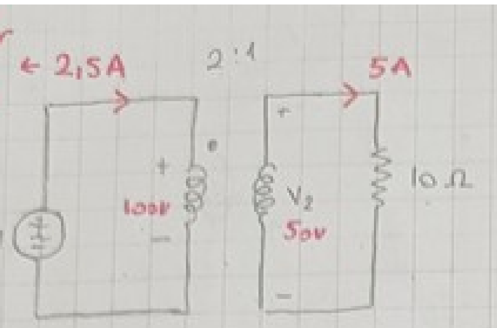
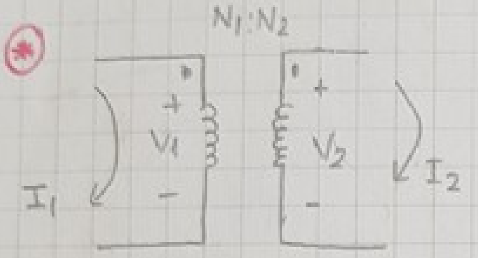
akımlar ters orantılı



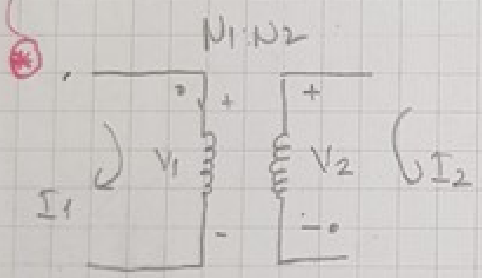


$$\frac{V_1}{V_2} = \frac{+N_1}{N_2}$$

(* noktaların yerinin işaretine göre belirlenir.)

$$\frac{I_1}{I_2} = -\left(\frac{-N_2}{N_1}\right) = \frac{N_2}{N_1}$$

(* akım her zaman (−) ile başlanır. Sonra karşısındakine geçilir, noktadan çıkarsa (−), girerse (+) yazılır)

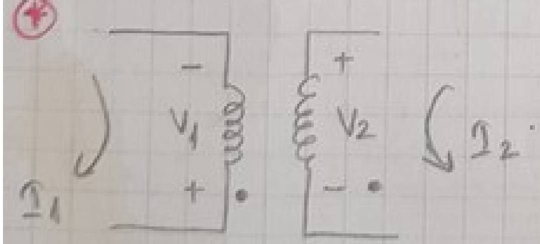


$$\frac{V_1}{V_2} = \frac{N_1}{-N_2}$$

(* N'lerde (V'lerde) noktanın işareti, I'larda noktanın giriş-çıkış durum

$$\frac{I_1}{I_2} = -\left(\frac{-N_2}{+N_1}\right) = +\frac{N_2}{N_1}$$

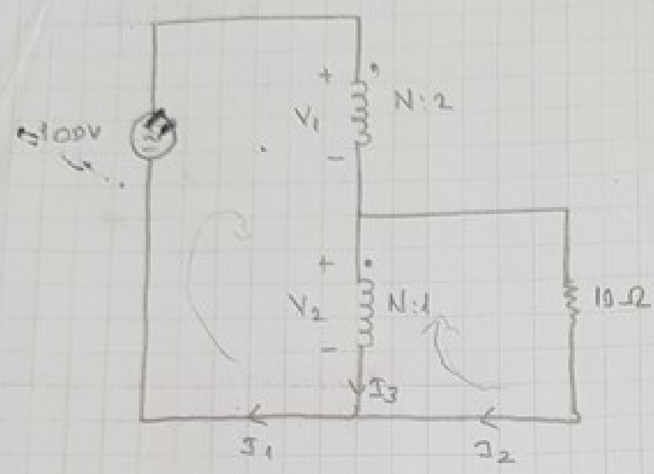
★ $\frac{I_1}{I_2} \Rightarrow -\left(\frac{I_1\text{'in karşısındaki}}{I_2\text{'nin karşısındaki}}\right)$



$$\frac{V_1}{V_2} = \frac{N_1}{-N_2} = -\frac{N_1}{N_2}$$

$$\frac{I_1}{I_2} = -\left(\frac{-N_2}{-N_1}\right) = -\frac{N_2}{N_1}$$

Step up:



① V'leri oranla, V'ler arasındaki bağıntıyı bul.

② I'ları oranla, I'lar arasındaki bağıntıyı bul.

③ 1. çevreyi al, V'leri bul. (V bakımından çevre)

④ 2. çevreyi al, I'ları bul. (I bakımından çevre)

1) $\dfrac{V_1+V_2}{V_2} = \dfrac{+2+1}{1} = \dfrac{3}{1}$ (N için noktaların işaretine bakıyorduk)

(1. çevrede ne var [illegible] bakma / 2. çevrede [illegible] ise bakma)

$\dfrac{V_1+V_2}{V_2} = \dfrac{3}{1} \Rightarrow \quad 3V_2 = V_1 + V_2$

$\boxed{V_1 = 2V_2}$

2) $\dfrac{I_1}{I_2} = -\left(\dfrac{-1}{2+1}\right) = \dfrac{1}{3} \qquad \boxed{I_2 = 3I_1}$

3) $V_1 + V_2 = 100$

$10I_2 - V_2 = 0 \rightarrow 10I_2 = V_2 \qquad 2V_2 + V_2 = 100 \Rightarrow \boxed{V_2 = 33{,}33V}$

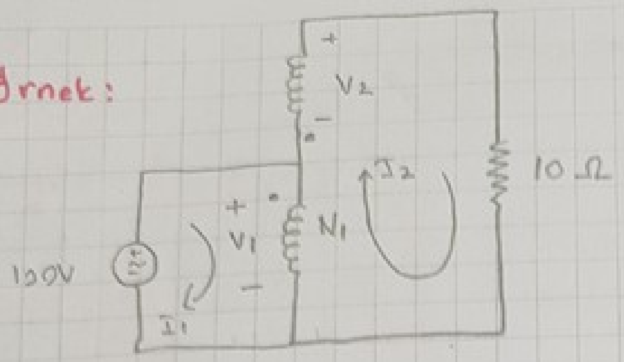
4) $I_2 = \dfrac{V_2}{10} \Rightarrow I_2 = \dfrac{33{,}33}{10} \Rightarrow \boxed{I_2 = 3{,}33}$

5) $I_1 = \dfrac{I_2}{3} = \dfrac{3{,}33}{3} = \boxed{1{,}11\ A}$

6) $V_1 = 2V_2 = 2 \cdot 33{,}33 = \boxed{66{,}66\ V}$

7) $I_1 = I_2 + I_3 \qquad I_3 = I_1 - I_2 = 1{,}11 - 3{,}33 = \boxed{-2{,}22\ A}$

Örnek:



$N_1 : 2$

$N_2 : 3$

$$\frac{V_1}{V_1+V_2} = \frac{N_1}{N_1-N_2} = \frac{2}{2-3} = -2$$

$$V_1 = -2V_1 - 2V_2$$

$$3V_1 = -2V_2 \Rightarrow \boxed{\frac{V_1}{V_2} = -\frac{2}{3}}$$

$$\frac{I_1}{I_2} = -\left(\frac{-N_1+N_2}{N_1}\right) \Rightarrow \frac{I_1}{I_2} = -\left(\frac{1}{2}\right) = -\frac{1}{2} \qquad \boxed{I_2 = -2I_1}$$

$$-100 + V_1 = 0 \Rightarrow \boxed{V_1 = 100V}$$

$$V_2 = -\frac{3}{2}V_1 = -\frac{3}{2}\cdot 100 = \boxed{-150}$$

$$-V_1 - V_2 + 10I_2 = 0$$

$$-100 - (-150) + 10I_2 = 0$$

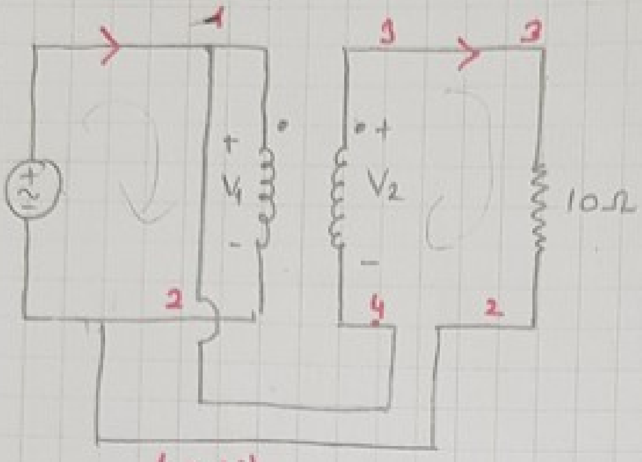
$$-100 + 150 + 10I_2 = 0$$

$$10I_2 = -50$$

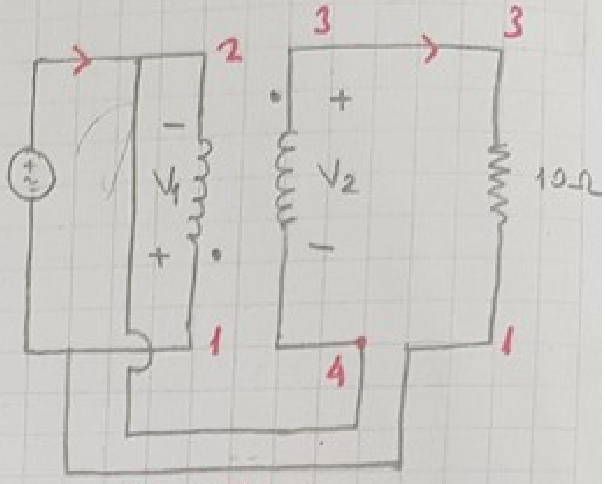
$$\boxed{I_2 = -5A}$$

$$I_1 = -\frac{1}{2}I_2 = \boxed{2{,}5\ A}$$

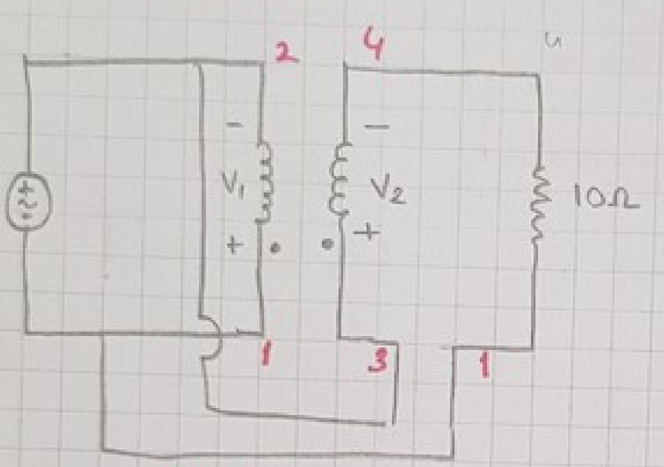
Örnek:



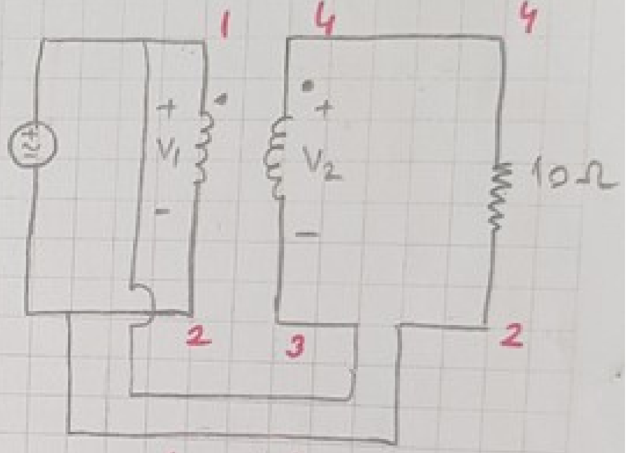
1. şekil



3. şekil



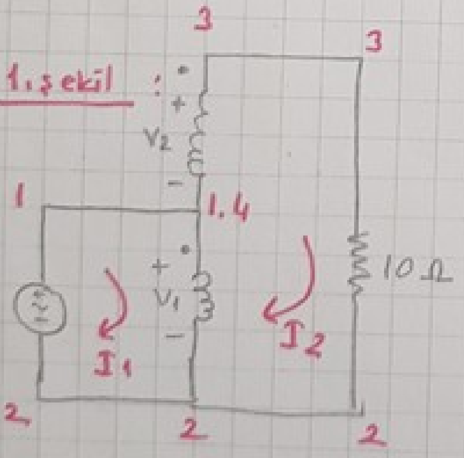
2. şekil



4. şekil

$N_1 : 4$
$N_2 : 1$

1. şekil :



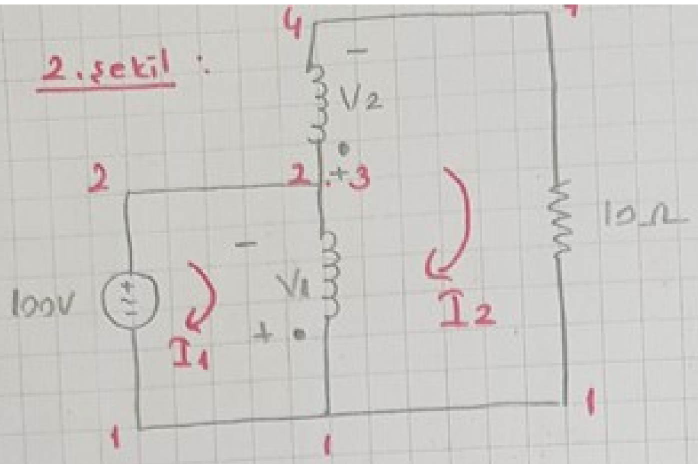
Sonuç: $V_1 = 100\,V$

$V_2 = 25\,V$

$I_1 = 15{,}625\,A$ $\left(12{,}5 + \frac{12{,}5}{4}\right)$

$I_2 = 12{,}5\,A$

2. şekil :



$$\frac{V_1}{V_1+V_2} = \frac{4}{4+1} = \frac{4}{5}$$

$$5V_1 = 4V_1 + 4V_2 \qquad \boxed{V_1 = 4V_2}$$

$$\frac{I_1}{I_2} = -\left(\frac{+N_1+N_2}{-N_1}\right) = -\frac{5}{-4} = \frac{5}{4} \qquad \boxed{4I_1 = 5I_2}$$

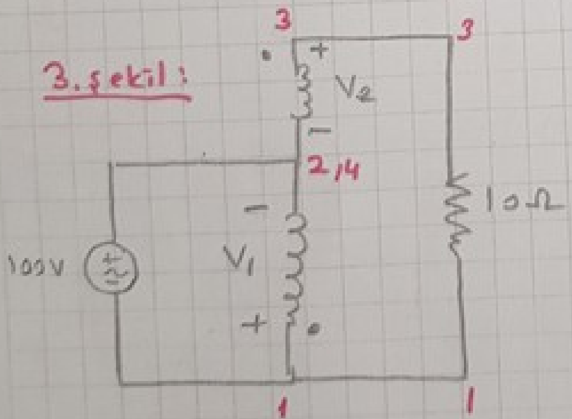
$$-100 - V_1 = 0$$

$$\boxed{V_1 = 100V} \qquad \boxed{V_2 = 25V}$$

$$+V_1 + V_2 + 10I_2 = 0$$

$$125 = -10I_2 \qquad \boxed{I_2 = -12{,}5A} \qquad I_1 = \frac{5I_2}{4} = \boxed{15{,}625\ A}$$

3. şekil :

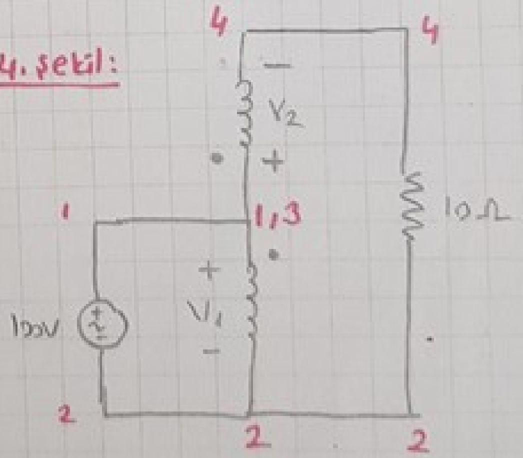


$V_1 = -100V$
$V_2 = -25V$

$I_1 = 5{,}625 \left(7{,}5 - \frac{7{,}5}{4}\right)$

$I_2 = 7{,}5A$

4. şekil:



$V_1 = 100V$

$V_2 = 25V$

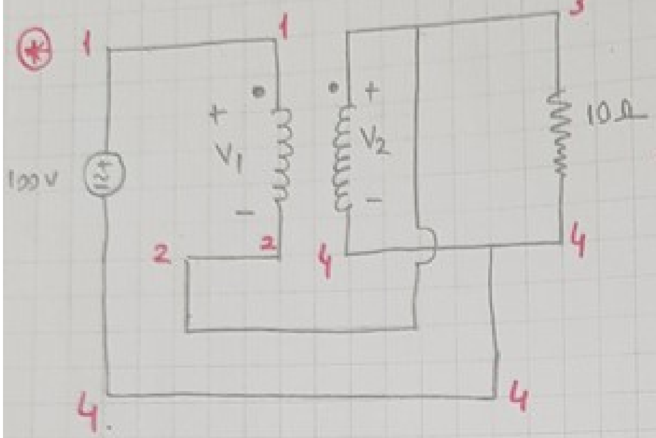
$I_1 = 5{,}625\ A$

$I_2 = 7{,}5\ A$

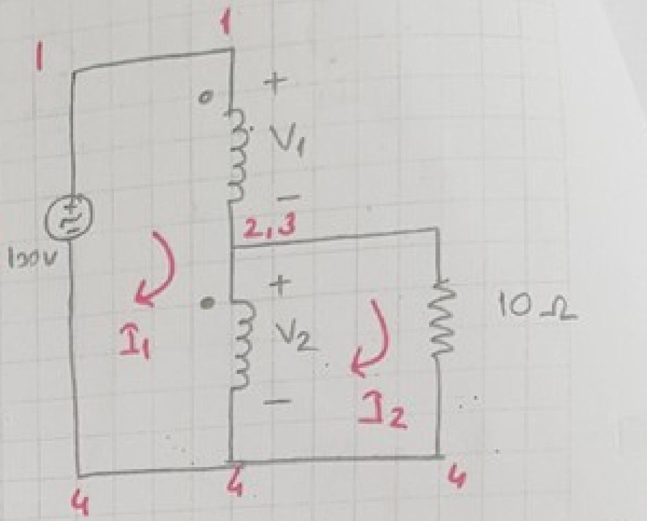
step down:

