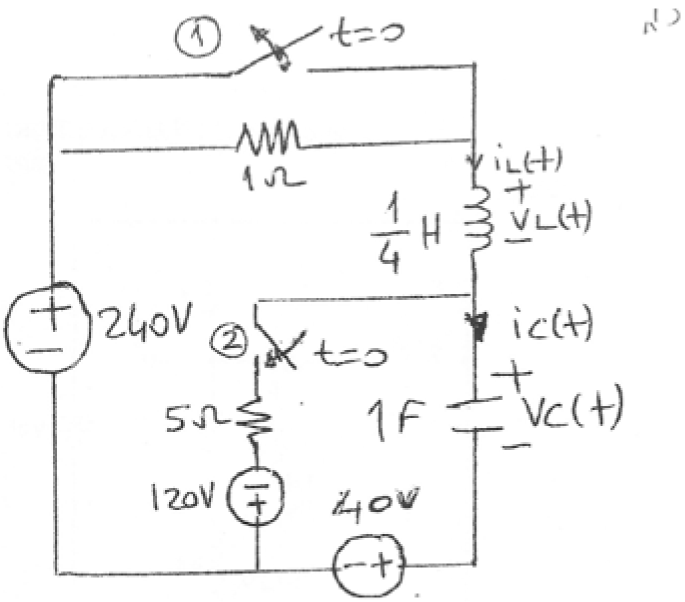
t =0 için A=-17.88

$Vc(t)=-17.88\times e^{-t/33.46}+56.96$

$Ic(t)=C\times\frac{dvc}{dt}$

$Ic(t)=5\times(-17.88)\times(-1/33.46)\times e^{\frac{-t}{33.46}}$

$Ic(t)=2.67\times e^{\frac{-t}{33.46}}$



3) 1 nolu anahtar uzun süre kapalı , 2 nolu anahtar ise uzun süre açık kaldıktan sonra
t = 0 anında 1 nolu anahtar açılıyor, 2 nolu anahtar ise kapatılıyor.
$t \geqslant 0$ için $I_L(t)$'yi ve $V_L(/t)$' yi yada $V_C(t)$'yi ve $I_C(/t)$' yi bulunuz.

$I_l(0^+)=I_l(0^-)0A$

$-V_c(0^+)+240-40=0$

$V_c(0^+)=Vc(0^-)=200V$

3 tane gerilim kaynağı var superpozisyon yapacağız;

1)240V devrede diğerleri kısa devre;

$I_{l1}(\infty)=\frac{240}{6}=40A$

2)120V devrede diğerleri kısa devre;

$I_{l2}(\infty)=\frac{120}{6}=20A$

3)40V devrede diğerleri kısa devre;

Kondansatör açık devre olduğu için $I_{l3}(\infty)=0A$

$I_{ltop}(\infty)=60A$

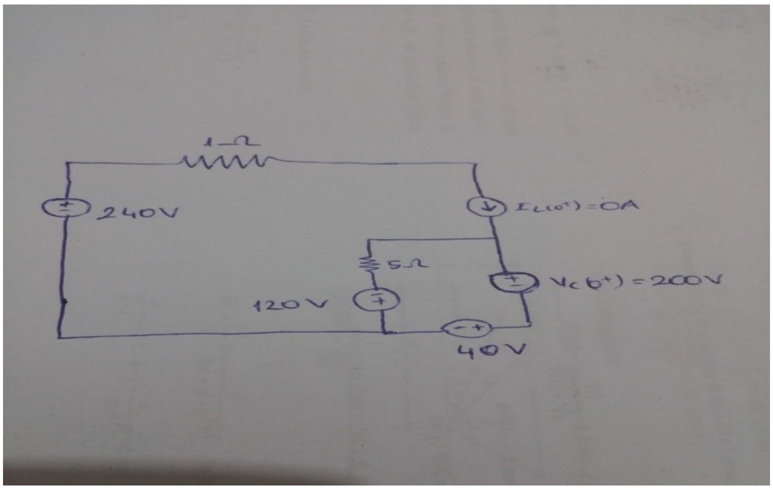
$-Vc(\infty)+5\times60-120-40=0$

$Vc(\infty)=140V$

2.durum için $Ic(^{0+})$ ve $V_l(0^{+)}$ bulacağız şimdi;

Bunu yaparken devrede bobin yerine bobin ile aynı yönlü $I_l(0^+)=0$ akımı yazılacak

Kondansatör yerine ise aynı yönlü $Vc(0^+)=200V$ yazılacak



İc(0+) ve Vc(0+) nın bulunması

1)240V devrede diğer gerilimler kısa devre akım kaynağı açık devre

$I_{c1}(0^+)=0$

2)200V devrede diğer gerilimler kısa devre akım kaynağı açık devre

Ic2(0+)=$\frac{-200}{5}$=-40A

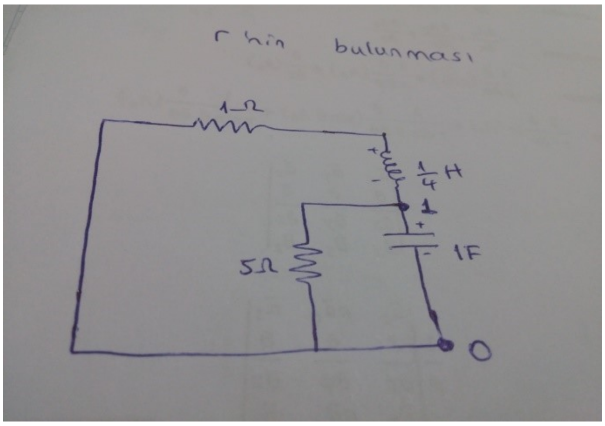
3)40V devrede diğer gerilimler kısa devre akım kaynağı açık devre;

Ic3(0+)=$\frac{-40}{5}$=-8A

4)120V devrede diğer gerilimler kısa devre akım kaynağı açık devre,

Ic4(0+)=$\frac{-120}{5}$=-24

$Ic_{top}(0^+)$=-72A



R nin bulunması

$$\frac{1}{\frac{1}{r}}+\frac{1}{5}+\frac{1}{1+\frac{r}{4}}=0$$

$5r^2+21r+24=0$

$R_{1,2}=-2.1\pm 0.62i$

$Vc(t)=e^{-2.1t}(A\cos(0.62t)+B\sin(0.62t))+140$

t=0 için 200=A+140

A=60

B yi bulmak için;

$Ic(t)=C\times\frac{dvc}{dt}$

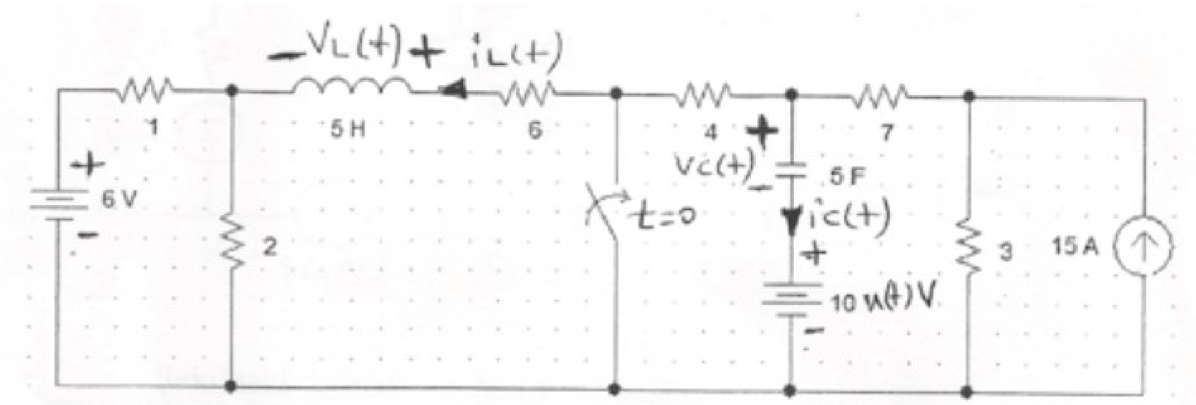
$Ic(t)=-2.1e^{-2.1t}\times 60\cos(0.62t)-e^{-2.1t}60\times 0.62\sin(0.62t)-$

$2.1Be^{-2.1t}\sin(0.62t)+Be^{-2.1t}\times 0.62\cos(0.62t)$

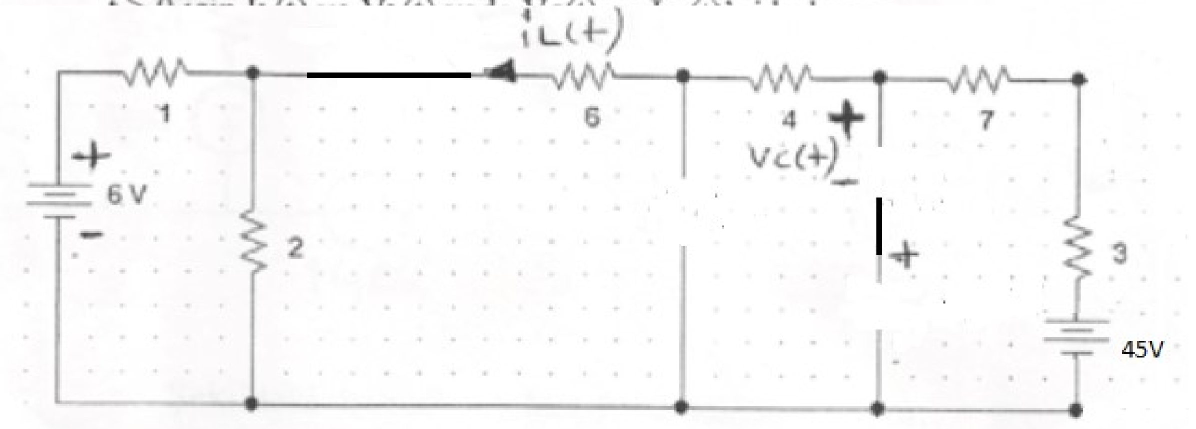
t=0 için B=87.1 çıkar.