$N_1 : 4$

$N_2 : 1$

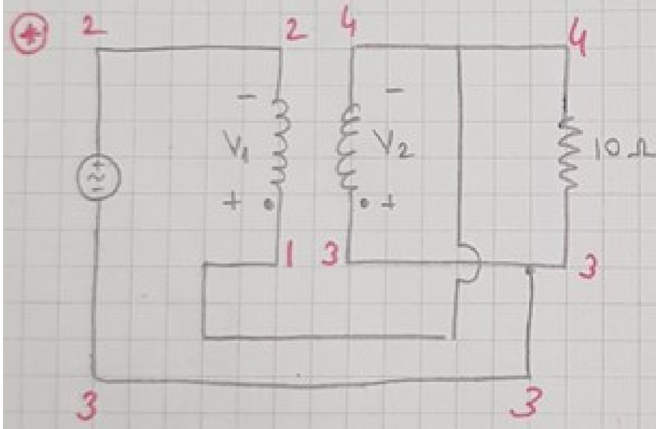


$\Rightarrow$

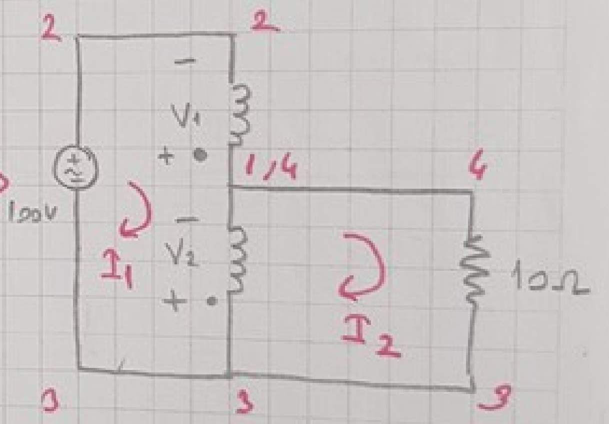


$V_1 = 80\ V$ $\quad I_1 = 2{,}5\ A\ \left(2+\frac{2}{4}\right)$

$V_2 = 20\ V$ $\quad I_2 = 2\ A$



$\Rightarrow$



$$\frac{V_1+V_2}{V_2} = \frac{+N_1+N_2}{+N_2} = \frac{4+1}{1} = 5 \Rightarrow 5V_2 = V_1+V_2$$

$$\boxed{V_1 = 4V_2}$$

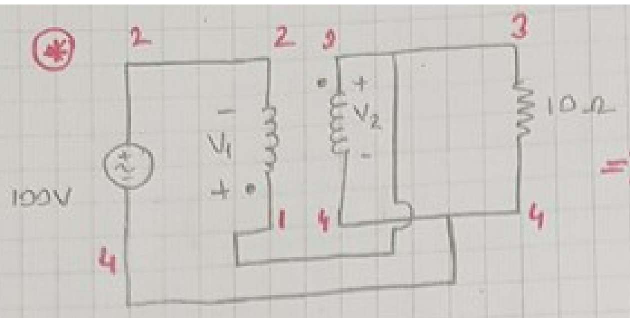
$$\frac{I_1}{I_2} = -\left(\frac{+N_2}{-N_1-N_2}\right) = -\left(\frac{1}{-4-1}\right) = \frac{1}{5}$$

$$\boxed{5\,I_1 = I_2}$$

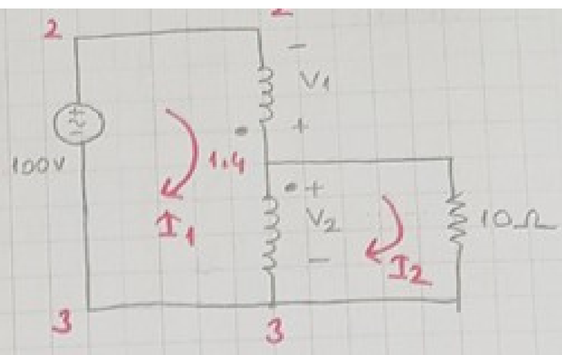
$$-100 - V_1 - V_2 = 0$$

$V_1 + V_2 = -100$ $\quad 5V_2 = -100V$ $\quad \boxed{V_2 = -20V}$ $\quad \boxed{V_1 = -80V}$

$+V_2 + 10 I_2 = 0 \Rightarrow 10 I_2 = 20$ $\quad \boxed{I_2 = 2\ A}$ $\quad \boxed{I_1 = \frac{I_2}{5} = 0{,}4A}$

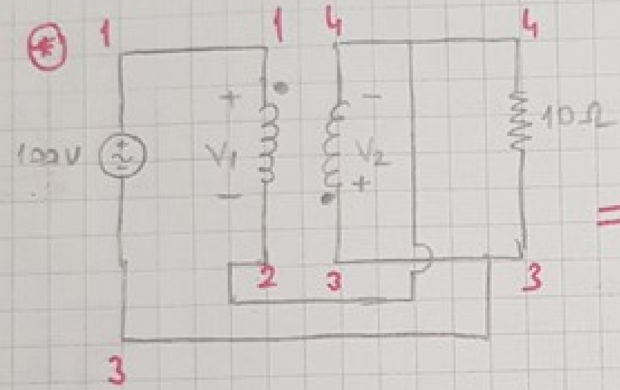


⇒

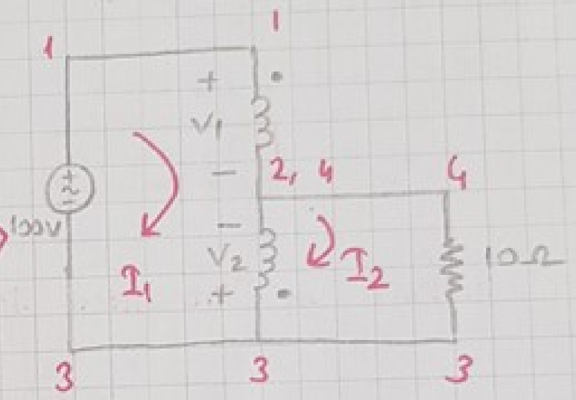


$$V_1 = -\frac{400}{3} \qquad I_1 = \frac{10}{9}$$

$$V_2 = -\frac{100}{3} \qquad I_2 = -\frac{10}{3}$$



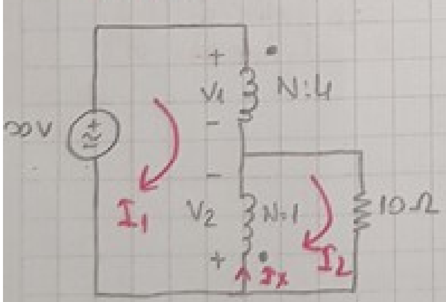
⇒



$$V_1 = \frac{400}{3} \qquad I_1 = \frac{10}{9}$$

$$V_2 = \frac{100}{3} \qquad I_2 = -\frac{10}{3}$$

Örnek:



$$\frac{V_1+V_2}{V_2} = \frac{4+1}{1} = 5 \qquad 5V_2 = V_1 + V_2 \qquad \boxed{V_1 = 4V_2}$$

$$\frac{I_1}{I_2} = -\left(\frac{+1}{+4-1}\right) = -\frac{1}{3} \qquad \boxed{I_2 = -3I_1}$$

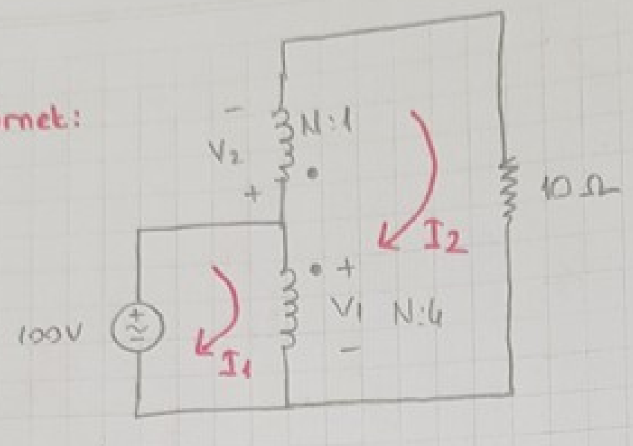
$-100+V_1-V_2=0 \quad V_1-V_2=100 \quad 4V_2-V_2=100 \quad 3V_2=100 \quad \boxed{V_2 = 33,33\text{ V}}$

$V_2+10I_2=0 \quad \cdot 10I_2=-V_2=-33,33 \Rightarrow \boxed{I_2=-3,33\text{A}} \rightarrow \boxed{I_1 = 1,11\text{ A}}$

$I_x = I_2 - I_1$ (yönlerine bakarak yazılır)

$I_x = -3,33 - 1,11 = \boxed{-4,44\text{ A}}$

Örnek:



$$\frac{V_1}{V_1+V_2} = \frac{4}{4+1} = \frac{4}{5}$$

$$4V_1 + 4V_2 = 5V_1$$

$$\boxed{4V_2 = V_1}$$

$$\frac{I_1}{I_2} = -\left(\frac{-4+1}{4}\right) = \frac{3}{4}$$

$$\boxed{3I_2 = 4I_1}$$

$$-100 + V_1 = 0$$

$$\boxed{V_1 = 100V} \Rightarrow \boxed{V_2 = 25V}$$

$$-V_1 + V_2 + 10I_2 = 0$$

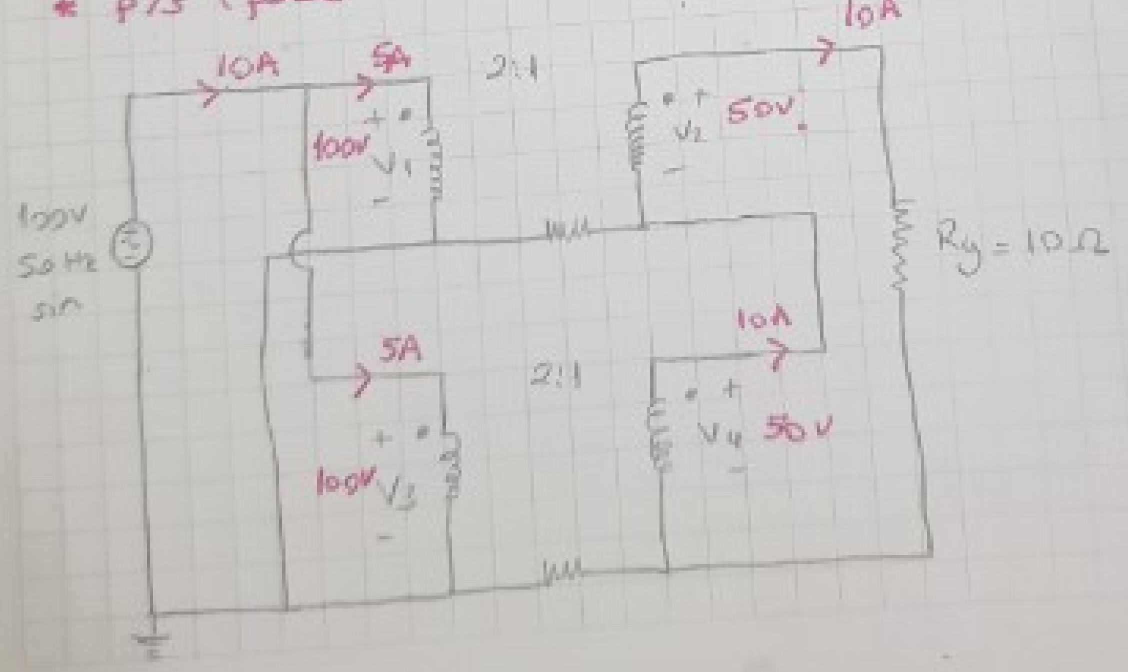
$$-100 + 25 + 10I_2 = 0$$

$$10I_2 = 75 \quad \boxed{I_2 = 7{,}5A} \Rightarrow I_1 = \frac{3 \cdot I_2}{4} = \frac{3 \cdot 7{,}5}{4} = \boxed{5{,}6}$$

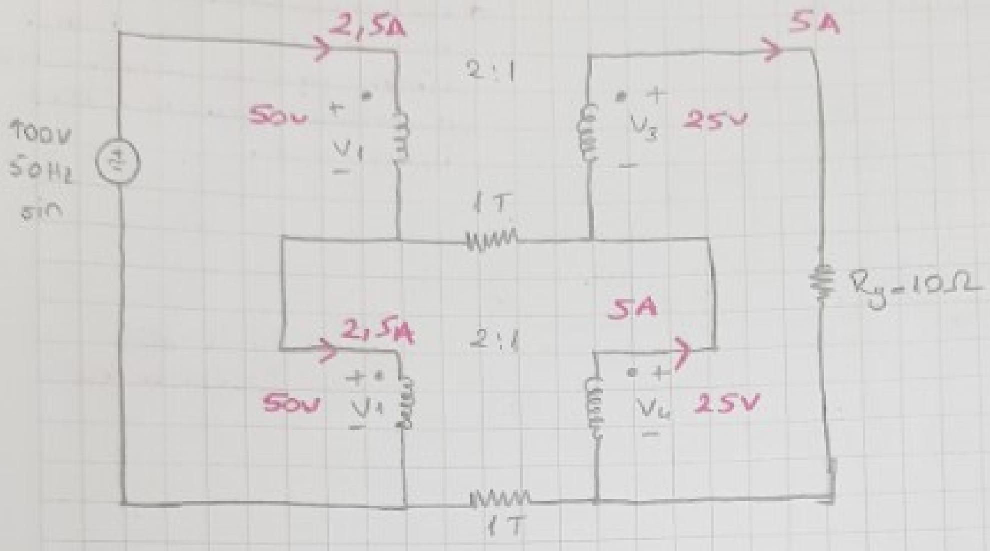
Trafo : (İndirici)

* P/P (paralel-paralel)

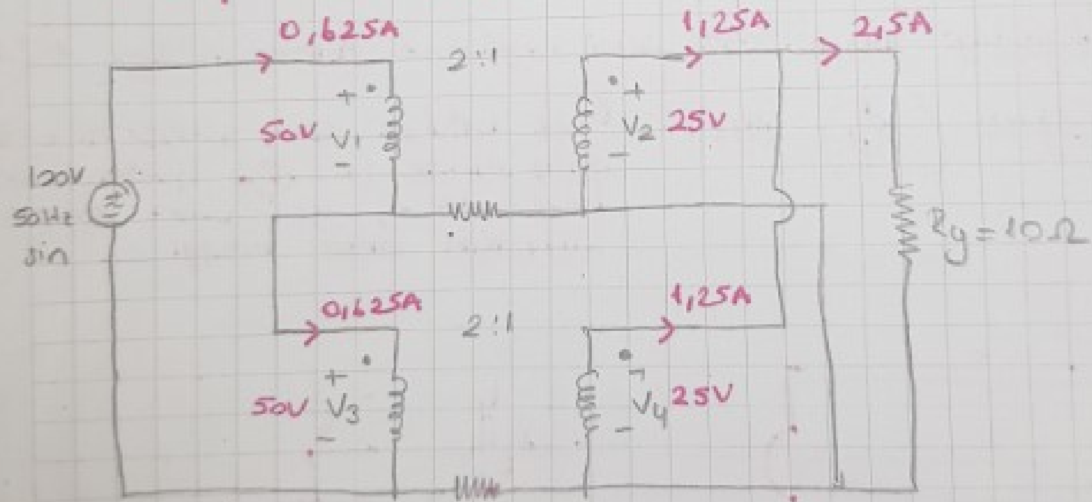
* P/S (paralel + seri)



s/s (seri - seri)

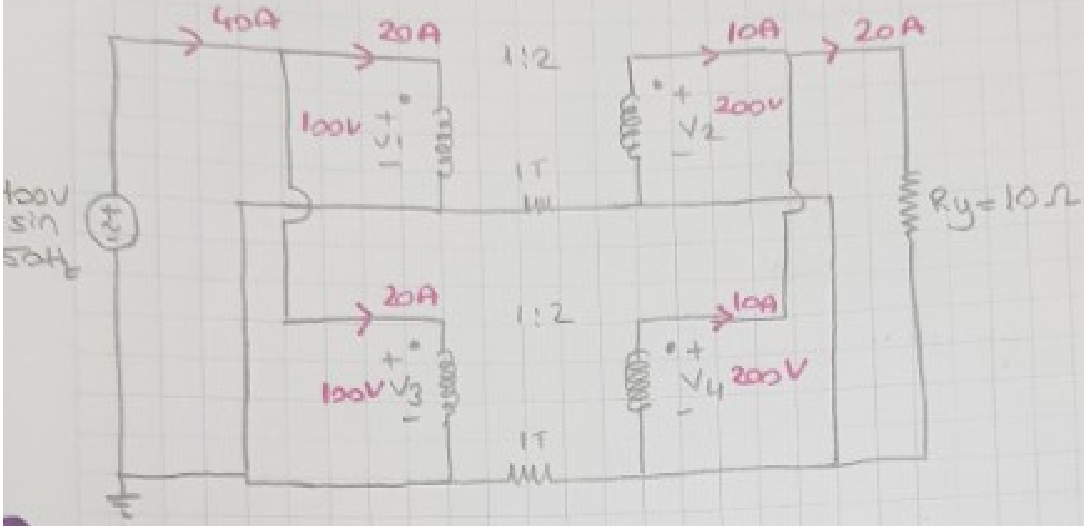


* s/p (seri - paralel)

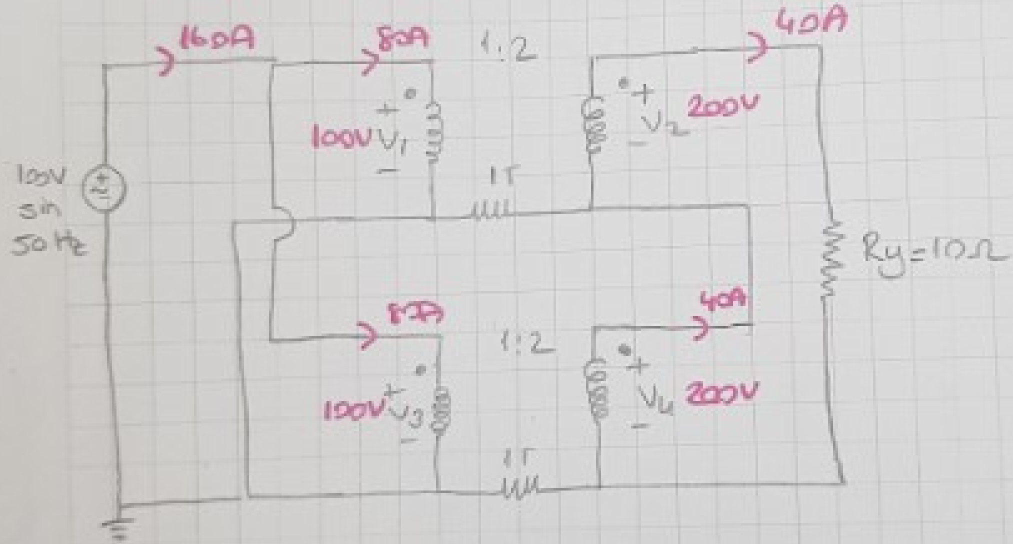


* Trafoların primer, sekonder, kaynak ve yük akımlarını bulunuz.

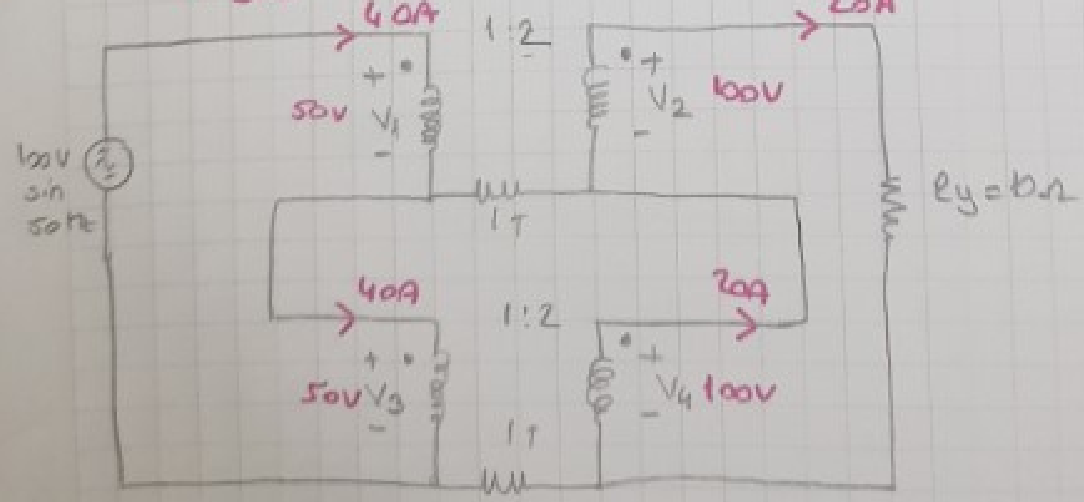
* P/P

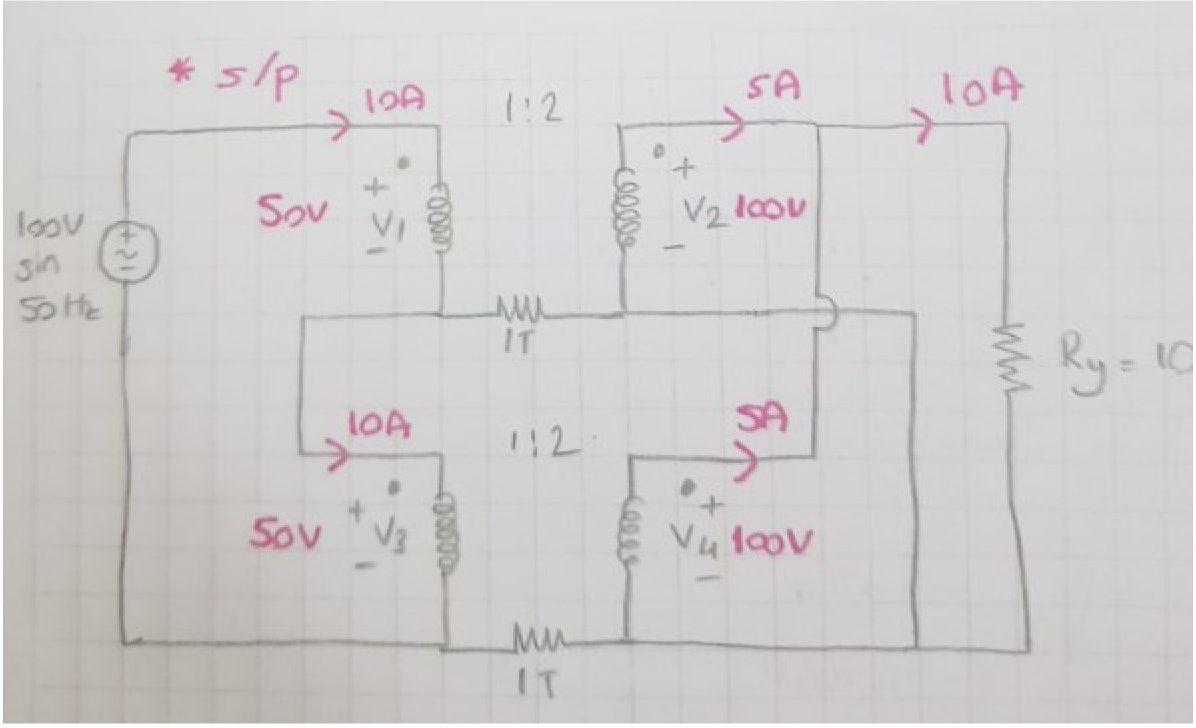

40A
20A
1:2
10A
20A
100V
V1
200V
V2
1T
100V sin 50Hz
Ry=10Ω
20A
1:2
10A
100V
V3
V4
200V
1T

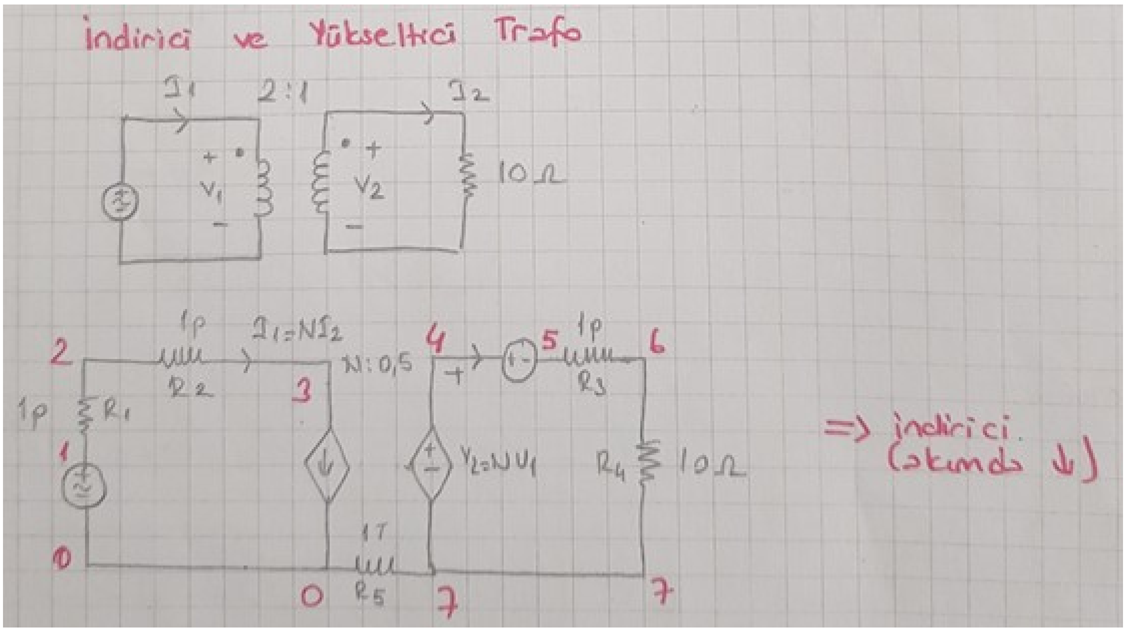
* P/S


160A
80A
1:2
40A
100V
V1
V2
200V
1T
100V sin 50Hz
Ry=10Ω
80A
1:2
40A
100V
V3
V4
200V
1T

* S/S


40A
1:2
20A
50V
V1
V2
100V
100V sin 50Hz
Ry=10Ω
1T
40A
1:2
20A
50V
V3
V4
100V
1T

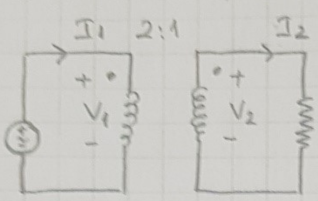

* s/p
10A
1:2
5A
10A
100V
sin
50Hz
50V
V1
V2
100V
1T
Ry = 10
10A
1:2
5A
50V
V3
V4
100V
1T

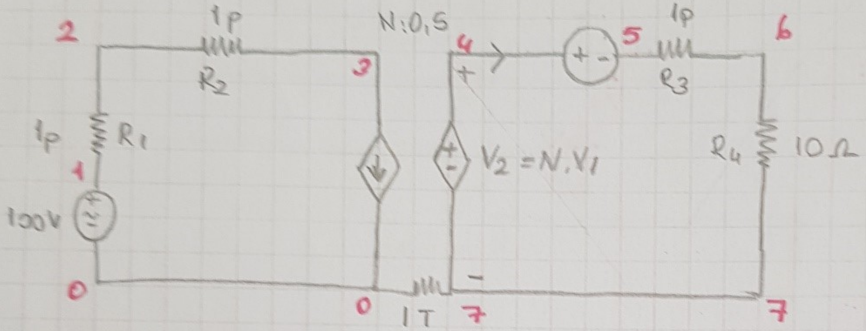

İndirici ve Yükseltici Trafo
I1
2:1
I2
V1
V2
10Ω
1p
I1=NI2
N:0,5
1p
2
R2
3
4
5
6
R3
1p
R1
1
V2=NV1
R4
10Ω
1T
0
R5
0
7
7
=> indirici.
(altında ↓)

(Daha önceki bir konu yanlışlıkla tekrar edildi)

İndirici ve Yükseltici Trafo :

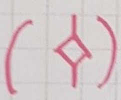
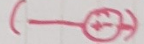
* Bağlantı şekli değişmeden trafonun katsayısını 1'den büyük yaparsak indirici trafo yükseltici ; yükseltici trafo indirici olur.

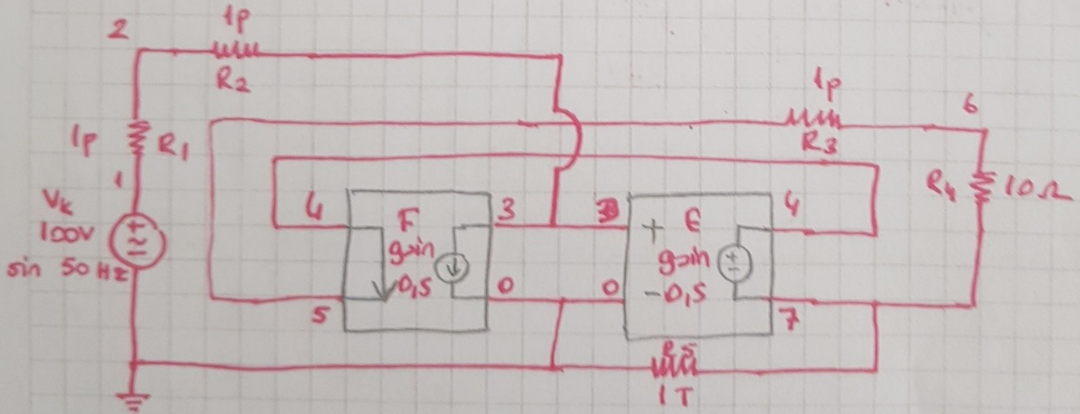




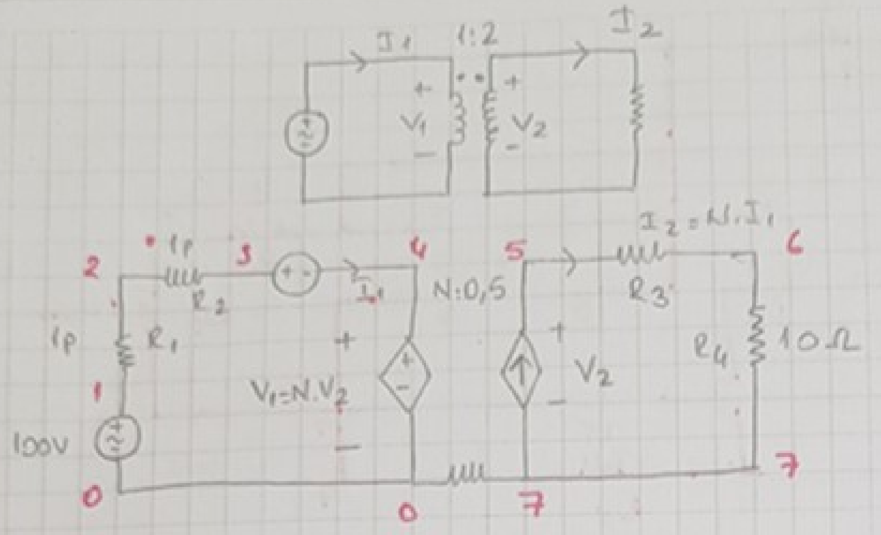
← indirici (akımda iniş)

(*) Bağımlı kaynaklar çıkıştır. Giriş suni kaynaksa durum voltajı 0'dır.

(*) Bağımlı kaynak () çıkıştır. Önce kutuları çiz, çıkışlarını belirt. Giriş içinse () ifadesini karşı taraf için al, geri kalan diğer çıkış için diğer bacağı al.

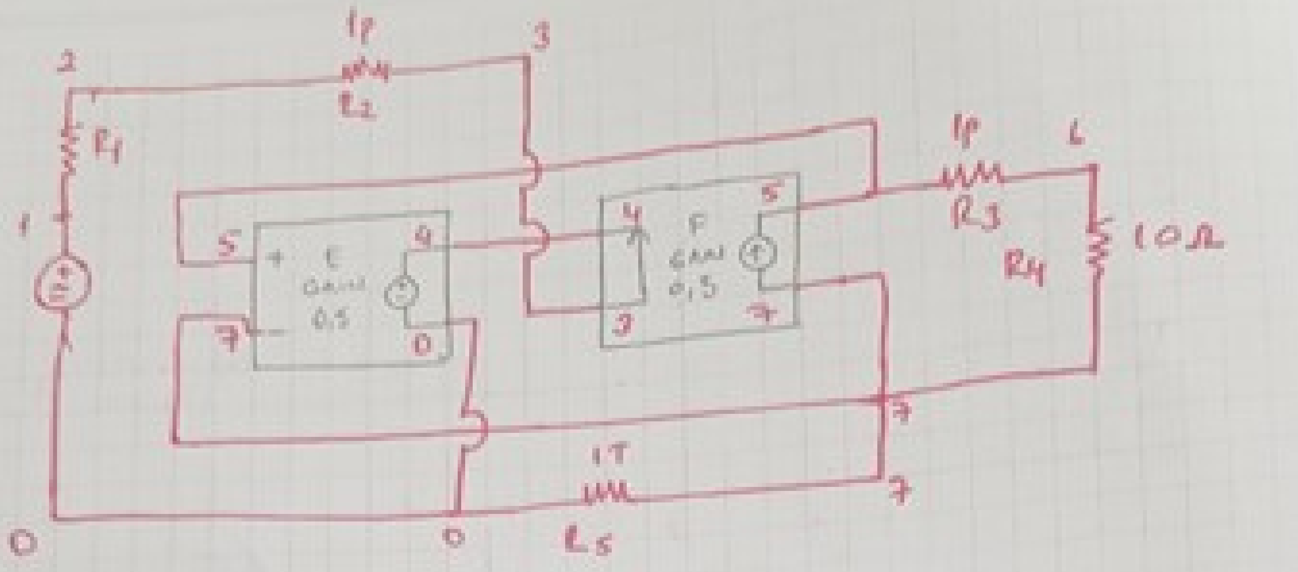


indirici



⇒ yükseltici (akımda ↑)

* Bağımlı kaynaklar çıkıştır. Giriş suni kaynaksa dummy voltage 0'dır.
* Bağımlı kaynak ( ) çıkıştır.
* Bağlantı şekli değişmeden trafonun kazancını 1'den büyük yaparsak indirici trafo - yükseltici ; yükseltici trafo - indirici olur.
* Önce kutuları çiz, sonra çıkışlarını belirt. Giriş için ise, yukarıdaki: ifadesini karşı taraf için al, geri kalan diğer çıkış için diğer bacağı al.



yükseltici

Separation of Core Losses

The core loss of a transformer depends upon the frequency and the maximum flux density when the volume and the thickness of the core laminations are given. The core loss is made up of two parts (*i*) hysteresis loss $W_h = PB^{1.6}_{max} f$ as given by Steinmetz's empirical relation and (*ii*) eddy current loss $W_e = QB^2_{max} f^2$ where Q is a constant. The total core-loss is given by

$$W_i = W_h + W_e = PB^{1.6}_{max} f^2 + QB^2_{max} f^2$$

If we carry out two experiments using two different frequencies but the same maximum flux density, we should be able to find the constants P and Q and hence calculate hysteresis and eddy current losses separately.

Example 32.29. *In a transformer, the core loss is found to be 52 W at 40 Hz and 90 W at 60 Hz measured at same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz.*

(Elect. Machines, Nagpur Univ. 1993)

Solution. Since the flux density is the same in both cases, we can use the relation

Total core loss $W_1 = Af + Bf^2$ or $W_i/f = A + Bf$

$\therefore$ $52/40 = A + 40B$ and $90/60 = A + 60B$; $\therefore$ $A = 0.9$ and $B = 0.01$

At 50 Hz, the two losses are

$$W_h = A_f = 0.9 \times 50 = 45 \text{ W} ; W_e = Bf^2 = 0.01 \times 50^2 = 25 \text{ W}$$

Example 32.30. *In a power loss test on a 10 kg specimen of sheet steel laminations, the maximum flux density and waveform factor are maintained constant and the following results were obtained:*

Frequency (Hz)	25	40	50	60	80
Total loss (watt)	18.5	36	50	66	104

Calculate the eddy current loss per kg at a frequency of 50 Hz.

(Elect. Measur. A.M.I.E. Sec B, 1991)

Solution. When flux density and wave form factor remain constant, the expression for iron loss can be written as

$$W_i = Af + Bf^2 \quad \text{or} \quad W_i/f = A + Bf$$

The values of W_i/f for different frequencies are as under :

f	25	40	50	60	80
W_i/f	0.74	0.9	1.0	1.1	1.3

The graph between f and W_i/f has been plotted in Fig. 32.44. As seen from it, $A = 0.5$ and $B = 0.01$

$\therefore$ Eddy current loss at 50 Hz $= Bf^2 = 0.01 \times 50^2 = 25$ W

$\therefore$ Eddy current loss/kg $= 25/10 = 2.5$ W

Example 32.31. *In a test for the determination of the losses of a 440-V, 50-Hz transformer, the total iron losses were found to be 2500 W at normal voltage and frequency. When the applied voltage and frequency were 220 V and 25 Hz, the iron losses were found to be 850 W. Calculate the eddy-current loss at normal voltage and frequency.*

(Elect. Inst. and Meas. Punjab Univ. 1991)

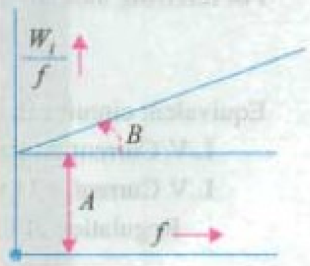


Fig. 32.44

Solution. The flux density in both cases is the same because in second case voltage as well as frequency are halved. Flux density remaining the same, the eddy current loss is proportional to f^2 and hysteresis loss $\propto f$.

Hysteresis loss $\propto f = Af$ and eddy current loss $\propto f^2 = Bf^2$

where A and B are constants.