$Vc(t)=e^{-2.1t}(60\cos(0.62t)+87.1\sin(0.62t))+140$

$Ic(t)=C\times\frac{dvc}{dt}=e^{-2.1t}(60\cos(0.62)-37.2\sin 0.62t)-182.1\sin(0.62t)+54\cos(0.62t)$



S-1) Şekildeki devrede anahtar t = 0 anında kapatılıyor.



1.Durum Devresi : 15 A lik akım kaynağı Gerilim kaynağına dönüştürülür. 15 x 3 = 45 Volt olur.

6 V Devredeyken

$$I_{L1}(0^+) = \frac{6}{1+\frac{20x2}{20+2}} x \frac{2}{22} = -0,193\ A$$

$$-0,193x7 - 0,193x3 - V_{C1}(0^+) = 0$$

$$V_{C1}(0^+) = -1,93\ V$$

45 V Devredeyken

$$I_{L2}(0^+) = \frac{45}{20+\frac{2x1}{2+1}} = 2,177\ A$$

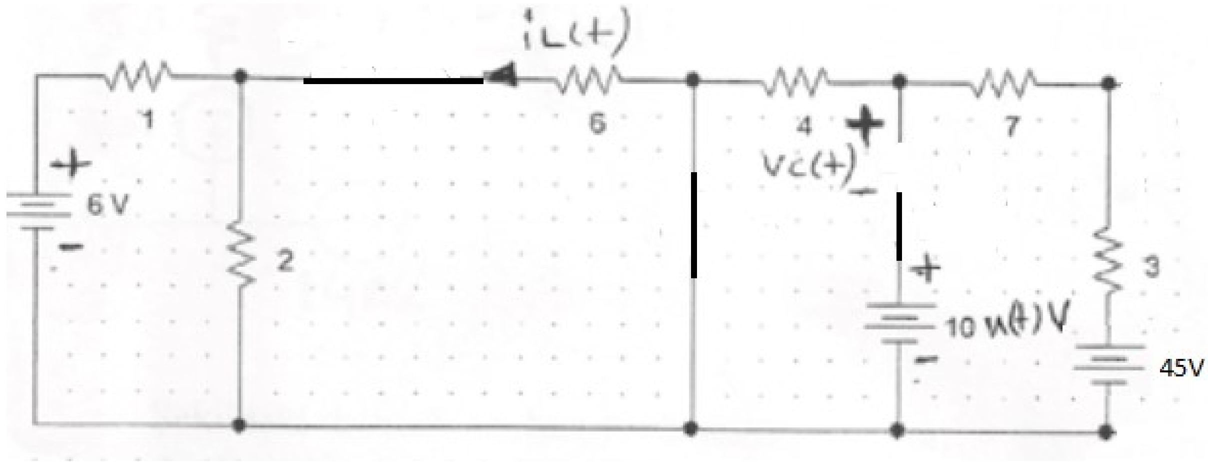
$$V_{C2}(0^+) + 3x2,177 + 7x2,177 - 45 = 0$$

$$V_{C2}(0^+) = 23,23\ V$$

$$I_L(0^+) = -0,193 + 2,177 = 1,984\ a$$

$$V_C(0^+) = -1,93 + 23,23 = 21,3\ V$$

2. Durum Devresi



6 V Devredeyken

$$I_{L1}(\infty) = \frac{6}{1 + \frac{2x6}{2+6}} x \frac{2}{8} = -0,6\ A$$

$$V_{C1}(\infty) = 0\ V$$

45 V Devredeyken

$$I_{L2}(\infty) = \frac{45}{14} = 3,214\ A$$

$$V_{C2}(\infty) + 10x3,214 - 45 = 0 \qquad\qquad V_{C2}(\infty) = 12,86\ V$$

10 V Devredeyken

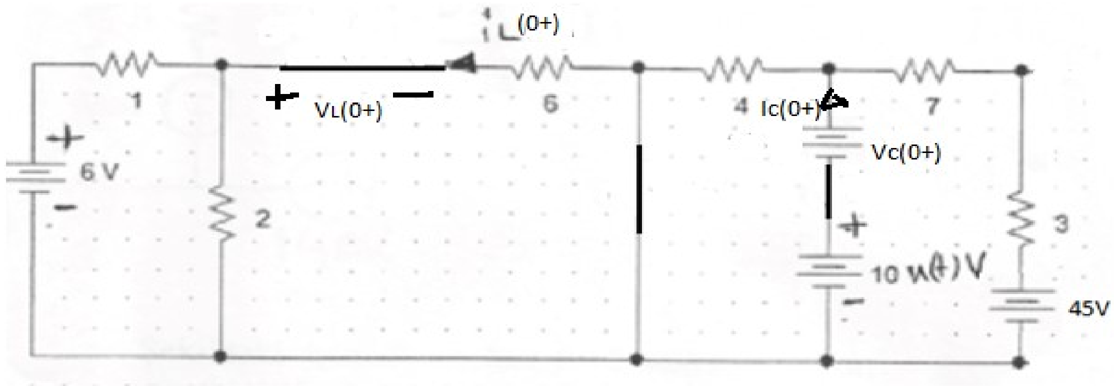
$$I_{L3}(\infty) = 0$$

$$V_{L3}(\infty) = -10\ \mathrm{V}$$

$$I_L(\infty) = -0,6 + 0 + 0 = -0,6\ A$$

$$V_C(\infty) = 0 + 12,86 - 10 = 2,86\ V$$

Sıfır Artı Eşdeğer Devresi ikinci durumda çizilir.



6 V Devredeyken

$$I_{C1}(0^{+}) = 0\ A$$

$$-V_{L1}(0^{+}) + 2x\frac{6}{3} = 0 \qquad V_{L1}(0^{+}) = 4\ V$$

$I_L(0^{+})$ = 1,984 A devredeyken

$$I_{C2}(0^{+}) = 0\ A$$

$$1{,}984x\frac{2x1}{2+1} + 6x1{,}984 - V_{L2}(0^{+}) = 0$$

$$V_{L2}(0^{+}) = 13{,}226\ V$$

Vc (0^{+}) = 21,3 V Devredeyken

$$I_{C3}(0^{+}) = \frac{21{,}3}{\frac{4x10}{4+10}} = 7{,}455\ A$$

$$V_{L3}(0^{+}) = 0\ V$$

10 V Devredeyken

$$I_{C4}(0^{+}) = \frac{10}{\frac{4x10}{4+10}} = 3{,}5\ A$$

$$V_{L4}(0^{+}) = 0\ V$$

45 V Devredeyken

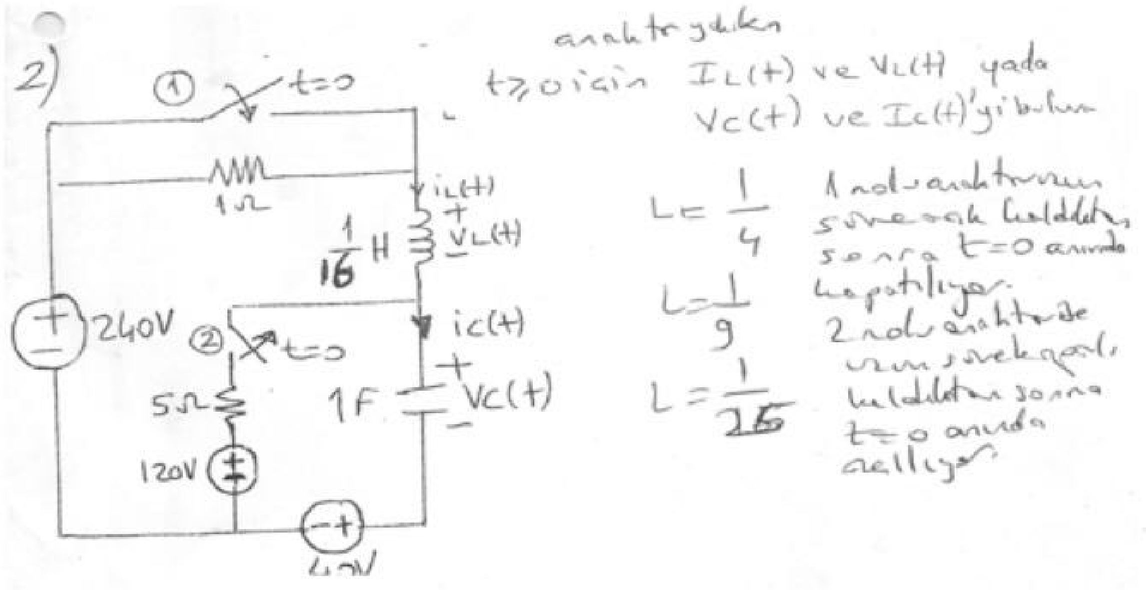
$$I_{C5}(0^{+}) = -4{,}5\ A$$

$$V_{L5}(0^{+}) = 0\ V$$

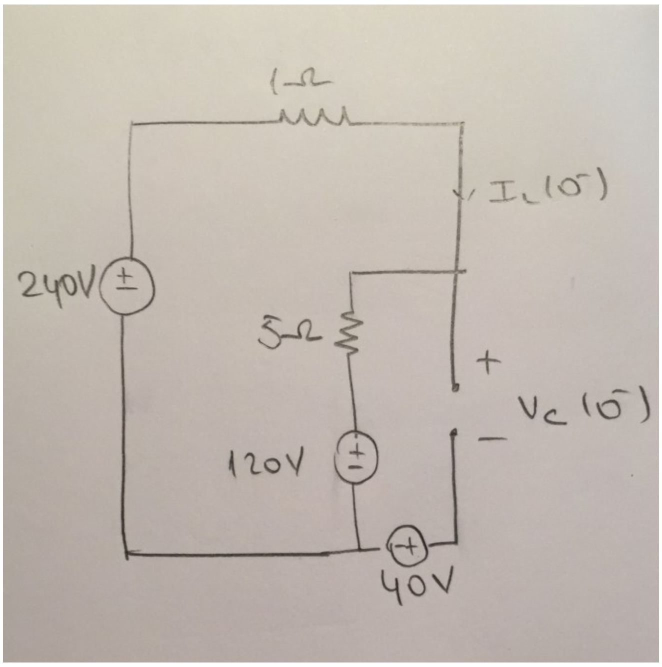
$$I_C(0^{+}) = 0 + 0 + 7{,}455 + 3{,}5 - 4{,}5 = 6{,}455\ A$$

$$V_L(0^{+}) = 4 + 13{,}226 + 0 + 0 + 0 = 17{,}226\ V$$

<u>Soru 1:</u>



1. Durum : (0^-) için

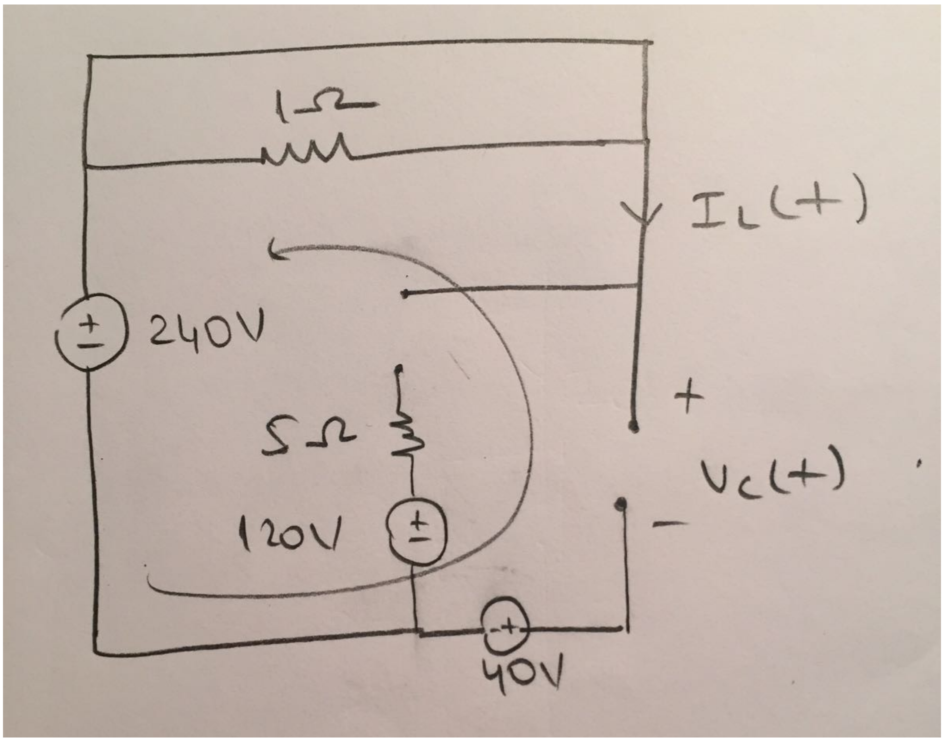


$I_L(0^-) = \frac{240-120}{5+1} = 20$ A

$V_C(0^-) = 5 \times 20 + 120 - 40$

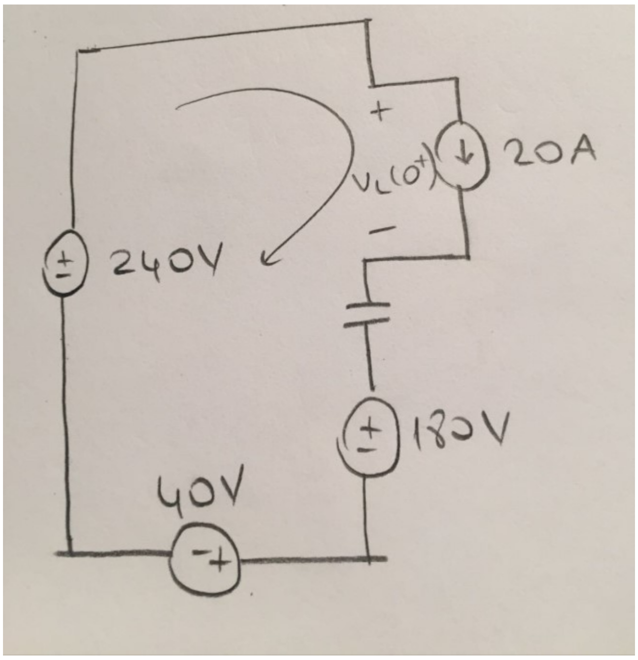
$V_C(0^-) = 180$ V $= V_C(0^+)$

2. Durum : (∞) için



$I_L(\infty)$ = 0 A
$V_C(\infty)$ = 1×0+240-40
$V_C(\infty) = 200\ V$

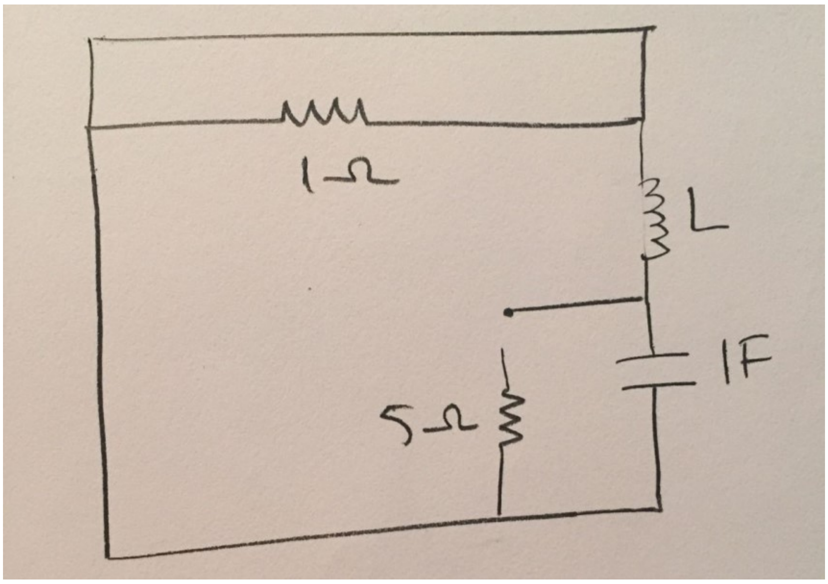
3. **Durum :** (0^+) için



$$I_{C_1}(0^+) = \ 20A$$

$$V_{L_1}(0^+) = 180 + 40 - 240 = -20\,V$$

Zaman Sabitini bulma:
(2. Durum için çizilir)



L=$\frac{1}{4}$ için:

$$\frac{1}{L.r} + \frac{1}{\frac{1}{C \times r}} = 0$$

$$\frac{1}{\frac{1}{4}.r} + \frac{1}{\frac{1}{1 \times r}} = 0$$

$$r^2 + 4 = 0$$

r=2j

(r+0+2j) (r+0-2j)=0

$$I_L(t) = e^{-0t}[A.cos2t + Bsin2t] + I_L(\infty)$$

t=0 için:

20=A.1+B.0+0

A=20

$$V_L(t) = L.\frac{dI_L(t)}{dt}$$

$V_L(t) = \frac{1}{4}$(A.2.(-sin2t)+2.B.cos2t)

t=0 için:

$-20 = \frac{1}{4}$(20.2.0+2.B.1)

B=-40

$$I_L(t) = e^{-0t}[20.cos2t + (-40)sin2t] + I_L(\infty)$$

$V_L(t) = \frac{1}{4}$(20.2.(-sin2t)+2. (-40).cos2t)

L=$\frac{1}{9}$ için:

$$\frac{1}{\frac{1}{9}.r} + \frac{1}{\frac{1}{1 \times r}} = 0$$

$$r^2 + 9 = 0$$

r=3j

(r+0+3j) (r+0-3j)=0

$$I_L(t) = e^{-0t}[A.cos3t + Bsin3t] + I_L(\infty)$$

t=0 için:

20=A.1+B.0+0

A=20

$$V_L(t) = L.\frac{dI_L(t)}{dt}$$

$V_L(t) = \frac{1}{9}$(A.3.(-sin3t)+3.B.cos3t)

t=0 için:

$-20 = \frac{1}{9}$(20.3.0+3.B.1)

B=-60

$$I_L(t) = e^{-0t}[20.cos3t + (-60)sin3t] + I_L(\infty)$$

$V_L(t) = \frac{1}{9}$(20.3.(-sin3t)+3. (-60).cos3t)

L=$\frac{1}{25}$ için:

$$\frac{1}{\frac{1}{25}\cdot r}+\frac{1}{\frac{1}{1\times r}}=0$$

$$r^2+25=0$$

r=5j

(r+0+5j) (r+0-5j)=0

$$I_L(t)=e^{-0t}[A.cos5t+Bsin5t]+I_L(\infty)$$

t=0 için:

20=A.1+B.0+0

A=20

$$V_L(t)=L.\frac{dI_L(t)}{dt}$$

$V_L(t)=\frac{1}{25}$(A.5.(-sin5t)+5.B.cos5t)

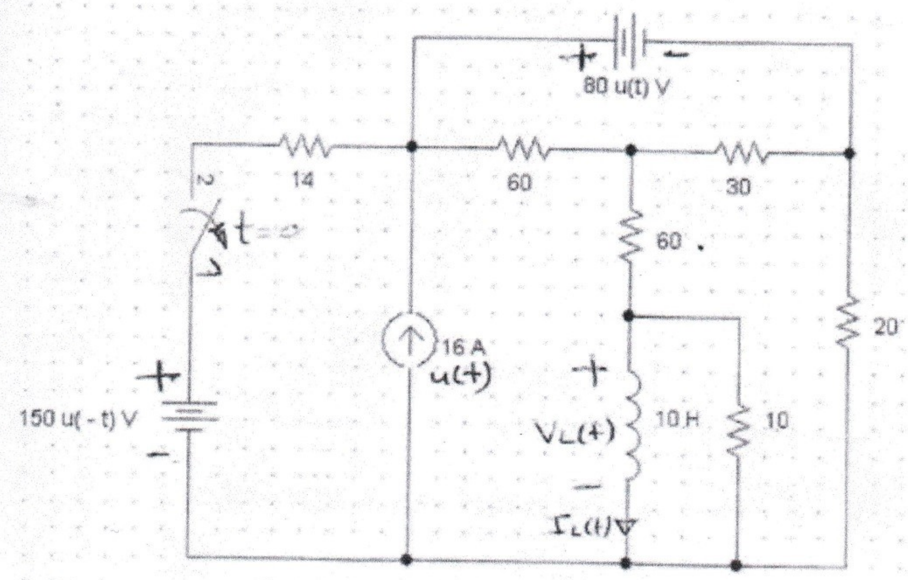
t=0 için:

$-20=\frac{1}{25}$(20.5.0+5.B.1)

B=-100

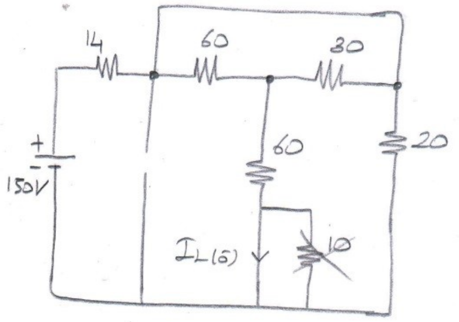
$$I_L(t)=e^{-0t}[20.cos5t+(-100)sin5t]+I_L(\infty)$$

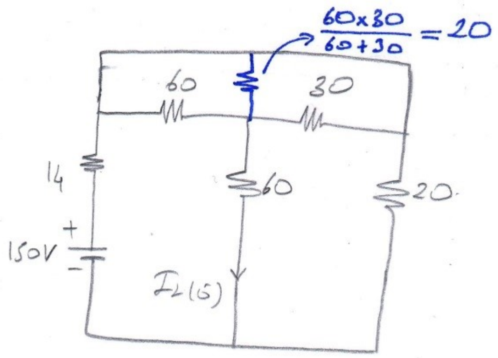
$V_L(t)=\frac{1}{25}$(20.5.(-sin5t)+5. (-100).cos5t)



i) IL(t) ve VL(t) 'yi bulunuz.

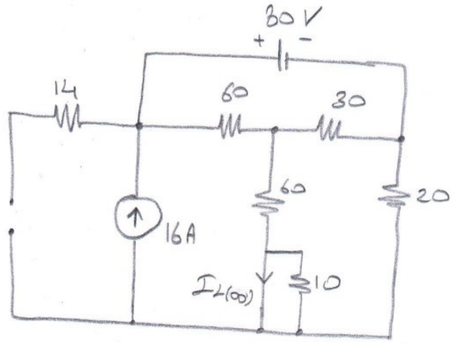
1. Durum Denklemi

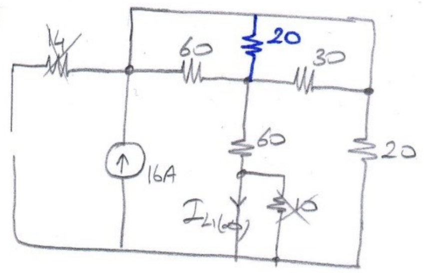




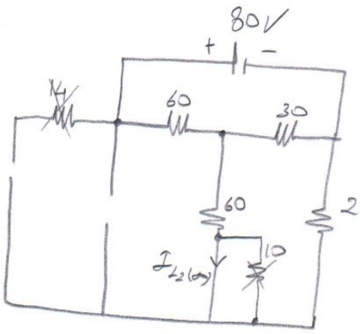
$$I_{L(S)} = \frac{150}{\frac{(20+60)\times 20}{(20+60)+20} + 14} \times \frac{20}{20+(20+60)} = 1A$$

2. Durum Denklemi

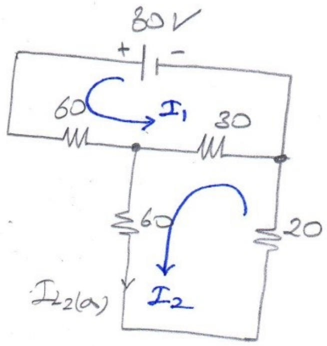




$$I_{L_1(\infty)} = 16 \cdot \frac{20}{20 + (20+60)} = 3{,}2\,A$$



$\Longrightarrow$



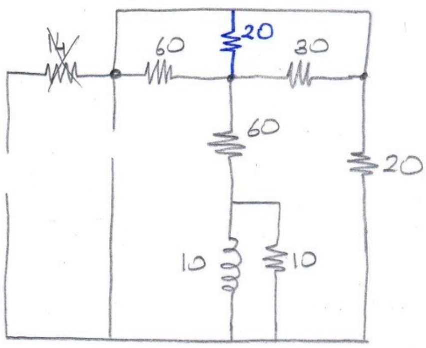
$$60 \cdot I_1 + 30(I_1 - I_2) = 80$$

$$60 \cdot I_2 + 20 I_2 + 30(I_2 - I_1) = 0$$

$$I_{L_2(\infty)} = 0{,}26\,A$$

$$I_{L(\infty)} = I_{L_1(\infty)} + I_{L_2(\infty)} = 3{,}2 + 0{,}26 = 3{,}46\,A$$