Total iron loss $\quad W_i = Af + Bf^2 \quad \therefore \quad \dfrac{W_i}{f} = A + Bf$ $\quad$ ...(*i*)

Now, when $\quad f = 50$ Hz ; $W_i = 2500$ W

and when $\quad f = 25$ Hz ; $W_i = 850$ W

Using these values in (*i*) above, we get, from Fig. 32.44

$$2{,}500/50 = A + 50\,B \text{ and } 850/25 = A + 25\,B \quad \therefore \quad B = 16/25 = 0.64$$

Hence, at normal p.d. and frequency

$$\text{eddy current loss} = Bf^2 = 0.64 \times 50^2 = \mathbf{1600\ W}$$

$$\text{Hystersis loss} = 2500 - 1600 = 900 \text{ W}$$

Example 32.32. *When a transformer is connected to a 1000-V, 50-Hz supply the core loss is 00 W, of which 650 is hysteresis and 350 is eddy current loss. If the applied voltage is raised to 00 V and the frequency to 100 Hz, find the new core losses.*

Solution. Hysteresis loss $W_h \propto B_{max}^{1.6} f = PB_{max}^{1.6} f$

Eddy current loss $W_e \propto B_{max}^2 f^2 = QB_{max}^2 f^2$

From the relation $E = 4.44 fNB_{max}$ A volt, we get $B_{max} \propto E/f$

Putting this value of B_{max} in the above equations, we have

$$W_h = P\left(\frac{E}{f}\right)^{1.6} f = PE^{1.6} f^{-0.6} \text{ and } W_e = Q\left(\frac{E}{f}\right)^2 f^2 = QE^2$$

In the first case, $E = 1000$ V, $f = 50$ Hz, $W_h = 650$ W, $W_e = 350$ W

$\therefore \quad 650 = P \times 1000^{1.6} \times 50^{-0.6} \quad \therefore \quad P = 650 \times 1000^{-1.6} \times 50^{0.6}$

Similarly, $350 = Q \times 1000^2 \quad \therefore \quad Q = 350 \times 1000^{-2}$

Hence, constants P and Q are known.

Using them in the second case, we get

$$W_h = (650 \times 1000^{-1.6} \times 50^{0.6}) \times 2000^{1.6} \times 100^{-0.6} = 650 \times 2 = 1{,}300 \text{ W}$$

$$W_e = (350 \times 1000^{-2}) \times 2{,}000^2 = 350 \times 4 = 1{,}400 \text{ W}$$

$\therefore$ Core loss under new condition is $= 1{,}300 + 1{,}400 =$ **2700 W**

Alternative Solution

Here, both voltage and frequency are doubled, leaving the flux density unchanged.

With 1000 V at 50 Hz

$$W_h = Af \text{ or } 650 = 50\,A\,;\ A = 13$$

$$W_e = Bf^2 \text{ or } 350 = B \times 50^2\,;\ B = 7/50$$

With 2000 V at 100 Hz

$$W_h = Af = 13 \times 100 = 1300 \text{ W and}$$

$$W_e = Bf^2 = (7/50) \times 100^2 = 1400 \text{ W}$$

$\therefore$ New core loss $= 1300 + 1400 =$ **2700 W**

Example 32.33. *A transformer with normal voltage impressed has a flux density of 1.4 Wb/m² and a core loss comprising of 1000 W eddy current loss and 3000 W hysteresis loss. What do these losses become under the following conditions ?*

(*a*) *increasing the applied voltage by 10% at rated frequency.*

(*b*) *reducing the frequency by 10% with normal voltage impressed.*

(*c*) *increasing both impressed voltage and frequency by 10 per cent.*

(Electrical Machinery-I, Madras Univ. 1985)

Solution. As seen from Ex. 32.32

$$W_h = PE^{1.6} f^{-0.6} \quad \text{and} \quad W_e = QE^2$$

From the given data, we have $3000 = PE^{1.6} f^{-0.6}$ ...(*i*)

and $1000 = QE^2$ ...(*ii*)

where E and f are the normal values of primary voltage and frequency.

(*a*) Here voltage becomes $= E + 10\%\,E = 1.1\,E$

The new hysteresis loss is $W_h = P(1.1\,E)^{1.6} f^{-0.6}$ ...(*iii*)

Dividing Eq. (*iii*) by (*i*), we get $\dfrac{W_h}{3000} = 1.1^{1.6};\ W_h = 3000 \times 1.165 =$ **3495 W**

The new eddy-current loss is

$$W_e = Q\,(1.1.\,E)^2 \quad \therefore \quad \frac{W_e}{1000} = 1.1^2$$

$$\therefore \qquad W_e = 1000 \times 1.21 = \mathbf{1210\ W}$$

(*b*) As seen from Eq. (*i*) above eddy-current loss would not be effected. The new hysteresis loss is

$W_h = \quad PE^{1.6}\,(0.9\,f)^{-0.6} \quad ...(iv)$

From (*i*) and (*iv*), we get $\dfrac{W_h}{3000} = 0.9^{-0.6}$, $W_h = 3000 \times 1.065 = \mathbf{3{,}196\ W}$

(*c*) In this case, both *E* and *f* are increased by 10%. The new losses are as under :

$$W_h = P\,(1.1\,E)^{1.6}\,(1.1\,f)^{-0.6}$$

$$\therefore \quad \frac{W_h}{3000} = 1.1^{1.6} \times 1.1^{-0.6} = 1.165 \times 0.944$$

$$\therefore \quad W_h = 3000 \times 1.165 \times 0.944 = \mathbf{3{,}299\ W}$$

As W_e is unaffected by changes in *f*, its value is the same as found in (*a*) above *i.e.* **1210 W**

Example 32.34. *A transformer is connected to 2200 V, 40 Hz supply. The core-loss is 800 watts out of which 600 watts are due to hysteresis and the remaining, eddy current losses. Determine the core-loss if the supply voltage and frequency are 3300 V and 60 Hz respectively.*

(Bharathiar Univ. Nov. 1997)

Solution. For constant flux density (*i.e.* constant V/f ratio), which is fulfilled by 2200/40 or 3300/60 figures in two cases,

$$\text{Core-loss} = Af + Bf^2$$

First term on the right-hand side represents hysteresis-loss and the second term represents the eddy-current loss.

At 40 Hz, 800 = 600 + eddy current loss.

Thus, $\quad Af = 600, \quad \text{or} \quad A = 15$

$\quad Bf^2 = 200, \quad \text{or} \quad B = 200/1600 = 0.125$

At 60 Hz, $\quad \text{core-loss} = 15 \times 60 + 0.125 \times 60^2$

$\quad = 900 + 450$

$\quad = 1350 \text{ watts}$

Example 1.5. *A single phase transformer is designed to operate at 240/120 V, 50 Hz. Calculate the secondary no load voltage and its frequency if the h.v. side of the transformer is connected to :*

(a) 240 V, 40 Hz; (b) 120 V, 25 Hz; (c) 120 V, 50 Hz ;
(d) 480 V, 50 Hz; (e) 240 V, dc.

Solution. Primary voltage V_1 at frequency f_1 is

$$V_1 = \sqrt{2}\pi f_1 \phi_{max1} N_1.$$

Let the primary voltage at frequency f_2 be V_{11} so that

$$V_{11} = \sqrt{2}\pi f_2 \phi_{max2} N_1$$

$$\therefore \quad \frac{V_{11}}{V_1} = \frac{f_2 \phi_{max2}}{f_1 \phi_{max1}} \qquad ...(i)$$

(a) From Eq. (i), $\dfrac{240}{240} = \dfrac{(40)(\phi_{max2})}{(50)(\phi_{max1})}$

or $\phi_{max2} = 1.25\, \phi_{max1}$.

Secondary no load voltage at frequency f_1 is

$$E_2 = \sqrt{2}\,\pi f_1 \phi_{max\,1} N_2$$

and at frequency f_2 is $E_{22} = \sqrt{2}\pi f_2 \phi_{max2} N_2$

$$\therefore \quad \frac{E_{22}}{E_2} = \frac{f_2 \phi_{max2}}{f_1 \phi_{max1}} \qquad ...(ii)$$

$$E_{22} = 120\left(\frac{40 \times 1.25\, \phi_{max1}}{50 \times \phi_{max1}}\right) = 120 \text{ volts at 40 Hz.}$$

This shows that the magnitude of the secondary voltage remains unaffected, even if the supply frequency changes.

(b) From Eq. (i), $\dfrac{120}{240} = \dfrac{(25)(\phi_{max2})}{(50)(\phi_{max1})}$ or $\phi_{max2} = \phi_{max1}$

From Eq. (ii), $E_{22} = 120 \times \dfrac{(25)(\phi_{max1})}{(50)(\phi_{max1})} = 60$ volts at 25 Hz.

(c) From Eq. (i), $\dfrac{120}{240} = \dfrac{(50)(\phi_{max2})}{(50)(\phi_{max1})}$ or $\phi_{max2} = 0.5\, \phi_{max1}$

From Eq. (ii), $E_{22} = 120 \times \dfrac{50 \times 0.5\, \phi_{max1}}{(50) \times (\phi_{max1})} = 60$ V at 50 Hz.

(d) From Eq. (i), $\dfrac{480}{240} = \dfrac{50 \times (\phi_{max2})}{50 \times (\phi_{max1})}$ or $\phi_{max2} = 2\phi_{max1}$.

For the same core area, the flux density is doubled, the magnetizing current becomes quite large (refer to B-H curve) and the transformer may get damaged.

(e) The direct current is alternating current at zero frequency. In other words, there is no change of flux. As a result, secondary induced emf, $E_{22} = 0$. Further the counter emf. E_1, which opposes the applied voltage V_1 is also zero. Therefore, the primary no load current is limited only by the primary winding resistance r_1. Since r_1 is quite small, the current $240/r_1$ will be tremendously high and transformer will definitely get burnt.

EXAMPLE 1.16. *In a transformer, the core loss is 100 W at 40 Hz and 72 W at 30 Hz. Find the hysteresis and eddy current losses at 50 Hz.*

SOLUTION. $$\frac{P_i}{f} = a + bf$$

$$\frac{100}{40} = a + 40\,b$$

and $$\frac{72}{30} = a + 30\,b$$

Solution of these equation gives

$$a = 2.1, \qquad b = 0.01$$

Therefore, hysteresis loss at 50 Hz

$$= af = 2.1 \times 50 = 105 \text{ W}$$

Eddy-current loss at 50 Hz

$$= bf^2 = 0.01 \times (50)^2 = 25 \text{ W}$$

EXAMPLE 1.17. *At 400 V and 50 Hz the total core loss of a transformer was found to be 2400 W. When the transformer is supplied at 200 V, and 25 Hz, the core loss is 800 W. Calculate the hysteresis and eddy current loss at 400 V and 50 Hz.*

SOLUTION. $$\frac{V_1}{f_1} = \frac{400}{50} = 8$$

$$\frac{V_2}{f_2} = \frac{200}{25} = 8$$

Since $$\frac{V_1}{f_1} = \frac{V_2}{f_2} = 8$$

the flux density B_m remains constant. Hence

$$\frac{P_i}{f} = a + bf$$

$$\therefore \quad \frac{2400}{50} = a + 50\,b$$

and $$\frac{800}{5} = a + 25\,b$$

Solving these equations, we get

$$a = 16 \qquad b = 0.64$$

Therefore, at 50 Hz

$$P_h = af = 16 \times 50 = 800 \text{ W}$$

$$P_e = bf^2 = 0.64 \times (50)^2 = 1600 \text{ W}$$

Example 1.24. *In a transformer, the core loss is found to be 52 watts at 40 Hz and 90 watts at 60 Hz ; both losses being measured at the same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz.*

Solution. From Eq. (1.47), the total core loss is

$$P_c = K_h f B_m^x + K_e f^2 B_m^2$$

For constant flux density $P_c = k_h f + k_e f^2$

where constant $k_h = K_h B_m^x$ and constant $k_e = K_e B_m^2$

$\therefore$ At 40 Hz, $52 = k_h (40) + k_e (40)^2$

and at 60 Hz, $90 = k_h (60) + k_e (60)^2$

or $52 = 40\, k_h + 1600\, k_e$

and $90 = 60\, k_h + 3600\, k_e$.

From the above two equations, k_h and k_e are found to be

$$k_h = \frac{9}{10} \text{ and } k_e = \frac{1}{100}.$$

Thus at 50 Hz,

the hysteresis loss $P_h = k_h f = \frac{9}{10}(50) = 45$ watts.

and eddy current loss $P_e = k_e f^2 = \frac{1}{100}(50)^2 = 50$ watts.

Example 1.25. *A 220 V, 60 Hz, single-phase transformer has hysteresis loss of 340 watts and eddy current loss of 120 watts. If the transformer is operated from 230 V, 50 Hz supply mains, then compute its total core loss. Assume Steinmetz's constant equal to 1.6.*

Solution. The operating voltage

$$V_1 = \sqrt{2}\pi f_1 B_{m1} A_i N \text{ and } V_{11} = \sqrt{2}\pi f_2 B_{m2} A_i N$$

$\therefore$ $$\frac{V_1}{V_{11}} = \frac{f_1 B_{m1}}{f_2 B_{m2}} \quad \text{or} \quad \frac{220}{230} = \left(\frac{60}{50}\right)\left(\frac{B_{m1}}{B_{m2}}\right)$$

$\therefore$ $$B_{m2} = \frac{(60)(230)}{(50)(220)} B_{m1} = 1.255\, B_{m1}.$$

From Eq. (1.46), hysteresis loss

$$P_h = K_h f B_m^x$$

$\therefore$ $$\frac{P_{h2}}{P_{h1}} = \frac{f_2 B_{m2}^x}{f_1 B_{m1}^x} = \frac{50}{60}(1.255)^{1.6}$$

or $$P_{h2} = (340)\left(\frac{5}{6}\right)(1.255)^{1.6} = (340)\left(\frac{5}{6}\right)(1.438) = 408 \text{ W}.$$

From Eq. (1.46), eddy current loss $P_e = K_e f^2 B_m^2$

$\therefore$ $$\frac{P_{e2}}{P_{e1}} = \left(\frac{f_2}{f_1}\right)^2 \left(\frac{B_{m2}}{B_{m1}}\right)^2 = \left(\frac{50}{60}\right)^2 (1.255)^2$$

or $$P_{e2} = (120)\left(\frac{5}{6}\right)^2 (1.255)^2 = 131.3 \text{ W}.$$

$\therefore$ Total core loss $P_{c2} = P_{h2} + P_{e2} = 408 + 131.3 = 539.3$ W.

Example 1.26. *The ohmic, hysteresis and eddy current losses in a transformer at 50 Hz are 1.6%, 0.9% and 0.6% respectively. For a Steinmetz's coefficient of 1.6, find*

(a) these losses at 60 Hz, for the same system voltage and current and

(b) the output at 60 Hz, for the total losses to remain the same as on 50 Hz.

Solution. (*a*) Subscripts 1 and 2 will be used to refer to 50 and 60 Hz quantities respectively.

Since the voltage and current at both the frequencies of 50 Hz and 60 Hz are the same, the output also remains the same.

The ohmic loss depends on the current and here it is given that the current at both the frequencies are equal. Therefore, ohmic losses in watts remain unchanged and for the same output, the percentage ohmic losses are again 1.6%. Thus

Ohmic loss at 50 Hz, P_{oh1} = Ohmic loss at 60 Hz, P_{oh_2} = 1.6%.

As before $$\frac{V_1}{V_2} = \frac{f_1 B_{m1}}{f_2 B_{m2}} \quad \text{or} \quad 1 = \frac{50\, B_{m1}}{60\, B_{m2}} \quad \text{or} \quad B_{m2} = \frac{5}{6} B_{m1}$$

From Eq. (1.46), hysteresis loss, $P_h = K_h f B_m^{\,x}$

$$\therefore \qquad \frac{P_{h2}}{P_{h1}} = \frac{f_2}{f_1}\left(\frac{B_{m2}}{B_{m1}}\right)^x$$

or $$P_{h2} = P_{h1}\left(\frac{60}{50}\right)\left(\frac{5}{6}\right)^{1.6} = P_{h1}(0.833)^{0.6} = 0.896\, P_{h1}$$

or $$\frac{P_{h2}}{\text{Output}} = 0.896\,\frac{P_{h1}}{\text{Output}} = 0.896\,(0.9) = 0.806\%$$

From Eq. (1.46), eddy current loss,

$$P_e = K_e f^2 B_m^{\,2}$$

$$\therefore \qquad \frac{P_{e2}}{P_{e1}} = \left(\frac{f_2}{f_1}\right)^2\left(\frac{B_{m2}}{B_{m1}}\right)^2$$

or $$P_{e2} = P_{e1}\left(\frac{60}{50}\right)^2\left(\frac{5}{6}\right)^2 = P_{e1}$$

For the same output, percentage P_{e1} = percentage P_{e2} = 0.6%.

Thus the ohmic, hysteresis and eddy current losses at 60 Hz are 1.6%, 0.806% and 0.6% respectively.

(*b*) The core loss depends on voltage and frequency only. Therefore, $P_{c1} = 1.5\%$ (= 0.9 + 0.6) and $P_{c2} = 1.406\% = (0.806 + 0.6)$ can't be changed for given values of voltage and frequency. For the total losses to remain the same, the ohmic loss alone can be varied.

$\therefore$ Total losses at 50 Hz = Total losses at 60 Hz

or
$$3.1(= 1.6 + 0.9 + 0.6) = 1.406 + \text{New ohmic losses.}$$

$\therefore$ Permissible value of ohmic losses at 60 Hz

$$= 3.1 - 1.406 = 1.694\%.$$

Since the ohmic losses are proportional to the square of the current, we have

$$\left(\frac{\text{New permissible current}}{\text{Original current}}\right)^2 = \frac{\text{New ohmic losses}}{\text{Original ohmic losses}} = \frac{1.694}{1.60}$$

$\therefore$ New permissible current

$$= \sqrt{\frac{1.694}{1.60}}\ (\text{original current}) = (1.028)\ (\text{original current})$$

For the same voltage, output at 60 Hz

$$= (1.028)\ (\text{output at 50 Hz})$$

It is, therefore, seen that output at a higher-frequency operation is increased for the same total loss.

Example 1.27. *An 11/0.4 kV, 25 Hz, single-phase transformer has ohmic, hysteresis and eddy current losses of 1.8, 0.8 and 0.3% respectively. What do these losses become if the transformer is operated from 22 kV, 50 Hz supply system. The current is assumed to remain the same in both the cases.*

Also calculate the efficiency in each case.

Solution. Subscripts 1 and 2 are used to denote 11 kV, 25 Hz and 22 kV, 50 Hz systems respectively.

At 50 Hz, the voltage is doubled $\left(= \frac{22}{11}\right)$. For the same current, therefore, the output P_2 at 50 Hz is double the output P_1 at 25 Hz, *i.e.* $P_2 = 2P_1$. It is given that

$$\frac{P_{oh1}}{P_1} = 1.8\%;\ \frac{P_{h1}}{P_1} = 0.8\%;\ \frac{P_{e1}}{P_1} = 0.3\%.$$

Ohmic loss. Since the current is same at both the frequencies and voltages, the ohmic losses in watts remain unaltered, *i.e.* $P_{oh2} = P_{oh1}$.