Zaman Sabiti Devresi



$$R_{eq} = \frac{(20+60+20)\times 10}{(20+60+20)+10} = \frac{1000}{110} = \frac{100}{11}$$

$$\tau = \frac{L}{R_{eq}} = \frac{10}{\frac{100}{11}} = \frac{110}{100} = \frac{11}{10}$$

$$I_{L(t)} = A.e^{-\frac{t}{\tau}} + I_{L(\infty)}$$

$$I_{L(t)} = A.e^{-\frac{t}{11/10}} + 3,46$$

$t=0$ için $I_{L(0^+)}$

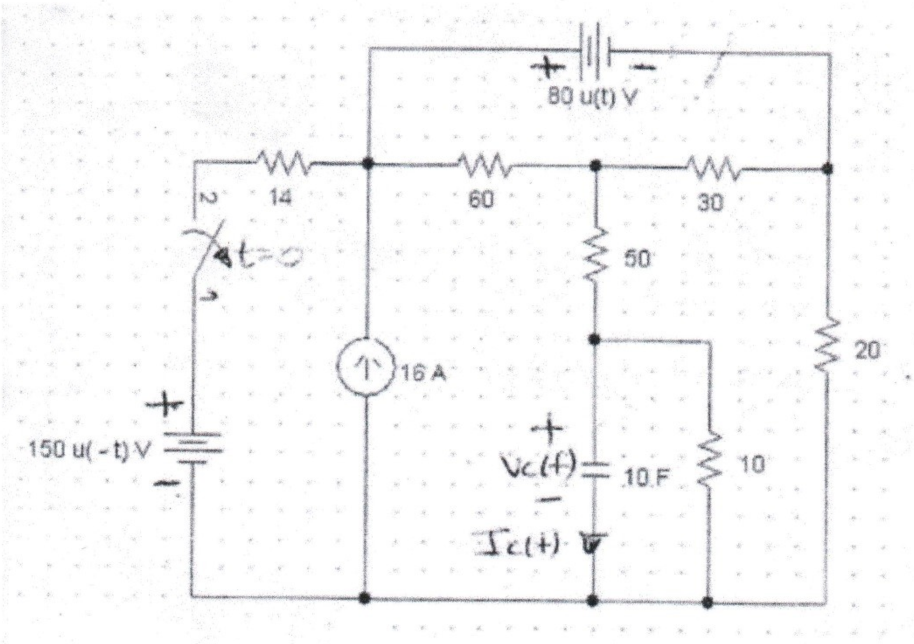
$$1 = A.1 + 3.46$$

$$-2,46 = A$$

$$I_{L(t)} = -2,46e^{-\frac{10}{11}t} + 3,46$$

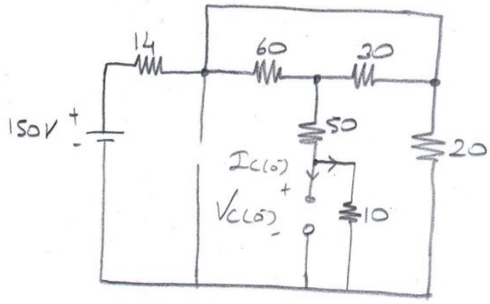
$$V_{L(t)} = L.\frac{di_{L(t)}}{dt} = 10.-2,46\left(\frac{-10}{11}\right).e^{-\frac{10}{11}t}$$

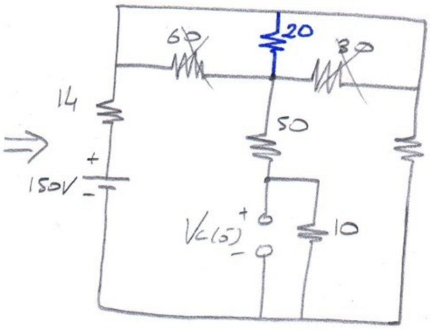
$$V_{L(t)} = 22,36e^{-\frac{10}{11}t} \qquad V_{L(0)} = 22,36 \text{ Volt}$$



2) Vc(t) ve Ic(t)'yi bulunuz.

1. Durum Denklemi

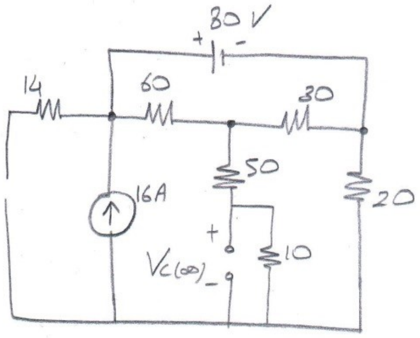


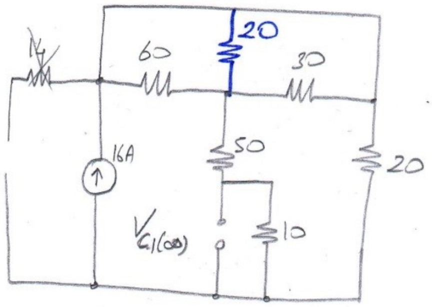


$$I_{C(0^-)} = \frac{150}{\dfrac{20 \times (20+50+10)}{20+(20+50+10)} + 14} \times \frac{20}{20+(20+50+10)} = 1A$$

$$V_{C(0^-)} = 1.10 = 10\ V = V_{C(0^+)}$$

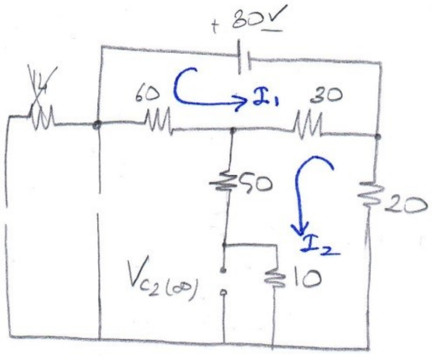
2. Durum Denklemi





$$I_{C1(\infty)} = 16 \cdot \frac{20}{20 + (20 + 50 + 10)} = 3{,}2A$$

$$V_{C1(\infty)} = 3{,}2 \cdot 10 = 32V$$



$$60 I_1 + 30(I_1 - I_2) = 30$$

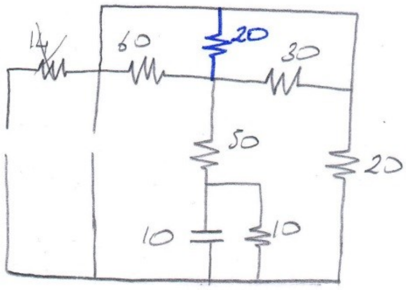
$$50 I_2 + 20 \cdot I_2 + 30 \cdot (I_2 - I_1) = 0$$

$$I_2 = I_{C2(\infty)} = 0{,}26\ A$$

$$V_{C2(\infty)} = 2{,}6\ \text{Volt}$$

$$V_{C(\infty)} = V_{C1(\infty)} + V_{C2(\infty)} = 32 + 2{,}6 = 34{,}6V$$

Zaman Sabiti Devresi



$$R_{eş} = \frac{10 \times (20+50+20)}{10+(20+50+20)} = \frac{900}{100} = 9$$

$$\tau = R_{eş} \cdot C = 9 \cdot 10 = 90$$

$$V_{C(t)} = A \cdot e^{-\frac{t}{\tau}} + V_{C(\infty)}$$

$$V_{C(t)} = A \cdot e^{-\frac{t}{90}} + 34{,}6$$

$t=0$ için $V_{C(0^+)} = 10V$

$$10 = A \cdot 1 + 34{,}6$$

$$A = -24{,}6$$

$$V_{C(t)} = -24{,}6 \cdot e^{-\frac{t}{90}} + 34{,}6$$

$$I_{C(t)} = C \cdot \frac{dV_{C(t)}}{dt} = 10 \cdot -24{,}6 \cdot \left(-\frac{1}{90}\right) \cdot e^{-\frac{t}{90}}$$

$$= 2{,}73 e^{-\frac{t}{90}} \qquad I_{C(\infty)} = 2{,}73\, A$$

Calculate the mesh currents in the circuit of Fig. 13.11.

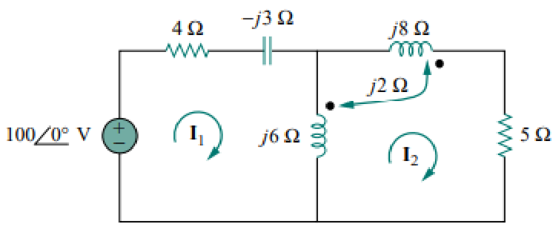


Figure 13.11 For Example 13.2.

Problem 13.5 Given the circuit in Figure 13.1, $V_1 = V_2 = 10$ volts, $R_1 = R_2 = 10$ ohms, $\omega L_1 = \omega L_2 = 10$, and $\omega M = 5$, find the coupling coefficient, k, the currents in the primary and secondary circuits, I_1 and I_2, and the power absorbed.

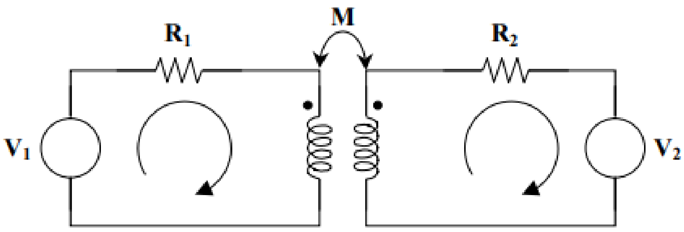


Figure 13.1

The coupling coefficient is $k = \dfrac{M}{\sqrt{L_1 L_2}}$.

Given values for ωM, ωL_1, and ωL_2, we need to modify the equation for k to be

$$k = \frac{\omega M}{\sqrt{(\omega L_1)(\omega L_2)}}.$$

$$k = \frac{5}{\sqrt{(10)(10)}} = \frac{5}{10} = \underline{\mathbf{0.5}}$$

Use mesh analysis to find I_1 and I_2.

Loop 1 : $\quad 10 = (10 + j10)I_1 - j5I_2$

Loop 2 : $\quad -10 = -j5I_1 + (10 + j10)I_2$

In matrix form,

$$\begin{bmatrix} 10 + j10 & -j5 \\ -j5 & 10 + j10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

where $\Delta = (10 + j10)(10 + j10) - (-j5)(-j5) = j200 + 25 = (25)(1 + j8)$.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 10 + j10 & j5 \\ j5 & 10 + j10 \end{bmatrix}}{(25)(1 + j8)} \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{10 + j10}{(25)(1 + j8)} & \dfrac{j5}{(25)(1 + j8)} \\ \dfrac{j5}{(25)(1 + j8)} & \dfrac{10 + j10}{(25)(1 + j8)} \end{bmatrix} \begin{bmatrix} 10 \\ -10 \end{bmatrix} = \begin{bmatrix} \dfrac{100 + j100 - j50}{(25)(1 + j8)} \\ \dfrac{j50 - 100 - j100}{(25)(1 + j8)} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{(50)(2 + j)}{(25)(1 + j8)} \\ \dfrac{(50)(-2 - j)}{(25)(1 + j8)} \end{bmatrix} = \begin{bmatrix} \dfrac{4 + j2}{1 + j8} \\ \dfrac{-4 - j2}{1 + j8} \end{bmatrix}$$

Thus,

$$I_1 = \frac{4 + j2}{1 + j8} = \frac{4.4721\angle 26.57°}{8.0623\angle 82.88°} = \underline{\mathbf{0.5547\angle -56.31°\ A}}$$

$$I_2 = \frac{-4 - j2}{1 + j8} = \frac{4.4721\angle -153.43°}{8.0623\angle 82.88°} = \underline{\mathbf{0.5547\angle 123.69°\ A}}$$

Now, find the power absorbed in the circuit. Look at the power absorbed by each element.

Starting with the primary circuit,

$$p_{V1} = -V\,I\cos\theta = -(10)(0.5547)\cos(0° - (-56.31°)) = -3.0769\ W$$

$$p_{R1} = |I_1|^2 R_1 = (0.5547)^2(10) = 3.0769\ W$$

$$p_1 = -(2.7735)(0.5547)\cos(213.69° - (-56.31)°) = 1.5385\cos(270°) = 0\ W$$

where p_1 is the power absorbed by the induced voltage of L_1.

Ending with the secondary circuit,

$$p_{V2} = V\,I\cos\theta = (10)(0.5547)\cos(0° - 123.69°) = -3.0769\ W$$

$$p_{R2} = |I_2|^2 R_2 = (0.5547)^2(10) = 3.0769\ W$$

$$p_2 = -(2.7735)(0.5547)\cos(33.69° - 123.69°) = 1.5385\cos(90°) = 0\ W$$

where p_2 is the power absorbed by the induced voltage of L_2.

<u>The voltage sources absorb –3.0769 watts, or deliver +3.0769 watts, the resistances absorb 3.0769 watts, and the induced voltages absorb 0 watts. The inductors do not absorb power.</u>

Problem 13.6 *[13.13]* Determine the currents I_1, I_2, and I_3 in the circuit of Figure 13.1. Find the energy stored in the coupled coils at $t = 2$ ms. Take $\omega = 1000$ rad/s.

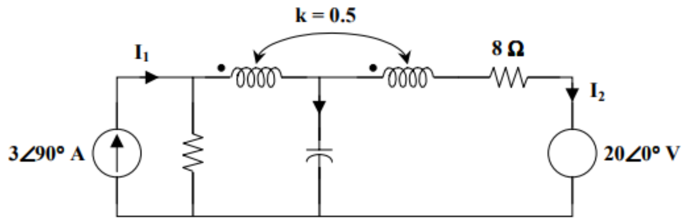
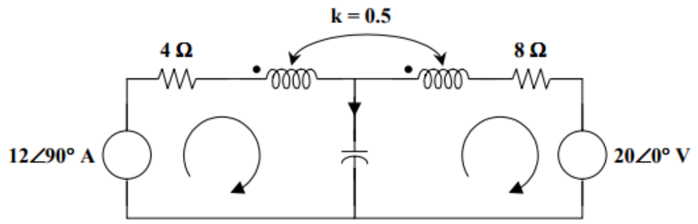


Figure 13.1

Transform the current source to a voltage source as shown below.



$$k = \frac{M}{\sqrt{L_1 L_2}} \qquad \text{or} \qquad M = \frac{k}{\sqrt{L_1 L_2}}$$

$$\omega M = k\sqrt{(\omega L_1)(\omega L_2)} = (0.5)(10) = 5$$

Using mesh analysis,

Mesh 1, $\quad j12 = (4 + j10 - j5)I_1 + j5I_2 + j5I_2 = (4 + j5)I_1 + j10I_2 \qquad (1)$

Mesh 2, $\quad 0 = 20 + (8 + j10 - j5)I_2 + j5I_1 + j5I_1$

$-20 = j10I_1 + (8 + j5)I_2 \qquad (2)$

From (1) and (2),

$$\begin{bmatrix} j12 \\ -20 \end{bmatrix} = \begin{bmatrix} 4 + j5 & j10 \\ j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 107 + j60, \qquad \Delta_1 = -60 - j296, \qquad \Delta_2 = 40 - j100$$

$$I_1 = \frac{\Delta_1}{\Delta} = \underline{\mathbf{2.462\angle 72.18° \ A}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \underline{\mathbf{0.878\angle -97.48° \ A}}$$

$$I_3 = I_1 - I_2 = \underline{\mathbf{3.329\angle 74.89° \ A}}$$

$$i_1(t) = 2.462\cos(1000t + 72.18°) \text{ A}$$
$$i_2(t) = 0.878\cos(1000t - 97.48°) \text{ A}$$

At $t = 2$ ms, $\qquad 1000t = 2 \text{ rad} = 114.6°$

$$i_1(2 \text{ ms}) = 2.462\cos(114.6° + 72.18°) = -2.445$$
$$i_2(2 \text{ ms}) = 0.878\cos(114.6° - 97.48°) = 0.8391$$

The total energy stored in the coupled coils is

$$w = 0.5L_1 i_1^2 + 0.5L_2 i_2^2 + M i_1 i_2$$

Since $\omega L_1 = 10$ and $\omega = 1000$,

$$L_1 = L_2 = 10 \text{ mH}, \qquad M = 0.5L_1 = 5 \text{ mH}$$

$$w = (0.5)(10)(-2.445)^2 + (0.5)(10)(0.8391)^2 + (5)(-2.445)(0.8391)$$
$$w = \mathbf{23.15 \ mJ}$$

Problem 13.7 Given the circuit in Figure 13.1, $V_1 = V_2 = 10$ volts, $R_1 = R_2 = 10$ ohms, $\omega L_1 = \omega L_2 = 10$, and $\omega M = 5$, find the coupling coefficient, k, the currents in the primary and secondary circuits, I_1 and I_2, and the power absorbed.

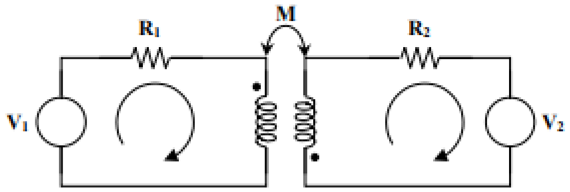


Figure 13.1

As seen in Problem 13.5,

$$k = \frac{\omega M}{\sqrt{(\omega L_1)(\omega L_2)}} = \frac{5}{\sqrt{(10)(10)}} = \frac{5}{10} = \underline{\mathbf{0.5}}$$

To find the currents, begin by finding an equivalent circuit which takes into account the coupling effects, i.e., the induced voltages.

Use mesh analysis to find I_1 and I_2.

Loop 1 : $\quad 10 = (10 + j10)\,I_1 + j5\,I_2$

Loop 2 : $\quad -10 = j5\,I_1 + (10 + j10)\,I_2$

In matrix form,

$$\begin{bmatrix} 10+j10 & j5 \\ j5 & 10+j10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

where $\Delta = (10 + j10)(10 + j10) - (j5)(j5) = j200 + 25 = (25)(1 + j8)$.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 10+j10 & -j5 \\ -j5 & 10+j10 \end{bmatrix}}{(25)(1+j8)}\begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{10+j10}{(25)(1+j8)} & \frac{-j5}{(25)(1+j8)} \\ \frac{-j5}{(25)(1+j8)} & \frac{10+j10}{(25)(1+j8)} \end{bmatrix}\begin{bmatrix} 10 \\ -10 \end{bmatrix} = \begin{bmatrix} \frac{100+j100+j50}{(25)(1+j8)} \\ \frac{-j50-100-j100}{(25)(1+j8)} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{(50)(2+j3)}{(25)(1+j8)} \\ \frac{(50)(-2-j3)}{(25)(1+j8)} \end{bmatrix} = \begin{bmatrix} \frac{4+j6}{1+j8} \\ \frac{-4-j6}{1+j8} \end{bmatrix}$$

Thus,

$$I_1 = \frac{4+j6}{1+j8} = \frac{7.2111\angle 56.31°}{8.0623\angle 82.88°} = \underline{\mathbf{0.8944\angle -26.57°\ A}}$$

$$I_2 = \frac{-4-j6}{1+j8} = \frac{7.2111\angle -123.69°}{8.0623\angle 82.88°} = \underline{\mathbf{0.8944\angle 153.43°\ A}}$$

Now, find the power absorbed in the circuit. Look at the power absorbed by each element.