Percentage ohmic loss at higher frequency

$$= \frac{P_{oh2}}{P_2} = \frac{P_{oh1}}{2P_1} = \frac{1}{2}(1.8) = 0.9\%.$$

Core loss. The voltage is related to f, B_m etc. by the expression

$$V = \sqrt{2}\,\pi f B_m A_i N$$

$$\therefore \quad \frac{V_1}{V_2} = \left(\frac{f_1}{f_2}\right)\left(\frac{B_{m1}}{B_{m2}}\right) \quad \text{or} \quad \frac{11{,}000}{22{,}000} = \left(\frac{25}{50}\right)\frac{B_{m1}}{B_{m2}}$$

or

$$\frac{1}{2} = \frac{1}{2}\left(\frac{B_{m1}}{B_{m2}}\right)$$

$$\therefore \quad B_{m2} = B_{m1}$$

The hysteresis loss $P_h = K_h f B_m{}^x$ watts

$$\therefore \quad \frac{P_{h2}}{P_{h1}} = \left(\frac{f_2}{f_1}\right)\left(\frac{B_{m2}}{B_{m1}}\right)^x$$

or

$$P_{h2} = P_{h1}\left(\frac{50}{25}\right)(1)^x = 2(1)^x P_{h1} = 2P_{h1}.$$

Percentage hysteresis loss at f_2, V_2 is

$$= \frac{P_{h2}}{P_2} = \frac{2P_{h1}}{2P_1} = 0.8\%.$$

The eddy current loss $P_e = K_e f^2 B_m{}^2$ watts

$$\therefore \quad \frac{P_{e2}}{P_{e1}} = \left(\frac{f_2}{f_1}\right)^2\left(\frac{B_{m2}}{B_{m1}}\right)^2$$

or

$$P_{e2} = P_{e1}\left(\frac{50}{25}\right)^2(1)^2 = 4\,P_{e1}$$

Percentage eddy current loss at f_2, V_2 is $\frac{P_{e2}}{P_2} = \frac{4P_{e1}}{2P_1}$

$$= 2\left(\frac{P_{e1}}{P_1}\right) = 2\,(0.3) = 0.6\%$$

Efficiency at f_1, V_1 is

$$= 1 - \frac{\text{Losses}}{\text{Output} + \text{Losses}} = 1 - \frac{\text{Losses/Output}}{1 + \text{Losses/Output}}$$

$$= 1 - \frac{\text{P.u. losses}}{1 + \text{P.u. losses}} = 1 - \frac{0.018 + 0.008 + 0.003}{1 + 0.018 + 0.008 + 0.003} = 1 - \frac{0.029}{1.029}$$

$$= 0.97182 \quad \text{or} \quad 97.182\%$$

Efficiency at f_2, V_2 is

$$= 1 - \frac{0.009 + 0.008 + 0.006}{1 + 0.009 + 0.008 + 0.006}$$

$$= 1 - \frac{0.023}{1.023} = 0.97755\% \quad \text{or} \quad 97.755\%.$$

Example 1.35. *An open circuit test when performed on the delta side of a bank of three-phase transformer gave the following data :*

Terminal voltage in V	*214*	*171.00*	*128.4*	*85.6*
Frequency in Hz	*50*	*40*	*30*	*20*
Power input in W	*100*	*72.5*	*50*	*30*

Determine the hysteresis and eddy current losses at :

(a) 60 Hz and

(b) 40 Hz.

Solution. First of all the readings are changed to per phase values, as given below :

Per phase voltage in V	214	171.00	128.40	85.6
Per phase power in W, *i.e.* P_c	33.3	24.2	16.67	10

It may be seen that the ratio $\frac{V}{f}$ has a constant value of 4.28. Therefore, Eq. (1.59) can be used for separating the hysteresis and eddy current losses.

The core loss per cycle, *i.e.* $\frac{P_c}{f}$ is calculated in a tabular form as follows :

$\frac{P_c}{f}$	0.667	0.605	0.556	0.50
f	50	40	30	20

In Fig. 1.31, $\frac{P_c}{f}$ is plotted against f. The straight line so obtained intersects the vertical axis at the point A. The intercept OA gives the value of K_1 equal to 0.39. The slope of the line AB can be obtained at any frequency, say 50 Hz.

$$\therefore \quad K_2\,(50) = 0.667 - 0.39 = 0.277$$

or $$K_2 = \frac{0.277}{50} = 0.00554.$$

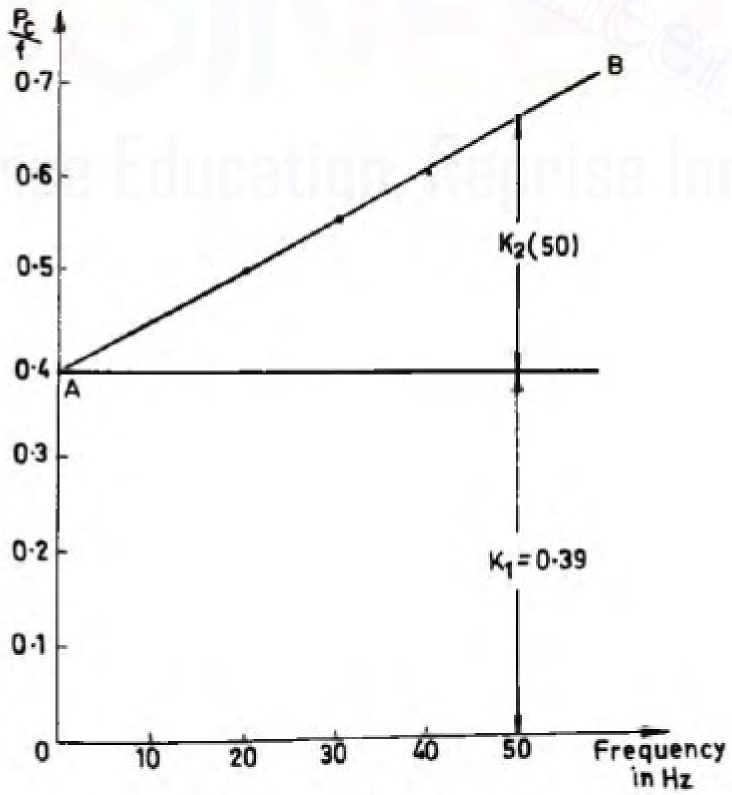


Fig. 1.31. Pertaining to Example 1.35.

From Eq. (1.59),

(a) P_h per phase = (0.39) (60) = 23.4 W

P_e per phase = (0.00554) $(60)^2$=19.95 W

∴ Total hysteresis and eddy-current losses at 60 Hz are 23.4 × 3 = 70.2 W and 59.85 W respectively.

(b) P_h per phase = (0.39) (40) = 15.6 W

P_e per phase = (0.00554) $(40)^2$ = 8.86 W.

Total hysteresis and eddy-current losses at 40 Hz are 46.8 W and 26.58 W respectively.

Example 1.12 : *1 kV / 2 kV transformer has 750 W hysteresis losses and 250 W eddy current losses. When the applied voltage is doubled and frequency is halfed, find the new losses.*

Solution : The hysteresis loss is given by,

$$P_h \propto B_m^{1.6} f$$

The eddy current loss is given by,

$$P_e \propto B_m^2 f^2$$

But $$B_m \propto \frac{V}{f}$$

$\therefore$ $$P_h \propto \left(\frac{V}{f}\right)^{1.6} \times f \quad \text{and} \quad P_e \propto \left(\frac{V}{f}\right)^2 \times f^2$$

$\therefore$ $$\frac{P_{h1}}{P_{h2}} = \left(\frac{V_1}{V_2}\right)^{1.6} \times \left(\frac{f_2}{f_1}\right)^{1.6} \times \frac{f_1}{f_2} \quad \text{and} \quad \frac{P_{e1}}{P_{e2}} = \left(\frac{V_1}{V_2}\right)^2 \times \left(\frac{f_2}{f_1}\right)^2 \times \left(\frac{f_1}{f_2}\right)^2$$

Now $V_2 = 2\ V_1$ and $f_2 = 0.5\ f_1$

$\therefore$ $$\frac{750}{P_{h2}} = (0.5)^{1.6} \times (0.5)^{1.6} \times 2 \quad \text{and} \quad \frac{250}{P_{e2}} = (0.5)^2 \times (0.5)^2 \times (2)^2$$

$\therefore$ $$P_{h2} = \mathbf{3446.095\ W} \quad \text{and} \quad P_{e2} = \mathbf{1000\ W}$$

$\therefore$ Total new iron loss = $P_{h2} + P_{e2}$ = **4446.095 W**

Example 1.16 : *A single phase transformer shows 63 W core losses at 40 Hz while 110 W at 60 Hz. Both the tests are performed at same value of maximum flux density in the core. Find hysteresis and eddy current losses at 50 Hz frequency.*

Solution : $P_{i1} = 63$ W, $f_1 = 40$ Hz, $P_{i2} = 110$ W, $f_2 = 60$ Hz, B_m is same.

As B_m is same, it can be absorbed in the constants A and B. Thus we can write,

$$P_h = Af \quad \text{while} \quad P_e = B f^2$$

$\therefore$ $$P_i = P_h + P_e = Af + Bf^2$$

$\therefore$ $$63 = A \times 40 + B \times (40)^2 \quad \text{...(1)}$$

and $$110 = A \times 60 + B \times (60)^2 \quad \text{...(2)}$$

Solving (1) and (2) we get,

A = 1.0584, B = 0.0129

Thus two losses at 50 Hz are,

$$P_h = Af = 1.0584 \times 50 = \mathbf{52.92\ W}$$

$$P_e = Bf^2 = 0.0129 \times 50^2 = \mathbf{32.25\ W}$$

Example 1.1

A single-phase transformer is designed to operate at 220/110 V, 60 Hz. What will be the effect on the transformer performance if the frequency reduces by 5% to 57 Hz and the primary voltage increases by 5% to 231 volts?

Solution:

The emf equation is

$$V_1 = 4.44\ \phi_{mp}\ f\ N_1 .$$

Now $\phi_{mp} = B_{mp}\ A_c$

where B_{mp} is peak value of the flux density in the core (wb/m^2)

A_c is cross-sectional area of the core in m^2.

Hence, for a given number of turns (N_1) and core area (A_c),

$$\frac{(231)_{57Hz}}{(220)_{60Hz}} = \frac{4.44\ (B_{mp})_{57Hz} \times A_c \times 57 \times N_1}{4.44\ (B_{mp})_{60Hz} \times A_c \times 60 \times N_1}$$

$$\frac{(B_{mp})_{57Hz}}{(B_{mp})_{60Hz}} = \frac{231 \times 60}{220 \times 57} = 1.10 .$$

Thus, with the reduced frequency and higher applied voltage, the flux density in the core increases. This increase will in turn increase the no-load current, the core loss and the noise level of the transformer. This example shows that the operating peak flux density in the core has to be correctly chosen depending on specified overexcitation conditions by the user.

Magnetic Hysteresis Loss

If an alternating voltage is connected to the magnetizing coil, as shown in Figure 1.8(a), the alternating magnetomotive force causes the magnetic domains to be constantly reoriented along the magnetizing axis. This molecular motion produces heat, and the harder the steel the greater the heat. The power loss due to hysteresis for a given type and volume of core material varies directly with the frequency and the nth power of the maximum value of the flux density wave. Expressed mathematically,

$$P_h = k_h \cdot f \cdot B_{max}^n \tag{1-11}$$

where:

P_h = hysteresis loss (W/unit mass of core)
f = frequency of flux wave (Hz)
B_{max} = maximum value of flux density wave (T)
k_h = constant
n = Steinmetz exponent[2]

The constant k_h is dependent on the magnetic characteristics of the material, its density, and the units used. The area enclosed by the hysteresis loop is equal to the hysteresis energy in joules/cycle/cubic-meter of material.

EXAMPLE 1.3 The hysteresis loss in a certain electrical apparatus operating at its rated voltage and rated frequency of 240 V and 25 Hz is 846 W. Determine the hysteresis loss if the apparatus is connected to a 60-Hz source whose voltage is such as to cause the flux density to be 62 percent of its rated value. Assume the Steinmetz exponent is 1.4.

Solution

From Eq. (1–11),

$$\frac{P_{h1}}{P_{h2}} = \frac{[k_h \cdot f \cdot B_{max}^n]_1}{[k_h \cdot f \cdot B_{max}^n]_2} \quad \Rightarrow \quad P_{h2} = P_{h1} \times \frac{[k_h \cdot f \cdot B_{max}^n]_2}{[k_h \cdot f \cdot B_{max}^n]_1}$$

$$P_{h2} = 846 \times \frac{60}{25} \times \left[\frac{0.62}{1.0}\right]^{1.4} = \underline{1.04\text{ kW}}$$

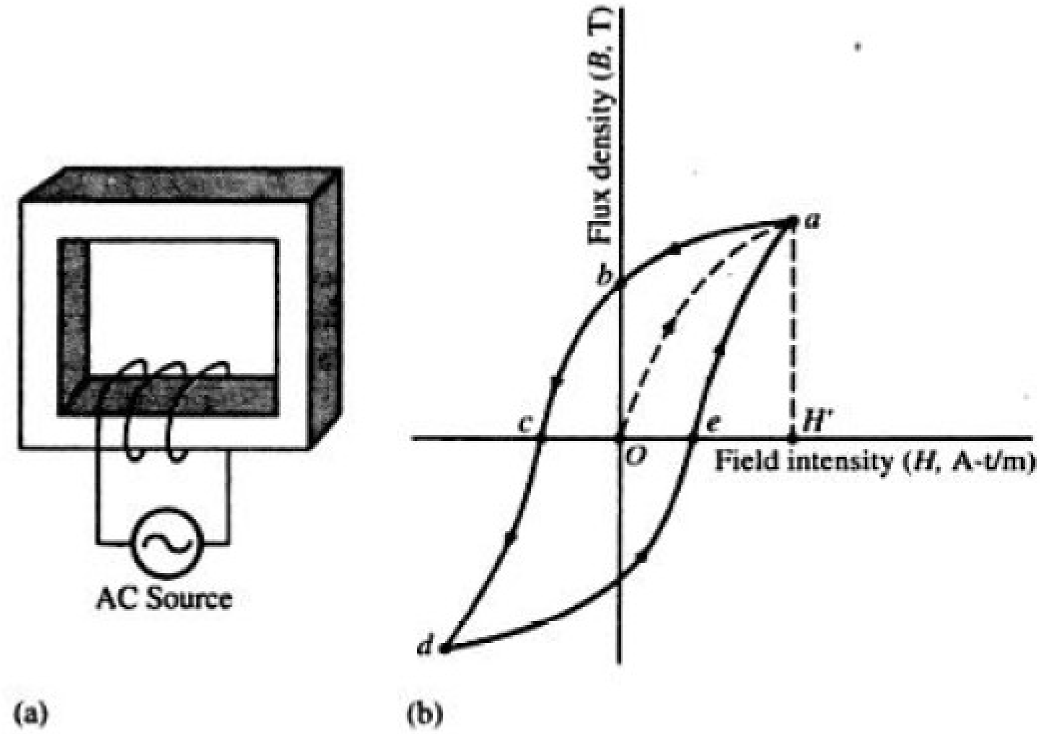


FIGURE 1.8
(a) Magnetic circuit with an alternating mmf; (b) representative hysteresis loop.

EXAMPLE 1–7 The eddy-current loss in a certain electrical apparatus operating at its rated voltage and rated frequency of 240 V and 25 Hz is 642 W. Determine the eddy-current loss if the apparatus is connected to a 60-Hz source whose voltage is such as to cause the flux density to be 62 percent of its rated value.

Solution
From Eq. (1–30),

$$\frac{P_{e1}}{P_{e2}} = \frac{[k_e f^2 B_{max}^2]_1}{[k_e f^2 B_{max}^2]_2} \quad \Rightarrow \quad P_{e2} = P_{e1} \times \left[\frac{f_2}{f_1}\right]^2 \times \left[\frac{B_{max,2}}{B_{max,1}}\right]^2$$

$$P_{e2} = 642 \times \left[\frac{60}{25}\right]^2 \times \left[\frac{0.62}{1.0}\right]^2 = \underline{1.42\ \text{kW}}$$

<u>Example 2.28</u>

A 1 kVA, 220/110 V, 400 Hz transformer is desired to be used at a frequency of 60 Hz. What will be the kVA rating of the transformer at reduced frequency?

Solution:

We know that $E_1 = V_1 = 4.44\ \phi_m\ N_1 f = 4.44\ B_m\ A_i\ N_1 f$

Assuming flux density in the core remaining unchanged, we have

$$V_1 \propto f$$

or $$\frac{V'_1}{V_1} = \frac{f'}{f}$$

or $$V'_1 = V_1 \times \frac{f'}{f} = 220 \times \frac{60}{400} = 33 \text{ volt}$$

As current rating of the transformer remains the same, the kVA rating is proportional to voltage,

∴ kVA rating of the transformer at 60 Hz,

$$\text{kVA}' = \frac{V'}{V} \times kVA = \frac{33}{220} \times 1 = \mathbf{0.15\ kVA}\ (Ans.)$$

Example 2.29

A 40 Hz transformer is to be used on a 50 Hz system. Assuming Steinmetz's coeff. as 2.6 and the losses at 40 Hz, 2.2%, 0.7% and 0.5% for copper, hysteresis and eddy currents, respectively, find (i) the losses on 50 Hz for the same supply voltage and current. (ii) the output at 50 Hz for the same total losses as on 40 Hz.

Solution:

Let W be the total power input to the transformer in both the cases in watt.

Copper loss = $I^2 \times$ total resistance

As long as current and supply voltage remain the same, copper loss will remain the same.

∴ Copper loss at 40 Hz or 50 Hz

$$= \frac{1.2}{100} \times W = 0.012\ W \text{ watt}$$

Hysteresis loss = $\eta\, B_{max}^{1.6} f$ watt/c.c. of the magnetic material.

For the same voltage induced per turn ($E/N = 4.44\, f\, \phi$) the product ϕf or $B_{max} f$ remains constant.

∴ $$B_{max1} f_1 = B_{\max_2} f_2$$

∴ $$\frac{B_{\max_1}}{B_{\max_2}} = \frac{f_2}{f_1} = \frac{50}{40} = 1.25$$

or $$B_{\max_2} = 0.8\ B_{\max_1}$$

∴ Hysteresis loss at 50 Hz

$$W_{h_2} = K_h = \eta B_{\max_2}^{1.6}\ f_2 \text{ watt/c.c.} = K_h\,(0.8\,B_{\max_1})^{1.6}\, 1.25\ f_1$$

$$= K_h\,(0.8)^{1.6} B_{\max_1}{}^{1.6} f_1 \times 1.25 = 1.25(0.8)^{1.6} \times K_h\, B_{\max_1}{}^{1.6} f_1 = 0.875 \times K_h\, B_{\max_1}{}^{1.6} f_1$$

But hysteresis loss at 40 Hz is 0.7%

$$W_{h_2} = 0.875 \times 0.7 = 0.6125\%$$

Eddy current loss, $W_e = B_{\max}^2 f^2 t^2$ watt per c.c. of the magnetic material.

$$\therefore \quad \frac{W_{e_1}}{W_{e_2}} = \frac{B_{\max_1}^2 f_1^2 t_1^2}{B_{\max_2}^2 f_1^2 t_2^2} \text{ But } t_1 = t_2$$

$$\therefore \quad \frac{W_{e_1}}{W_{e_2}} = \frac{B_{\max_1}^2 f_1^2}{B_{\max_2}^2 f_2^2} = \left(\frac{B_{\max_1}}{B_{\max_2}}\right)^2 \left(\frac{f_1}{f_2}\right)^2 = \left(\frac{5}{4}\right)^2 \times \left(\frac{4}{5}\right)^2 = 1$$

$$\therefore \quad W_{e_1} = W_{e_2}$$

Hence eddy current loss will remain the same.