Starting with the primary circuit,

$$p_{V1} = -V\,I\cos\theta = -(10)(0.8944)\cos(0° - (-26.57°)) = -7.9994\ W$$

$$p_{R1} = |I_1|^2 R_1 = (0.8944)^2(10) = 7.9995\ W$$

$$p_1 = (4.4720)(0.8944)\cos(243.43° - (-26.57°)) = 3.9998\cos(270°) = 0\ W$$

where p_1 is the power absorbed by the induced voltage of L_1.

Ending with the secondary circuit,

$$p_{V2} = V\,I\cos\theta = (10)(0.8944)\cos(0 - 153.43°) = -7.9994\ W$$

$$p_{R2} = |I_2|^2 R_2 = (0.8944)^2(10) = 7.9995\ W$$

$$p_2 = (4.4720)(0.8944)\cos(63.43° - 153.43°) = 3.9998\cos(-90°) = 0\ W$$

where p_2 is the power absorbed by the induced voltage of L_2.

The voltage sources absorb –7.9994 watts, or deliver +7.9994 watts, the resistances absorb 7.9995 watts, and the induced voltages absorb 0 watts. The inductors do not absorb power.

Problem 13.1 Given the circuit in Figure 13.1 and $k = 1$, find I_1 and I_2.

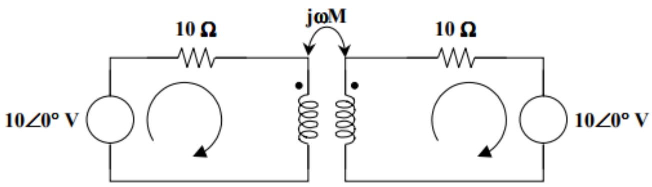


Figure 13.1

- **Carefully DEFINE the problem.**
  Each component is labeled completely. The problem is clear.

- **PRESENT everything you know about the problem.**
  We know all values of the independent sources. We also know the values of all the elements.

  In order to find the equivalent circuit containing the induced voltages, we need to know the mutual inductance, M.

  We know that the coupling coefficient is

$$k = \frac{M}{\sqrt{L_1 L_2}} = 1$$

Then,

$$M = k\sqrt{L_1 L_2} = \sqrt{L_1 L_2}$$

Now, using mesh analysis,

Loop 1 :
$$-10 + 10I_1 + j10I_1 - j10I_2 = 0$$
$$(10 + j10)I_1 - j10I_2 = 10$$
$$(1 + j)I_1 - jI_2 = 1$$

Loop 2 :
$$-j10I_1 + j10I_2 + 10I_2 + 10 = 0$$
$$-j10I_1 + (10 + j10)I_2 = -10$$
$$-jI_1 + (1 + j)I_2 = -1$$

In matrix form,

$$\begin{bmatrix} 1+j & -j \\ -j & 1+j \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

or

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 1+j & j \\ j & 1+j \end{bmatrix}}{\Delta}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

where $\Delta = (1 + j)^2 - (-j)^2 = (1 + j2 + j^2) - j^2 = 1 + j2$.

From the circuit in Figure 13.1, we can see that $j\omega L_1 = j\omega L_2 = j10$.

Thus,

$$\omega L_1 = \omega L_2 = 10 \text{ and } L_1 = L_2 = L.$$

Hence,

$$M = \sqrt{L_1 L_2} = L \text{ and } j\omega M = j\omega L = j10.$$

Now,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1+j}{1+j2} & \frac{j}{1+j2} \\ \frac{j}{1+j2} & \frac{1+j}{1+j2} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Therefore,

$$I_1 = \frac{1+j-j}{1+j2} = \frac{1}{1+j2} = \frac{1\angle 0°}{\sqrt{5}\angle 63.43°} = 0.4472\angle -63.43° \text{ A}$$
$$I_2 = \frac{j-(1+j)}{1+j2} = \frac{-1}{1+j2} = \frac{1\angle 180°}{\sqrt{5}\angle 63.43°} = 0.4472\angle 116.57° \text{ A}$$

➢ **EVALUATE the solution and check for accuracy.**
Use KVL to check the solution.

The equation produced by KVL of the left loop is

$$-10 + 10 I_1 + j10 I_1 - j10 I_2 = 0$$

The equation produced by KVL of the right loop is

$$10 - j10 I_1 + j10 I_2 + 10 I_2 = 0$$

Inserting the values for I_1 and I_2 results in valid equations. Thus, our check for accuracy was successful.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$I_1 = \underline{\mathbf{0.4472\angle -63.43° \ A}}$ $\qquad$ $I_2 = \underline{\mathbf{0.4472\angle 116.57° \ A}}$

Problem 13.2 *[13.1]* For the three coupled coils in Figure 13.1, calculate the total inductance.

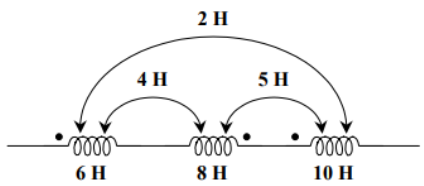


Figure 13.1

For coil 1, $\quad L_1 - M_{12} + M_{13} = 6 - 4 + 2 = 4$

For coil 2, $\quad L_2 - M_{21} - M_{23} = 8 - 4 - 5 = -1$

For coil 3, $\quad L_3 + M_{31} - M_{32} = 10 + 2 - 5 = 7$

$$L_T = 4 - 1 + 7 = \underline{\mathbf{10\ H}}$$

or

$$L_T = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{12}$$
$$L_T = 6 + 8 + 10 - (2)(4) - (2)(5) + (2)(2)$$
$$L_T = 6 + 8 + 10 - 8 - 10 + 4 = \underline{\mathbf{10\ H}}$$

Problem 13.3 For the frequency domain circuit shown in Figure 13.1, determine the value of $v_{out}(t)$ for $v_{in}(t) = 10\cos(377t)$ and a coupling coefficient $k = 0.8$.

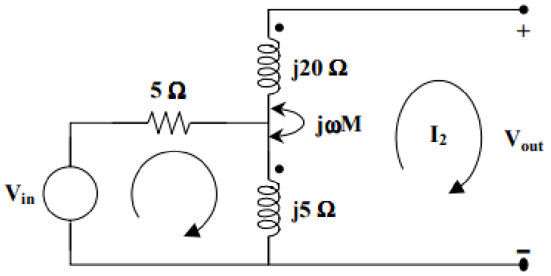


Figure 13.1

Before an equivalent circuit can be drawn, we must determine the value of ωM. Using $k = 0.8$,

$$k = \frac{M}{\sqrt{L_1 L_2}}.$$

Because the circuit is in the frequency domain rather than the time domain, we know the value of ωL rather than the value of L. So, transform the equation for k to include ω. Then,

$$k = \frac{\omega M}{\sqrt{(\omega L_1)(\omega L_2)}}.$$

Hence,

$$\omega M = k\sqrt{(\omega L_1)(\omega L_2)} = (0.8)\sqrt{(5)(20)} = 8$$

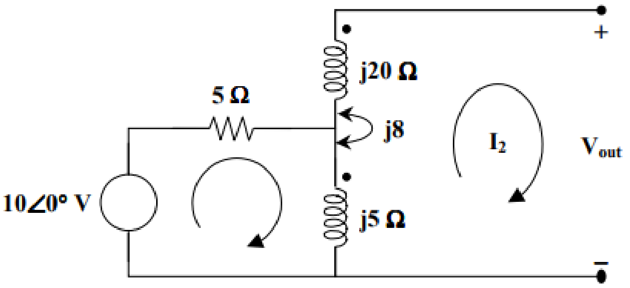
We also need to transform the voltage source from the time domain to the frequency domain. Let's assume a reference of

$$A\cos(377t + \phi).$$

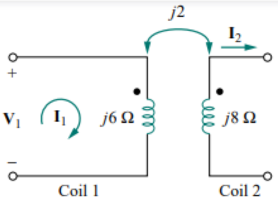
Then,

$$V_{in} = 10\angle 0°.$$

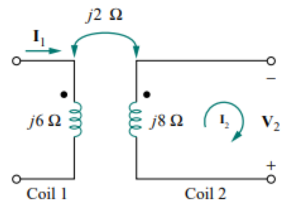
The circuit can be redrawn as



Using the dot convention, we can draw an equivalent circuit to incorporate the induced voltages from the coupling effects.