Thus, the three losses at 50 Hz will be

Copper loss = **1.2%**

Hysteresis loss = **0.6125%** (*Ans.*)

Eddy current loss = **0.5%**

Example 2.42

The iron losses of a 400 V, 50 Hz transformer are 2500 W. These losses are reduced to 850 W when the applied voltage is reduced to 200 V, 25 Hz. Determine the eddy current loss at normal frequency and voltage.

Solution:

We know, $E = 4.44\, N f A_i B_m$

$$B_m \propto \frac{E}{f} \text{ (since all other quantities are constant)}$$

As $\frac{E_1}{f_1} = \frac{400}{50} = 8$ and $\frac{E_2}{f_2} = \frac{200}{25} = 8$; B_m is same in both the cases

$\therefore$ Hysteresis loss, $W_h \propto f = Pf$ (where P is a constant)

Eddy current loss, $W_e \propto f^2 = Qf^2$ (where Q is a constant)

$$\text{Total iron loss, } W_i = W_h + W_e = Pf + Qf^2 \text{ or } \frac{W_i}{f} = P + Qf \quad \text{...(2.83)}$$

When $f = 50$ Hz; $W_i = 2500$

$$\therefore \quad \frac{2500}{50} = P + Q \times 50 \text{ or } P + 50\,Q = 50 \quad \text{...(2.84)}$$

When $f = 25$ Hz; $W_i = 850$

$$\therefore \quad \frac{850}{25} = P + Q \times 25 \text{ or } P + 25Q = 34 \quad \text{...(2.85)}$$

Subtracting eq. (*iii*) from (*ii*), we get,

$$25\,Q = 16 \text{ or } Q = 0.64$$

From eq. (*ii*), we get, $P + 50 \times 0.64 = 50;\ P = 18$ ...(2.86)

Eddy current loss at normal frequency and voltage.

$$W_e = Qf^2 = 0.64 \times 50 \times 50 = \mathbf{1600\ W}\ (Ans.)$$

Example 2.43

A transformer has hysteresis and eddy current loss of 700 W and 500 W, respectively when connected to 1000 V, 50 Hz supply. If the applied voltage is raised to 2000 V and frequency to 75 Hz, find the new core losses.

Solution:

Here, $V_1 = 1000$ V; $f_1 = 50$ Hz; $W_i = 1200$ W; $W_{h1} = 700$ W

$$W_{e1} = 500 \text{ W};\ V_2 = 2000 \text{ V};\ f_2 = 75 \text{ Hz}$$

We know, $$W_h \propto B_m^{1.6} \times f = PB_m^{1.6} \times f \quad \text{...(2.87)}$$

$$W_e \propto B_m^2 \times f^2 = Q\,B_m^2 \times f^2 \quad \text{...(2.88)}$$

Induced emf, $E = 4.44\,Nf\,B_m A_i$ volt

or $$B_m \propto \frac{E}{f} \qquad (\because 4.44\,NA_i \text{ are constant})$$

Substituting this value in eqs. (2.87) and (2.88), respectively, we get,

$$W_h = P\left(\frac{E}{f}\right)^{1.6} \times f = PE^{1.6} f^{-0.6} \quad \text{...(2.89)}$$

$$W_e = Q\left(\frac{E}{f}\right)^{2} \times f^2 = QE^2 \quad \text{...(2.90)}$$

Case-I: When $E = V_1 = 1000$ V and $f = f_1 = 50$ Hz

$$W_{h1} = P(V_1)^{1.6} f_1^{-0.6}$$

$$700 = P(1000)^{2.6} \times (50)^{-0.6}$$

$$700 = P \times 63096 \times 0.0956 \text{ or } P = 0.116$$

$$W_{e1} = Q \times E^2$$

$$500 = Q \times (1000)^2 \text{ or } Q = 5 \times 10^{-4}$$

Case-II: When $E = V_2 = 2000$ V and $f = f_2 = 75$ Hz

$$W_{h2} = P(V_2)^{1.6} f_2^{-0.6} = 0.116 \times (2000)^{2.6} \times (75)^{-0.6}$$

$$= 0.116 \times 192.27 \times 10^3 \times 0.075 = 1664 \text{ W}$$

$$W_{e2} = Q \times (V_2)^2 = 5 \times 10^{-4} \times (2000)^2 = 2000 \text{ W}$$

New core losses, $W_{i2} = W_{h2} + W_{e2} = 1664 + 2000 = \mathbf{3664\ W}$ (*Ans.*)

Example 2.44

The hysteresis and eddy current loss of a ferromagnetic sample at a frequency of 50 Hz is 25 watts and 30 watts, respectively, when the flux density of 0.75 tesla. Calculate the total iron loss at a frequency of 400 Hz, when the operating flux density is 0.3 tesla.

Solution:

At frequency, $f_1 = 50$ Hz: 25 W; $W_{h1} = W_{e1} = 30$ W; $B_{m1} = 0.75$ tesla

frequency, $f_2 = 400$ Hz; $B_{m2} = 0.3$ tesla

$$W_{h1} = PB_{m1}^{1.6} f_1 \text{ or } 25 = \text{P} \times (0.75)^{2.6} \times 50$$

or $$P = \frac{25}{(0.75)^{1.6} \times 50} = 0.7922\,; \quad W_{e1} = QB_{m1}^2 f_1^2$$

or $$30 = Q \times (0.75)^2 \times (50)^2 \text{ or } Q = \frac{30}{(0.75)^2 \times (50)^2} = 0.02133$$

$$W_{h2} = PB_{m2}^{1.6} \times f_2 = 0.7922 \times (0.3)^{2.6} \times 400 = 46.16 \text{ W}$$

$$W_{e2} = QB_{m2}^2 \times f_2^2 = 0.02133 \times (0.3)^2 \times (400)^2 = 307.2 \text{ W}$$

Total iron losses, $P_i = W_{h2} + W_{e2} = 46.16 + 307.2 = \mathbf{353.36\ W}$ (*Ans.*)

Example 2.45

The following test results were obtained when a 10 kg specimen of sheet steel laminated core is put on power loss test keeping the maximum flux density and wave form factor constant.

Frequency (in Hz)	*25*	*40*	*50*	*60*	*80*
Total loss (in watt)	*18.5*	*36*	*50*	*66*	*104*

Calculate the current loss per kg at frequency of 50 Hz.

Solution:

At a given flux density and waveform factor, total iron losses are given as

$$P_i = P_h + P_e = A\,f + Bf^2 \text{ or } \frac{P_i}{f} = A + Bf$$

Total iron loss/cycle i.e., P_i/f for various values of frequency is given below:

f	25	40	50	60	80
P_i/f	0.74	0.9	2.0	2.1	2.3

A graph is plotted between P_i/f and f as illustrated in Fig. 2.50. From graph $A = 0.5$ and $B = 0.01$

Eddy current loss at 50 Hz = $Bf^2 = 0.01 \times (50)^2 = 25$ watt

Eddy current loss per kg at 50 Hz = $\frac{25}{10} = \mathbf{2.5\ watt}$ (*Ans.*)

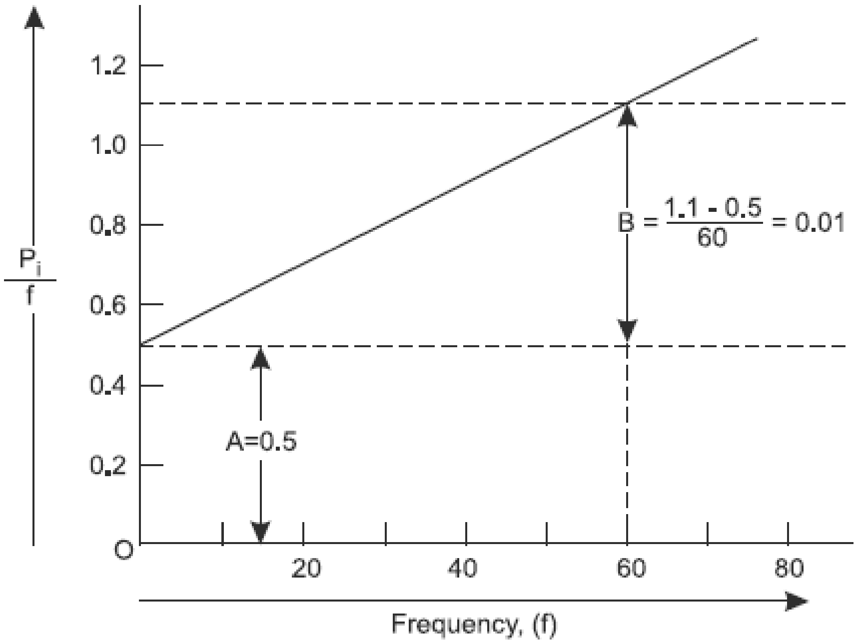


Fig. 2.50 Curve for frequency vs iron losses in a given transformer

2.16. A 25-Hz, 120-V/30-V, 500-VA transformer is to be used on a 60-Hz source. If the core flux density is to remain unchanged, determine (*a*) the maximum permissible primary voltage, and (*b*) the new (60-Hz) rated secondary voltage and current.

(*a*) By (*2.9*), the primary voltage will vary directly with frequency. Hence,

$$\text{maximum primary voltage} = \frac{60}{25}(120) = 288 \text{ V}$$

(*b*)

$$\text{rated } V_2 = \frac{60}{25}(30) = 72 \text{ V}$$

$$\text{rated } I_2 = \frac{500}{30} = 16.67 \text{ A} \quad \text{(same as at 25 Hz)}$$

3.45 The flux in a magnetic core is varying sinusoidally at a frequency of 600 c/s. The maximum flux density B_{max} is 0.6 Wb/m^2. The eddy current loss then is 16 W. Find the eddy current loss in this core, when the frequency is 800 c/sec, and the flux density is 0.5 Wb/m^2 (Tesla).

Solution

We know, eddy current loss $\propto B_{max}^2 \times f$

$$\text{at 600 c/sec: } P_{e_1} \propto (0.6)^2 \times 600 \qquad \text{(i)}$$

$$\text{at 800 c/sec: } P_{e_2}\text{(say)} \propto (0.5)^2 \times 800 \qquad \text{(ii)}$$

Dividing equation (ii) by equation (i) gives:

$$\frac{P_{e_2}}{16} = \frac{(0.5)^2 \times 800}{(0.6)^2 \times 600} \quad [\because P_{e_1} \text{ is 16 W}]$$

$$= 9.259 \times 10^{-1}$$

$$\therefore \quad P_{e_2} = 16 \times 9.259 \times 10^{-1} \text{ W}$$

$$= 14.8148 \text{ W.}$$

....

Example 4-2 (Section 4-4)

A sample of iron having a volume of 33 cm^3 is subjected to a magnetizing force varying sinusoidally at a frequency of 400 Hz. The hysteresis loop is plotted using the following scales: 1 cm represents 300 At/m, and 1 cm represents 0.2 weber/m^2. The area of the hysteresis loop is 57.5 cm^2. Find the hysteresis loss in watts.

Solution

Let P_h' denote the energy loss represented by Eq. 4-17. This is the energy loss per unit volume for one cycle.

$$P_h' = \oint H\,dB = \left(57.5 \frac{\text{cm}^2}{\text{cycle}}\right)\left(\frac{300 \text{ At/m}}{1 \text{ cm}}\right)\left(\frac{0.2 \text{ weber/m}^2}{1 \text{ cm}}\right) = 3450 \frac{\text{joules}}{\text{m}^3 \text{ cycle}}$$

The hysteresis loss of Eq. 4-18 is P_h' multiplied by the volume and the frequency.

$$P_h = P_h' \mho f$$

$$= \left(3450 \frac{\text{joules}}{\text{m}^3 \text{ cycle}}\right)\left(400 \frac{\text{cycles}}{\text{sec}}\right)(33 \text{ cm}^3)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3$$

$$= 45.5 \text{ watts}$$

Example 4-3 (Section 4-4)

The flux in a magnetic core is alternating sinusoidally with a frequency of 400 Hz. The maximum flux density is 0.6 weber/m^2. The eddy-current loss is 28 w. Find the eddy-current loss in this core when the frequency is 300 Hz and the maximum flux density is 0.7 weber/m^2.

Solution

Let $k_e' = k_e \mho \tau^2$

Use Eq. 4-19 to find k_e'.

$$k_e' = \frac{P_{e1}}{f_1^2 B_1^2} = \frac{28}{(400 \times 0.6)^2} = 4.86 \times 10^{-4} \ \frac{\text{watts m}^4}{\text{Hz}^2 \ \text{weber}^2}$$

For the new frequency and new maximum flux density, we can find

$$P_{e2} = k_e' f_2^2 B_2^2 = (4.86 \times 10^{-4})(300)^2 (0.7)^2 = 21.4 \text{ w}$$

Example 4-4 (Section 4-4)

The total core losses (hysteresis plus eddy current) for a sheet steel core are found to be 500 w at 25 Hz. When the frequency is increased to 50 Hz and the maximum flux density is kept constant, the total core loss becomes 1400 w. Find the hysteresis and eddy-current losses for both frequencies.

Solution

Since $B_{\max}$ is constant, Eq. 4-20 can have the following form:

$$P_c = Af + Bf^2 \text{ where } A = k_h'(B_{\max})^n \text{ and } B = k_e'(B_{\max})^2$$

For a frequency of 25 Hz, the core loss is

$$P_{c1} = 500 = A(25) + B(25)^2$$

For a frequency of 50 Hz, the core loss is

$$P_{c2} = 1400 = A(50) + B(50)^2$$

Solve the two equations to find $A = 12$ and $B = 0.32$. Now, we can find the individual losses.

$$P_{h1} = Af_1 = 300 \text{ w} \qquad P_{e1} = Bf_1^2 = 200 \text{ w}$$

$$P_{h2} = Af_2 = 600 \text{ w} \qquad P_{e2} = Bf_2^2 = 800 \text{ w}$$

8.65 A 200 V, 60 Hz single-phase transformer has hysteresis and eddy current losses of 250 W and 90 W respectively. If the transformer is now energized from 230 V, 50 Hz supply, calculate its core losses. Assume Steinmmetz's constant equal to 1.6.

Solution

If W_h and W_e are the hysteresis and eddy current loss respective then

$$W_h = K_h f B_m^x \quad \text{and} \quad W_e = K_e f^2 B_m^2$$

where K_h and K_e are constants, f is the frequency, B_m is maximum flux density and x is the Steinmetz constant.

Core loss = $(V_1 I_e \cos\theta_o)$ = 700 W, where $\cos\theta_o$ is the no load p.f. and V_1 is the voltage of the primary winding.

So, $$\cos\theta_o = \frac{700}{2400 \times 0.64} = 0.456$$

The core loss component of exciting current

$$= I_e \cos\theta_o = 0.64 \times 0.456 = 0.292 \text{ A.}$$

The magnetizing component of exciting current

$$= I_e \sin\theta_o = 0.64 \times \sin(\cos^{-1} 0.456) = 0.569 \text{ A.}$$

14.178 A single-phase transfoprmer on open circuit gave the following test results:

216 V	45 Hz	58.2 W
264 V	55 Hz	73.2 W

Calculate the eddy current and hysteresis losses separately at 240 V, 50 Hz.

Solution

For the first test, $\dfrac{V}{f} = \dfrac{216}{45} = 4.8$

For the second test, $\dfrac{V}{f} = \dfrac{264}{55} = 4.8$

At 240 V, $\dfrac{V}{f} = \dfrac{240}{50} = 4.8$

Since $V = E = 4.44\, f N \phi_m$

then $\dfrac{V}{f} = K\phi_m$ where K is a constant (K = 4.44 N).

As V is constant, the flux and flux density are constant.

We know, hysteresis loss $(P_h) \propto f$ or, $P_h = af$

01. In a transformer zero voltage regulation is achieved at a load power factor which is

At leading power factor the voltage regulation can be negative or zero. This can be found from this equation % regulation = $\varepsilon_x \cos\theta - \varepsilon_r \sin\theta$

02. A transformer has resistance and reactance in per unit as 0.01 and 0.04 respectively. Its voltage regulation for 0.8 power factor lagging and leading will be

Voltage regulation for lagging power factor = (R cosθ + X sinθ)×100 Voltage regulation for 0.8 lagging power factor = (0.01×0.8+0.04×0.6)×100 = 3.2% Voltage regulation for leading power factor = (R cosθ - X sinθ)×100 Voltage regulation for 0.8 leading power factor = (0.01×0.8 - 0.04×0.6)×100 = -1.6%

Eddy current loss and hysteresis loss are almost independent of load, significantly depending on supply voltage and frequency. As the flux density or flux is constant for a given voltage and frequency, eddy current loss and hysteresis loss remain constant at any load. Therefore, these losses are called constant losses. Copper loss varies as the square of load current and called variable loss.

04. In a transformer, hysteresis and eddy current losses depend upon

$$\text{Eddy current loss } P_e \propto K_e B_m^2 f^2$$
$$\text{Hysteresis loss } P_h \propto K_h B_m^{1.6} f$$

Therefore, in a transformer, hysteresis and eddy current losses depend upon both maximum flux density and supply frequency.

In a transformer, the core loss is found to be 52 watts at 40 Hz and 90 Watts at 60 Hz; both losses being measured at the same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz.

05. In a transformer operating at constant voltage if the input frequency increases, the core loss

$$Eddy\ current\ loss\ P_e \propto K_e B_m^2 f^2$$
$$Hysteresis\ loss\ P_h \propto K_h B_m^{1.6} f$$
$$B_m \propto \frac{v}{f}$$

$$1.\ When\ B_m\ is\ constant$$
$$P_e \propto K_e f^2\ and\ P_h \propto K_h f$$
$$2.\ When\ B_m\ is\ not\ constant$$
$$P_e \propto K_e(\frac{V}{f})^2 \times f^2 \propto K_e,\ V^2$$
$$P_h \propto K_h(\frac{V}{f})^{1.6} \times f \propto K_h V^{1.6} \times f^{-0.6}$$

06. If the frequency of input voltage of a transformer is increased keeping the magnitude of voltage unchanged, then

$$Eddycurrentloss P_e = K_e B_m^2 f^2$$
$$Hysteresis\ loss\ P_h = K_h B_m^{1.6} f^2$$
$$When\ flux\ density\ is\ not\ constant$$
$$P_e = k_e V^2\ and\ P_h = K_h V^{1.6} \times f^{-0.6}$$

Therefore, hysteresis loss will decrease but eddy current loss will remain same.

$$Eddy\ current\ loss\ P_e = K_e B_m^2 f^2$$

Eddy current loss is directly proportional to supply frequency. Therefore, for dc source, frequency is zero and eddy current loss is also zero.

08. A transformer has hysteresis loss of 30 W, at 240 V, 60 Hz. The hysteresis loss at 200 V, 50 Hz will be

$$Hysteresis\ loss\ P_h \propto K_h B_m^{1.6} f$$
$$When\ B_m\ is\ constant\ P_h = K_h f$$
$$For\ given\ problem,\ B_{m1} = \frac{240}{60} = 4\ and\ and\ B_{m2} = \frac{200}{50} = 4$$
$$B_{m1} = B_{m2}$$
$$therefore,\ \frac{P_{h2}}{P_{h1}} = \frac{f_2}{f_1}$$
$$P_{h2} = \frac{50}{60} \times 30 = 25\ \text{W}$$

09. A single phase transformer when supplied from 220 V, 50 Hz has eddy current loss of 50 W. If the transformer is connected voltage of 330 V, 75 Hz, the eddy current loss will be

$$Eddy\ current\ loss\ P_e = K_e B_m^2 f^2$$
$$When\ B_m\ is\ constant\ P_e = K_e f^2$$
$$For\ given\ problem\ B_{m1} = \frac{220}{50} = 4.4\ and\ B_{m2} = \frac{330}{75} = 4.4$$
$$therefore,\ B_{m1} = B_{m2}$$
$$\frac{P_{e2}}{P_{e1}} = (\frac{f_2}{f_1})^2$$
$$P_{e2} = (\frac{75}{50})^2 \times 50 = 112.5\ W$$

10. The full load copper loss and iron loss of transformer are 6400W and 5000W respectively. The copper loss and iron loss at half load will be respectively?

Iron losses do not depend on the load, iron losses remain constant for any load. Therefore iron losses are considered as constant losses. Copper losses vary as square of load current and these are considered as variable losses.

$$Copper\ loss\ at\ half\ load$$
$$= (\frac{1}{2})^2 \times full\ load\ copper\ loss$$
$$= \frac{1}{4} \times 6400 = 1600\ W$$

07. 1 KVA, 230 V, 50 Hz, single phase transformer has an eddy current loss of 30 watts. The eddy current loss when the transformer is excited by a dc source of same voltage will be

Core losses in a transformer is the sum of hysteresis loss and eddy current loss.

hysteresis loss is proportional to frequency and eddy current loss is proportional to square of the frequency.

Hence we can write :

core loss = A f + B f^2

where A and B are constants.

from the given data we have;

52 = A 40 + B 1600..........eq 1

and

90 = A 60 + B 3600..............eq 2

Solving both the equation we get,

A = 36/40

B = 1/100

Hence putting these values in the equation with frequency 50 hz,

core loss = 36*50/40 + 2500/100

= 45 + 25

= 70 watts

Hence core loss at 50hz is **70 watts**.

Example 1.77. *A 4 kVA, 50 Hz, single-phase transformer has a ratio 200/400 V. The data taken on the l.v. side at the rated voltage show that the open circuit input wattage is 80 W. The mutual inductance between the primary and secondary windings is 1.91 H. What value will be the current taken by the transformer, if the no-load test is conducted on the h.v. side at rated voltage ? Neglect the effect of winding resistances and leakage reactances.* (GATE, 1995)

Solution. Open-circuit input wattage = core loss in transformer = 80 W

But core loss $= I_{c1} V = 80$ W

Core loss in a transformer remains unaltered whether it is energized from l.v. side or h.v. side.

$\therefore$ Core-loss current when energized from h.v. side,

$$I_{c2} = \frac{80}{400} = 0.2 \text{ A}$$

In a transformer, $E = \sqrt{2}\,\pi f N \phi_{max} = \sqrt{2}\,\pi f \psi_{max}$

$\therefore$ Maximum value of flux linkages ψ_{max} with l.v. winding $= \dfrac{V_1}{\sqrt{2}\pi f}$

Mutual inductance, $M = \dfrac{\text{Flux linkages with l.v. winding}}{\text{Current in h.v. winding}} = \dfrac{V_1}{\sqrt{2}\,\pi f} \cdot \dfrac{1}{\sqrt{2}\, I_{m2}}$

where I_{m2} is the magnetizing current in h.v. winding.

$$\therefore \qquad I_{m2} = \frac{200}{\sqrt{2}\,\pi \times 50} \cdot \frac{1}{\sqrt{2} \times 1.91} = 0.3333 \text{ A}$$

The current taken by transformer when energized on h.v. side, as per Eq. (1.18), is

$$I_e = \sqrt{I_{c2}^2 + I_{m2}^2} = [0.2^2 + 0.3333^2]^{1/2} = \mathbf{0.3887\ A.}$$

Example 1.61. *A 10 kVA, 2300/230 V, single-phase transformer has the following parameters :*

$r_1 = 10\ \Omega$, $\quad r_2 = 0.10\ \Omega$, $\quad l_1 = 40$ mH, $\quad l_2 = 4 \times 10^{-4}$ H, $\quad M = 10$ H.

Subscripts 1 and 2 indicate h.v. and l.v. windings respectively.

(a) Find the self-impedances of primary and secondary windings.

(b) Find the values of the equivalent-circuit parameters referred to (i) the primary and (ii) the secondary.

(c) The primary of this transformer is energised from 2300 V, 50 Hz source. If its secondary is connected to a load of impedance $5 + j5\ \Omega$, find the secondary terminal voltage.

Solution. (*a*) Primary self-inductance,

$$L_1 = L_{m1} + l_1 = \frac{N_1}{N_2} M + l_1 = \frac{2300}{230} \times 10 + 0.04 = 100.04 \text{ henrys.}$$

Secondary self-inductance,

$$L_2 = L_{m2} + l_2 = M \frac{N_2}{N_1} + l_2 = 10 \times \frac{1}{10} + 4 \times 10^{-4} = 1.0004 \text{ H}$$

(*b*) Parameters referred to primary winding are :

$$r_1 = 10\ \Omega\ ; \qquad r_2 = 0.1 \times \left(\frac{N_1}{N_2}\right)^2 = 0.1 \times (10)^2 = 10\ \Omega$$

$$l_1 = 40 \text{ mH}\ ; \qquad l_2 = 4 \times 10^{-4} \left(\frac{N_1}{N_2}\right)^2 = 4 \times 10^{-4}\ (10)^2 = 40 \text{ mH.}$$

$$L_{m1} = M \frac{N_1}{N_2} = 10 \times 10 = 100 \text{ H.}$$

Parameters referred to secondary winding are :

$$r_1 = 10 \times \left(\frac{1}{10}\right)^2 = 0.10\ \Omega\ ;\ r_2 = 0.1\ \Omega$$

$$l_1 = 40 \times 10^{-3} \left(\frac{1}{10}\right)^2 = 4 \times 10^{-4} \text{ H}\ ; \quad l_2 = 4 \times 10^{-4} \text{ H.}$$

$$L_{m2} = M \frac{N_2}{N_1} = 10 \times \frac{1}{10} = 1.00 \text{ H.}$$

Example 1.62. *A single-phase two winding transformer gave the following test results :*

(i) H.V. winding (590) turns when energised from 230 V, 50 Hz supply, takes a no-load current of 0.35 A and induced e.m.f. across open circuited l.v. winding is 110 V.

(ii) L.V. winding (295 turns) when energised from 115 V, 50 Hz supply takes a no-load current of 0.72 A and induced e.m.f. across open circuited h.v. winding is 226 V.

Calculate (a) self-inductances of h.v. and l.v. windings (b) the mutual inductance between h.v. and l.v. windings (c) coupling factors k_1 and k_2 for h.v. and l.v. windings respectively and the coefficient of coupling k.

Neglect core loss and winding resistances.

Solution. Self-inductance $L = \frac{\psi}{i}$

Now

$$E = \sqrt{2}\pi f N\phi_{max} = \sqrt{2}\pi f \psi_{max}$$

Here ψ_{max} are the maximum flux-linkages.

(*a*) ∴ Maximum value of flux-linkages with h.v. winding

$$\psi_{m1} = \frac{V_1}{\sqrt{2}\pi f} = \frac{230}{\sqrt{2}(\pi)\,(50)}$$

∴ Self-inductance of h.v. winding $L_1 = \frac{\psi_{m1}}{\sqrt{2} I}$

$$= \left[\frac{230}{\sqrt{2}(\pi)\,(50)}\right] \times \left[\frac{1}{\sqrt{2} \times 0.35}\right] = 2.092 \text{ H.}$$

Similarly self-inductance of l.v. winding,

$$L_2 = \left[\frac{115}{\sqrt{2}(\pi)\,(50)}\right] \times \left[\frac{1}{\sqrt{2} \times 0.72}\right] = 0.5084 \text{ H.}$$

(*b*) The maximum value of mutual flux linkages

$$= \frac{E_2}{\sqrt{2}\,\pi f}$$

$\therefore$ Mutual inductance $M = \left(\frac{E_2}{\sqrt{2}\pi f}\right) \times \frac{1}{\sqrt{2}\,I} = \left(\frac{110}{\sqrt{2}\,\pi\,(50)}\right) \times \left[\frac{1}{\sqrt{2}\,(0.35)}\right] = 1 \text{ H.}$

Alternatively, $M = \left[\frac{226}{\sqrt{2}\,\pi \times 50}\right] \times \left[\frac{1}{\sqrt{2} \times 0.72}\right] = 1 \text{ H.}$

(*c*) Coupling factor $k_1 = \frac{N_1}{N_2} \cdot \frac{M}{L_1} = \frac{590}{295} \times \frac{1}{2.092} = 0.956.$

Coupling factor $k_2 = \frac{N_2}{N_1} \cdot \frac{M}{L_2} = \frac{295}{590} \times \frac{1}{0.5084} = 0.9835$

Coefficient of coupling $k = \sqrt{k_1 k_2} = \sqrt{0.956 \times 0.9835} = 0.9696.$

Check. $M = k\sqrt{L_1 L_2} = 0.9696\sqrt{(2.092)\,(0.5084)} = 1.00 \text{ H.}$

Example 1.63. *The self and mutual inductances of a two-winding transformer are*

$$L_1 = 4 \text{ mH},\ L_2 = 6 \text{ mH},\ M_{12} = M_{21} = 1.8 \text{ mH}.$$

Calculate the current which would flow in the winding 1 when this winding is connected to a 130-volt, (500/π) Hz supply and the load of 0.2 mH inductance is connected across the winding 2. Assume power losses in the windings and the magnetic circuit to be negligible. (I.E.S., 1982)

Solution. The voltage equation for the primary winding, in terms of rms values, can be obtained from Eq. (1.84) as

$$\overline{V}_1 = r_1 \overline{I}_1 + j\omega L_1 \overline{I}_1 - j\omega M I_2 \qquad \text{...(1.101)}$$

Similarly for the secondary winding, from Eq. (1.85),

$$\overline{V}_2 = j\omega M I_1 - j\omega L_2 I_2 - r_2 I_2 \qquad \text{...(1.102)}$$

Substitution of the values in Eqs. (1.101) and (1.102), with V_1 as reference phasor, gives

$$130 + j0 = j2\pi\,(500/\pi)\,4 \times 10^{-3} I_1 - j\,2\pi\,(500/\pi)\,1.8 \times 10^{-3} I_2$$

and $j2\pi\,(500/\pi) \times 0.2 \times 10^{-3} I_2 = j\,2\pi(500/\pi)\,1.8 \times 10^{-3}\ I_1 - j\,2\pi\left(\frac{500}{\pi}\right)6 \times 10^{-3} I_2$

or $130 = j\,4\,I_1 - j\,1.8\,I_2$

and $0 = j\,1.8 I_1 - j\,6.2\,I_2$

from above, $I_2 = \frac{1.8}{6.2} I_1$

Simultaneous solution for I_1 gives $I_1 = 37.384$ A.

This example can also be solved alternatively by referring to the equivalent circuit of Fig. 1.65 (*b*). Here all the given parameters are referred to primary, *i.e.* take $a = 1$ in Fig. 1.65 (*b*).

$\therefore$ $L_1 - aM = 4 - 1.8 = 2.2$ mH

$aM = 1.8$ mH

$$a^2L_2 - aM = 6 - 1.8 = 4.2 \text{ mH}$$

Load inductance L referred to primary $= a^2L = 0.2$ mH

Total inductance seen by the primary applied voltage

$$= (L_1 - aM) + \frac{(aM)\,[a^2L_2 - aM + a^2L]}{aM + a^2L_2 - aM + a^2L}$$

$$= 2.2 \times 10^{-3} + \frac{(1.8 \times 10^{-3})\,(4.2 + 0.2) \times 10^{-3}}{(1.8 + 4.2 + 0.2) \times 10^{-3}} = 3.4774 \times 10^{-3} \text{ H}$$

Total reactance at the primary terminals

$$= 2\pi \times \frac{500}{\pi} \times 3.4774 \times 10^{-3} = 3.4774\ \Omega$$

$\therefore$ Current in the primary winding 1 $= \dfrac{130}{3.4774} = 37.384$ A.

Example 1.64. *An ideal audio-frequency transformer couples a 60-ohm resistive load to an electronic circuit which is represented by a constant voltage source of 5 V in series with an internal resistance of 3000 Ω.*

(a) Determine the transformer turns ratio so that maximum power transfer takes place from source to the load.

(b) Find the load current, voltage and power under the conditions of maximum power transfer.

Solution. (*a*) For maximum power transfer, the load resistance of 60 Ω when referred to the primary side must be equal to the source resistance of 3000 Ω.

$$\therefore \qquad 3000 = \left(\frac{N_1}{N_2}\right)^2 \times 60$$

or $$\frac{N_1}{N_2} = \sqrt{50} = 7.071$$

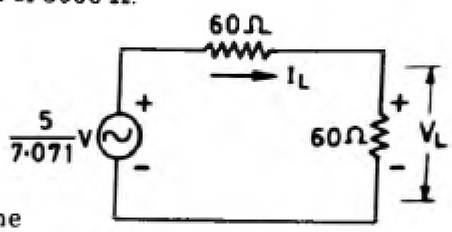


Fig. 1.72. Pertaining to Example 1.64.

(*b*) Referring all the quantities to load side, the equivalent circuit is as shown in Fig. 1.72. The source voltage on load side is (5/7.071) V and the source resistance is 60 Ω.

$\therefore$ Load current, $\quad I_L = \dfrac{5}{7.071 \times 120} = 5.893$ mA

Load voltage, $\quad V_L = 5.893 \times 10^{-3} \times 60 = 0.3536$ V

Load power $\quad = I_L^2\, R_L = (5.893 \times 10^{-3})^2 \times 60 = 2.084$ mW.

Example 1.65. *An audio-frequency transformer has the following parameters :*

$r_1 = 20\ \Omega,\quad l_1 = 1\ mH,\quad R_2 = 0.5\ \Omega,\quad l_2 = 0.025\ mH,\quad M_{12} = M_{21} = 0.2\ H$

Iron losses are neglected.

This transformer couples a load of 50 Ω to a voltage source of 5 V whose internal resistance is 2000 Ω.

(a) Find the turns ratio for maximum power transfer to load.

(b) Compute the load voltage at the following frequencies :

(i) 100 Hz (LF), (ii) 5000 Hz (IF) and (iii) 15,000 Hz (HF).

Solution. (*a*) For maximum power transfer

$$2000 = \left(\frac{N_1}{N_2}\right)^2 \times 50$$

$$\therefore \qquad \frac{N_1}{N_2} = \sqrt{40} = 6.3245$$

(*b*) $r_2' = 0.5\ (40) = 20\ \Omega,\ l_2' = (0.025)\ (40) = 1$ mH,

$R_L' = (50)\ (40) = 2000\ \Omega,\ R_g = 2000\ \Omega$

$$\therefore \qquad R_s' = R_g + r_1 + r_2' + R_L' = 4040\ \Omega$$

$$R_p' = \frac{(2020)\ (2020)}{4040} = 1010\ \Omega$$

$L_1 \cong M_{12} = M_{21} = 0.2$ H, $l_{eq} = l_1 + l_2' = 2$ mH.

(*i*) At 100 Hz (LF), from Eq. (1.103),

$$\frac{V_L}{E_g} = \frac{1}{\sqrt{40}} \cdot \frac{2000}{4040\left[1 + \left(\dfrac{1010}{2\pi \times 100 \times 0.2}\right)^2\right]^{1/2}} = 0.00966432$$

$$\therefore \qquad V_L = 0.04832\ \text{V}.$$

(*ii*) At 5000 Hz (IF), from Eq. (1.107),

$$\frac{V_L}{E_g} = \frac{1}{\sqrt{40}} \cdot \frac{2000}{4040} = 0.0782742$$

$$\therefore \qquad V_L = 0.3914\ \text{V}.$$

(*iii*) At 15,000 Hz (HF), from Eq. (1.108),

$$\frac{V_L}{E_g} = \frac{1}{\sqrt{40}} \cdot \frac{2000}{4040} \cdot \frac{1}{\left[1 + \left(\dfrac{2\pi \times 15{,}000 \times 1 \times 10^{-3}}{4040}\right)^2\right]^{1/2}} = 0.0782529$$

$$\therefore \qquad V_L = 0.3913\ \text{V}.$$

EXAMPLE 3.29 *A transformer has turn ratio of a = 10. The results of two open-circuit tests conducted on the transformer are given below:*

(a) The primary on application of 200 V draws 4 A with secondary open circuited which is found to have a voltage of 1950 V.

(b) The secondary on application of 2000 V draws 0.41 A with the primary open circuited.

Calculate L_1 and L_2 and coupling coefficient. What is the voltage of primary in part (b).

SOLUTION

(a)

$$X_m = \frac{240}{4} = 50\ \Omega,\ X_m = 2\pi f L_1$$

$$L_1 = \frac{200}{2\pi \times 50} = 0.159\ \text{H}$$

$$1950 = \sqrt{2}\ \pi N_2 \phi_{\max} = \sqrt{2}\ \pi \psi_{\max}$$

$$\psi_{\max} = \frac{1950}{\sqrt{2}\pi} = 8.78\ \text{Wb–T}$$

$$M = \frac{\psi_{\max}}{i_1(\max)} = \frac{8.78}{\sqrt{2} \times 4} = 1.55\ \text{H}$$

(b)

$$E_1 = \sqrt{2}\ \pi f N_2 \phi_{\max} = \sqrt{2}\ \pi f \psi_{\max}$$

$$\frac{\psi_{\max}}{i_2(\max)} = M, \quad \psi_{\max} = \sqrt{2} \times 0.42 \times 1.55$$

$\therefore$

$$E_1 = \sqrt{2}\ \pi \times 50 \times \sqrt{2} \times 0.41 \times 1.55 = 199.6\ \text{A}$$

$$L_2 = \frac{2000}{\sqrt{2}\pi \times 50} \cdot \frac{1}{\sqrt{2} \times 0.41} = 15.53\ \text{H}$$

Coupling coefficient,

$$k = \frac{1.55}{\sqrt{0.159 \times 15.53}} = 0.986$$

EXAMPLE 3.30 *A 150 kVA transformer 2400/240 V rating has the following parameters:*

$$R_1 = 0.2\ \Omega, \qquad R_2 = 2 \times 10^{-3}$$
$$X_1 = 0.45\ \Omega, \qquad X_2 = 4.5 \times 10^{-3}$$
$$R_i = 10\ k\Omega \qquad X_m = 1.6\ k\Omega \quad \text{(referred to HV)}$$

Calculate the leakage inductances, magnetizing inductance, mutual inductance and self-inductances.