(a) $\mathbf{V}_1 = -2j\mathbf{I}_2$



(b) $\mathbf{V}_2 = -2j\mathbf{I}_1$

Figure 13.12 For Example 13.2; redrawing the relevant portion of the circuit in Fig. 13.11 to find mutual voltages by the dot convention.

$$-100 + \mathbf{I}_1(4 - j3 + j6) - j6\mathbf{I}_2 - j2\mathbf{I}_2 = 0$$

or

$$100 = (4 + j3)\mathbf{I}_1 - j8\mathbf{I}_2 \tag{13.2.1}$$

Similarly, to figure out the mutual voltage in coil 2 due to current $\mathbf{I}_1$, consider the relevant portion of the circuit, as shown in Fig. 13.12(b). Applying the dot convention gives the mutual voltage as $\mathbf{V}_2 = -2j\mathbf{I}_1$. Also, current $\mathbf{I}_2$ sees the two coupled coils in series in Fig. 13.11; since it leaves the dotted terminals in both coils, Eq. (13.18) applies. Therefore, for mesh 2, KVL gives

$$0 = -2j\mathbf{I}_1 - j6\mathbf{I}_1 + (j6 + j8 + j2 \times 2 + 5)\mathbf{I}_2$$

or

$$0 = -j8\mathbf{I}_1 + (5 + j18)\mathbf{I}_2 \tag{13.2.2}$$

Putting Eqs. (13.2.1) and (13.2.2) in matrix form, we get

$$\begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j3 & -j8 \\ -j8 & 5 + j18 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

14.33 Write three mesh equations for the circuit shown in Fig. 14.20.

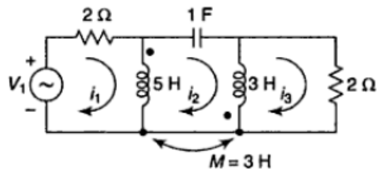


Fig. 14.20

Solution

The mutual inductance and the self inductances are replaced by their impedances and the corresponding circuit is shown in Fig. 14.21.

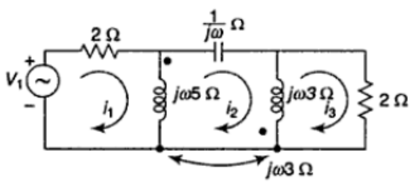


Fig. 14.21

Applying KVL in the first mesh (leftmost mesh),

$$2i_1 + j\omega 5(i_1 - i_2) + 3j\omega(i_3 - i_2) = V_1$$

or $$(2 + 5j\omega)i_1 - 8j\omega i_2 + 3j\omega i_3 = V_1 \tag{i}$$

Applying KVL in the second mesh (middle mesh),

$$5j\omega(i_2 - i_1) + 3j\omega(i_2 - i_3) + \frac{1}{j\omega}i_2 + 3j\omega(i_2 - i_3) + 3j\omega(i_2 - i_1) = 0$$

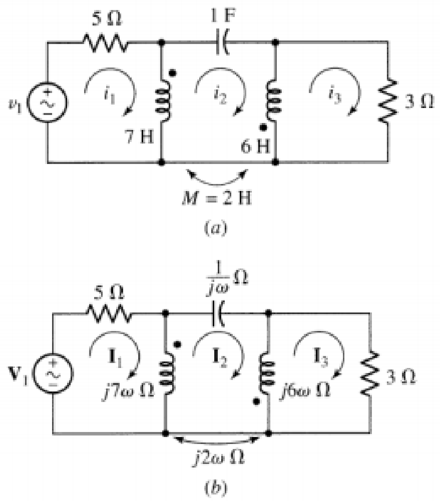
or $$-8j\omega i_1 + \left(14\,j\omega + \frac{1}{j\omega}\right) i_2 - 6j\omega i_3 = 0 \qquad \text{(ii)}$$

Applying KVL in the third mesh (rightmost mesh),

$$3j\omega(i_3 - i_2) + 3j\omega(i_1 - i_2) + 2i_3 = 0$$

or $$3j\omega i_1 - 6j\omega i_2 + (2 + 3j\omega)i_3 = 0. \qquad \text{(iii)}$$

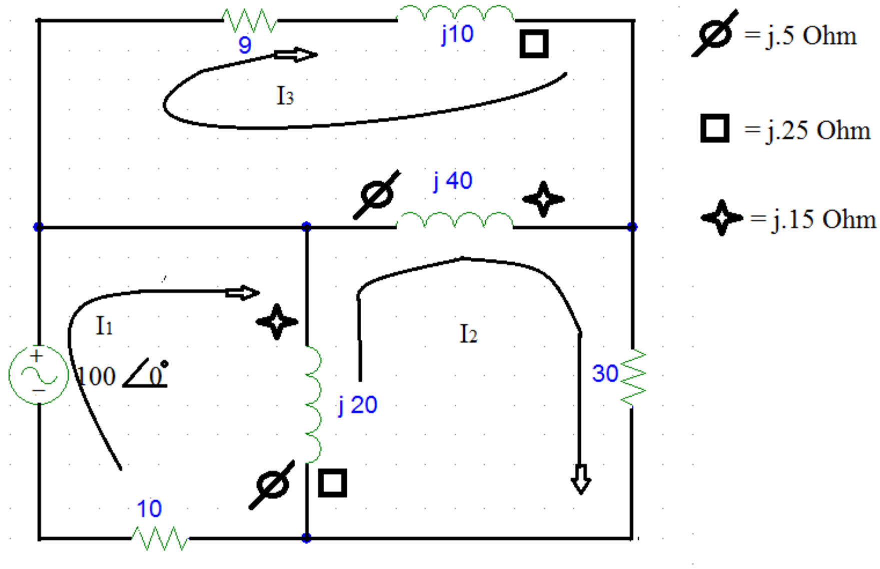
Write a complete set of phasor equations for the circuit of Fig. 13.10*a*.



■ **FIGURE 13.10** (*a*) A three-mesh circuit with mutual coupling. (*b*) The 1 F capacitance as well as the self- and mutual inductances are replaced by their corresponding impedances.

The circuit contains three meshes, and the three mesh currents have already been assigned. Once again, our first step is to replace both the mutual inductance and the two self-inductances with their corresponding impedances as shown in Fig. 13.10*b*. Applying Kirchhoff's voltage law to the first mesh, a positive sign for the mutual term is assured by

https://www.youtube.com/watch?v=X4YfWgwL3r4&feature=youtu.be



S- 4-B) I1, I2 ve I3 akımlarını matrisel biçimde yazınız.

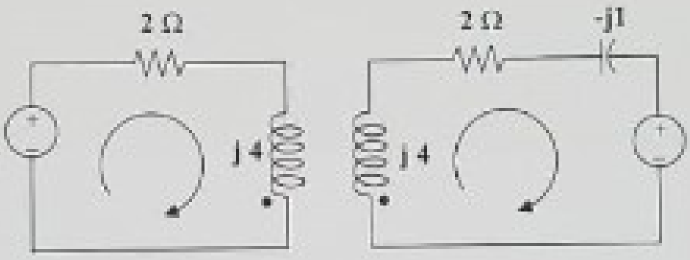
$10.I_1 + j.20.(I_1 - I_2) - j.5.I_2 + j.5.I_3 + j.25.I_3 - j.15.I_2 + j.15.I_3 = 100\angle 0^0$ 10puan

$30.I_2 + j.20.(I_2 - I_1) + j.40.(I_2 - I_3) - j.5.I_1 + 2.\, j.5.I_2 - j.5.I_3 - j.25.I_3 - j.15.I_1 + 2.\, j.15.I_2 - j.15.I_3 = 0$ 10puan

$(9 + j10).I_3 + j.40.(I_3 - I_2) + j.5.I_1 - j.5.I_2 + j.25.I_1 - j25.I_2 + j.15.I_1 - j.15I_2 = 0$ 10puan

$$\begin{vmatrix} 10 + j.20 & -j.20 - j.5 - j.15 & j.5 + j.25 + j.15 \\ -j.20 - j.5 - j.15 & 30 + j.20 + j.40 + j.10 + j.20 & j.40 - j.5 - j.25 - j.15 \\ j.5 + j.25 + j.15 & -j.40 - j.5 - j.25 - j.15 & 9 + j.10 + j.40 \end{vmatrix} . \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 100\angle 0^0 \\ 0 \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} 10 + j.20 & -j.40 & j.45 \\ -j.40 & 30 + j.100 & -j.85 \\ j.45 & -j.85 & 9 + j.50 \end{vmatrix} . \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 100 \\ 0 \\ 0 \end{vmatrix}$$ 5puan



For loop 1,

$$8\angle 30^\circ = (2 + j4)I_1 - jI_2 \qquad (1)$$

For loop 2,

$$((j4 + 2 - j)I_2 - jI_1 + (-j2) = 0$$

or

$$I_1 = (3 - j2)I_2 - 2 \qquad (2)$$

Substituting (2) into (1),

$$8\angle 30^\circ + (2 + j4)2 = (14 + j7)I_2$$

$$I_2 = (10.928 + j12)/(14 + j7) = 1.037\angle 21.12^\circ$$

$$V_x = 2I_2 = \underline{\mathbf{2.074\angle 21.12^\circ}}$$