SOLUTION

$$a = \frac{N_1}{N_2} \approx \frac{2400}{240} = 10$$

$$X_1 = 2\pi f l_1, l_1 = \frac{0.45}{314} \times 10^{-3} = 0.01433 \text{ mH}$$

$$X_2 = 2\pi f l_2, l_2 = \frac{4.5 \times 10^{-3}}{314} = 0.01433 \text{ mH}$$

Magnetizing inductance

$$2\pi f L_{m1} = X_m = 1.6 \times 10^3$$
$$L_{m1} = 5.096 \text{ H}$$

Self inductances

$$l_1 = L_1 - L_{m1}$$
$$L_1 = 5.096 + 0.01433 \times 10^{-3} = 5.096 \text{ H}$$

$$L_{m1} = aM, M = \frac{L_{m1}}{a} = \frac{5.096}{10} = 0.5096 \text{ H}$$

$$l_2 = L_2 - \frac{M}{a}$$

$$L_2 = l_2 + \frac{M}{a} = 0.01433 \times 10^{-3} + \frac{0.5096}{10}$$

$$= 0.05098 \text{ H}$$

Coupling factor
$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.5096}{\sqrt{5.096 \times 0.05098}} = 0.09998 \approx 1$$

3.61 Two coils with terminals T_1, T_2 and T_3, T_4 respectively are placed side by side. Measured separately, the inductance of the first is 1200 μH and that of the second coil is 800 μH. With T_2 joined with T_3 (Fig. 3.41), the total inductance between the two coils is 2500 μH. What is the mutual inductance? If T_2 is joined with T_4 instead of T_3, what would be the value of equivalent inductance of the two coils?

Solution

Given $L_1 = 1200$ μH, $L_2 = 800$ μH, $T_{14} = 2500$ μH.

Let the mutual inductance between the two coils be M, then total inductance $L_1 + L_2 + 2M$. In the first case (refer (Fig. 3.41)

$$T_{14} = L_1 + L_2 + 2M$$

$$2500 = 1200 + 800 + 2M$$

$$\therefore \quad M = \frac{500}{2} = 250 \text{ μH.}$$

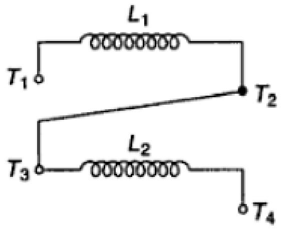


Fig. 3.41 ***Connection of two coils, 1st case***

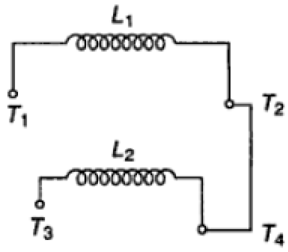


Fig. 3.42 ***Connection of two coils, 2nd case***

If T_{13} is the total inductance in the second case, then

$$T_{13} = L_1 + L_2 - 2M \qquad \text{(See Fig. 3.42)}$$

$$= 1200 + 800 - 2 \times 250$$

$$= 1500 \text{ μH.}$$

....

3.18 The combined inductance of the two coils connected in series is 0.60 H and 0.40 H, depending on the relative directions of currents in the coils. If one of the coils, when isolated, has a self-inductance of 0.15 H, then find: (a) the mutual inductance, and (b) the co-efficient of coupling K.

Solution

$$L_{\text{additive}} = L_1 + L_2 + 2M$$

$$0.60 = 0.15 + L_2 + 2M \qquad \text{(i)}$$

$$L_{\text{subtractive}} = L_1 + L_2 - 2M$$

$$0.40 = 0.15 + L_2 - 2M \qquad \text{(ii)}$$

adding equations (i) and (ii), we have

$$1.0 = 0.3 + 2L_2.$$

$$\therefore \quad L_2 = \frac{(1.0 - 0.3)}{2} = 0.35 \text{ H}$$

Substituting this value of L_2 in equation (i),

$$0.60 = 0.15 + 0.35 + 2M$$

or

$$M = 0.05 \text{ H}$$

(b) Co-efficient of coupling, $K = \dfrac{M}{\sqrt{L_1 L_2}}$

$$= \frac{0.05}{\sqrt{0.15 \times 0.35}} = 0.218 = 0.22.$$

14.31 Two coils having self-inductances of 0.3 H and 0.5 H are connected in series across a 230 V, 50 Hz supply. What current will flow if the coupling co-efficient of the coils is 0.45?

Solution

Mutual inductance $M = \sqrt{L_1 L_2} = 0.45\sqrt{0.3 \times 0.5} = 0.1743$

When connected in series the equivalent impedance is given by

$$L = L_1 + L_2 \pm 2M = 0.3 + 0.5 \pm 2 \times 0.1743 = 1.1486 \text{ H or } 0.4514.$$

Hence $X_L = 100\pi \times 1.1486 = 360.84\ \Omega$

or $X_L = 100\pi \times 0.4514 = 141.8\ \Omega$

Current is, $\dfrac{230}{360.84}\text{ A} = 0.6374\text{ A}$

or $\dfrac{230}{141.8}\text{ A} = 1.622\text{ A}.$

14.32 Two coils are connected in series with same polarities and the combined inductance is found to be 0.567 H. When the coils are connected in series with reverse polarities then the combined inductance is 0.267 H. The self-inductance of one coil is 0.3 H. Determine the mutual inductance and the coupling coefficient.

Solution

Let L_1 and L_2 be the self-inductances of the two coils and M the mutual inductance. Then

$L_1 + L_2 + M = 0.567$

and $L_1 + L_2 - M = 0.267$

Hence $L_1 + L_2 = 0.417$

But $L_1 = 0.3$ H,

$\therefore$ $L_2 = 0.417 - 0.3 = 0.117$ H

and $M = 0.417 - 0.267 = 0.15$ H

We know, $M = K\sqrt{L_1 L_2}$, where K is the coupling co-efficient

Hence $K = \dfrac{0.15}{\sqrt{0.3 \times 0.117}} = 0.8.$

3.25 Three coils are connected in series. Their self-inductances are L_1, L_2 and L_3. Each coil has a mutual inductance M with respect to the other coil. Determine the equivalent inductance of the connection. If $L_1 = L_2 = L_3 = 0.3$ H and $M = 0.1$ H, calculate the equivalent inductance. Consider that the fluxes of the coil are additive in nature.

Solution

Let the current i and v_1, v_2, v_3 be the voltage across the three coils.

$$\therefore \quad v_1 = L_1\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$v_2 = L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$v_3 = L_3\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3 + 6M)\frac{di}{dt}$$

$\because$ Equivalent inductance $= L_1 + L_2 + L_3 + 6M.$

Now putting the values of L_1, L_2, L_3 and M, we have

$$\text{Equivalent inductance} = 0.3 + 0.3 + 0.3 + 6 \times 0.1 = 0.9 + 0.6 = 1.5\text{ H}.$$

3.17 Two coils are connected in parallel as shown in Fig. 3.17. Calculate the net inductance of the connection.

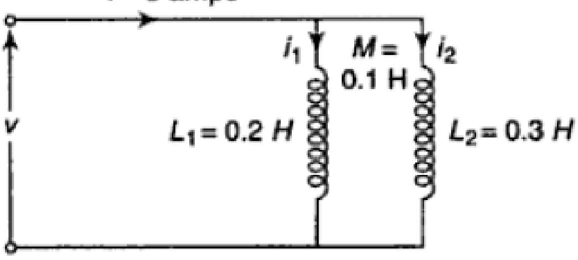


Fig. 3.17

Solution

The net inductance in the given circuit

$$= \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

$$= \frac{0.2 \times 0.3 - (0.1)^2}{0.2 + 0.3 + 2 \times 0.1}$$

$$= \frac{0.06 - .01}{0.7} = \frac{0.05}{0.7} = 0.0714 \text{ H.}$$

....

3.24 Two coils of inductance 8 H and 10 H are connected in parallel. If their mutual inductance is 4 H, determine the equivalent inductance of the combination if (a) mutual inductance assists the self-inductance, (b) mutual inductance opposes the self-inductance.

Solution

It is given that

$$L_1 = 8 \text{ H}, L_2 = 10 \text{ H}, M = 4 \text{ H}$$

(a) $$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{8 \times 10 - 4^2}{8 + 10 - 2 \times 4} = \frac{80 - 16}{18 - 8} = 6.4 \text{ H.}$$

(b) $$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{8 \times 10 - 4^2}{8 + 10 + 2 \times 4} = \frac{80 - 16}{26} = 2.46 \text{ H.}$$

....

3.19 Pure inductors each of inductance 3 H are connected as shown in Fig. 3.18. Find the equivalent inductance of the circuit.

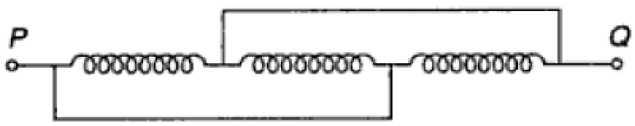


Fig. 3.18 ***The equivalent inductance of the circuit***

Solution

Since all three are in parallel. Hence the equivalent inductance is $L/3 = 3/3 = 1$ H.

....

14.27 Determine the total energy stored in the passive network shown in Fig. 14.16 at $t = 0$. Assume $K = 0.5$ and terminals x and y (i) open circuited (ii) short circuited.

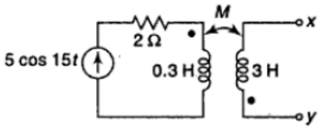


Fig. 14.16

Solution

$$M = K\sqrt{L_1 L_2} = 0.5\sqrt{0.3 \times 3}\ \text{H} = 0.474\ \text{H}.$$

Let us consider the two mesh currents i_1 and i_2 are flowing the clockwise direction in the two meshes.

From Fig. 14.16, we have

$$i_1 = 5\angle 0°\ \text{A}.$$

(i) When x and y are open circuited $i_2 = 0$

Hence total energy stored is $\frac{1}{2}L_1 i_1^2 = \frac{1}{2} \times 0.3 \times 5^2 = 3.75$ J.

(ii) When x and y are short circuited,
$i_1(t) = 5\cos 15t$ and voltage v_{xy} across xy is 0.

Hence, $\quad v_{xy} = 3\dfrac{di_2}{dt} + 0.474\dfrac{di_1}{dt} = 0$

or, $\quad \dfrac{di_2}{dt} = -\dfrac{0.474}{3}\dfrac{d}{dt}(5\cos 15t) = \dfrac{0.474}{3} \times 5 \times 15 \sin 15t = 11.85 \sin 15t.$

Hence $i_2(t) = \displaystyle\int_{-\infty}^{t} 11.85 \sin 15t\, dt = -0.75\cos 15t$ (assuming zero initial point)

Energy stored is

$$\left[\frac{1}{2}\times 0.3\times 5^2 + \frac{1}{2}\times 3\times (0.75)^2 + 0.474\times 5(-0.75)\right] = 2.817\ \text{J}.$$

14.28 In the circuit shown in Fig. 14.17 $L_1 = 2$ H, $L_2 = 5$ H and $M = 1.8$ H. Find the expression for the energy stored after the circuit is connected to a dc voltage of 30 V. Assume M to be positive.

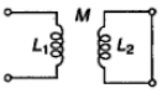


Fig. 14.17

Solution

If i_1 and i_2 be the currents in the two coils, we can write

$$30 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \qquad \text{(i)}$$

$$0 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \qquad \text{(ii)}$$

From Eq. (ii), we get

$$\frac{di_2}{dt} = -\frac{M}{L_2}\frac{di_1}{dt}$$

∴ From Eq. (i), we get

$$30 = \frac{di_1}{dt}L_1 + M\left(-\frac{M}{L_2}\frac{di_1}{dt}\right) = \frac{di_1}{dt}\left(L_1 - \frac{M^2}{L_2}\right) = \frac{di_1}{dt}\,\frac{L_1 L_2 - M^2}{L_2}$$

The equivalent inductance $= \dfrac{L_1 L_2 - M^2}{L_2} = \dfrac{2\times 5 - (1.8)^2}{5} = 1.352$ H.

Solution

Current $\qquad i = \left(15 \sin \dfrac{\pi}{3} t\right)$ A

∴ Voltage (instantaneous) across the resistor is $\left(0.5 \times 15 \sin \dfrac{\pi}{3} t\right)$ V.

i.e. $\qquad v_R = \left(7.5 \sin \dfrac{\pi}{3} t\right)$ V.

Also, voltage across the inductor is given by

$$v_L = L\cdot\frac{di}{dt} = 5\frac{d}{dt}\left(15 \sin\frac{\pi}{3}t\right) = 75\cdot\frac{\pi}{3}\cos\frac{\pi}{3}t = \left(25\pi\cos\frac{\pi}{3}t\right)\text{V}.$$

Power across the resistor is

$$i^2 R = \left(225 \sin^2 \frac{\pi}{3}t\right)0.5 = 112.5 \sin^2\left(\frac{\pi}{3}t\right)\text{W}.$$

The energy stored by the inductor is maximum when the current through it is maximum.

Current is maximum when $\left(\sin^2 \dfrac{\pi}{3} t\right) = 1$

i.e. $1 - \cos\left(\dfrac{2\pi}{3}t\right) = 2$ or, $\cos\dfrac{2\pi}{3}t = -1 = \cos\pi$

∴ $\qquad \dfrac{2\pi}{3}t = \pi$ or, $t = \dfrac{3}{2}$ s.

Hence energy stored in the inductor is maximum at t = 3/2s. In another 3/2s energy will be recovered from the inductor.

Hence in $\left(\frac{3}{2}+\frac{3}{2}=3\,\text{s}\right)$ energy dissipated in the resistor is $\int_0^3\left(112.5\sin^2\frac{\pi}{3}t\right)dt$

$$= \frac{112.5}{2}\int_0^3\left(1-\cos\frac{2\pi}{3}t\right)dt$$

$$= \frac{112.5}{2}\left[t-\frac{\sin\frac{2\pi}{3}t}{\frac{2\pi}{3}}\right]_0^3 = \frac{112.5}{2}\left[3-\frac{\sin 2\pi}{\frac{2\pi}{3}}\right] = 168.75\text{ J.}$$

......

14.33 Write three mesh equations for the circuit shown in Fig. 14.20.

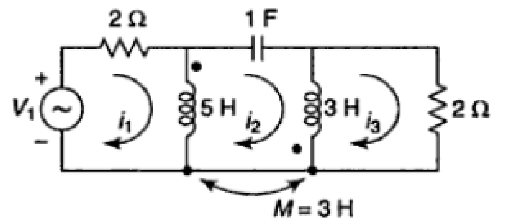


Fig. 14.20

Solution

The mutual inductance and the self inductances are replaced by their impedances and the corresponding circuit is shown in Fig. 14.21.

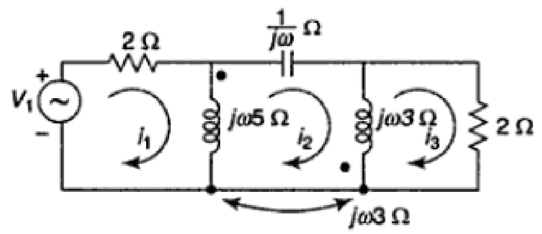


Fig. 14.21

Applying KVL in the first mesh (leftmost mesh),

$$2i_1 + j\omega 5(i_1 - i_2) + 3j\omega(i_3 - i_2) = V_1$$

or $\quad (2 + 5j\omega)i_1 - 8j\omega i_2 + 3j\omega i_3 = V_1 \qquad$ (i)

Applying KVL in the second mesh (middle mesh),

$$5j\omega(i_2 - i_1) + 3j\omega(i_2 - i_3) + \frac{1}{j\omega}i_2 + 3j\omega(i_2 - i_3) + 3j\omega(i_2 - i_1) = 0$$

or $\quad -8j\omega i_1 + \left(14\,j\omega + \frac{1}{j\omega}\right) i_2 - 6j\omega i_3 = 0 \qquad$ (ii)

Applying KVL in the third mesh (rightmost mesh),

$$3j\omega(i_3 - i_2) + 3j\omega(i_1 - i_2) + 2i_3 = 0$$

or $\quad 3j\omega i_1 - 6j\omega i_2 + (2 + 3j\omega)i_3 = 0. \qquad$ (iii)

....

3.20 A current of 10 A when flowing through a coil of 2000 turns establishes a flux of 0.6 milliwebers. Calculate the inductance L of the coil.

iolution

Given $I = 10$ A

$N = 2000$ turns

$\phi = 0.6 \times 10^{-3}$ wb

L = to be calculated.

We have $$L = \frac{N\phi}{I} = \frac{2000 \times 0.6 \times 10^{-3}}{10}$$

$= 0.12$ H. ......

3.21 Determine the inductance L of a coil of 500 turns wound on an air cored torroidal ring having a mean diameter of 300 mm. The ring has a circular cross section of diameter 50 mm.

Solution

Given $N = 500$ turns

Mean diameter, $D = 300 \text{ mm} = 300 \times 10^{-3}$ m

$l = \pi D = \pi \times 300 \times 10^{-3}$ m

$= 0.942$ m

Cross-sectional diameter $d = 50$ mm

$= 50 \times 10^{-3}$ m

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (50 \times 10^{-3})^2}{4}$$

$= 1.963 \times 10^{-3}\ \text{m}^2$.

For air cored torroidal ring, $\mu_r = 1$ and L = is to be calculated.

We have, inductance $L = \dfrac{N^2}{\text{Reluctance}}$

$$\left[\because \quad L = \frac{\mu_o \mu_r A N^2}{l} \right.$$

$$= \frac{N^2}{l/\mu_o \mu_r A} = \frac{N^2}{\text{Reluctance}}$$

$$\left. \text{where Reluctance} = \frac{l}{\mu_o \cdot \mu_r \cdot A}. \right]$$

(The concept of reluctance is explained in article 3.18 and 3.24)

Here $$\text{Reluctance} = \frac{\pi \times 300 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 1.963 \times 10^{-4}}$$

$= 3.818 \times 10^8$ AT/Wb

$$\therefore \quad L = \frac{N^2}{\text{Reluctance}} = \frac{500 \times 500}{3.818 \times 10^8}$$

$= 0.000654$ H

$= 6.54 \times 10^{-4}$ H. ....

3.22 Two coils having 80 and 350 turns respectively are wound side by side on a closed iron circuit of mean length 2.5 m with a cross-sectional area of 200 cm^2. Calculate the mutual inductance between the coils. Consider relative permeability of iron as 2700.

Solution

Given
$$N_1 = 80 \text{ turns}$$
$$N_2 = 350 \text{ turns}$$
$$l = 2.5 \text{ m}$$
$$A = 200 \text{ cm}^2 = 200 \times 10^{-4} \text{ m}^2$$
$$\mu_r = 2700$$
$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$
$$M = \text{to be calculated.}$$

We have $$M = \frac{N_1 \cdot N_2}{\text{Reluctance}}$$ [for two coils of turns N_1 and N_2]

where reluctance $= \dfrac{l}{\mu_o \cdot \mu_r \cdot A}$ (Ref. article 3.18)

$$= \frac{2.5}{4\pi \times 10^{-7} \times 2700 \times 200 \times 10^{-4}}$$

$$= 36860 \text{ AT/Wb}$$

$\therefore$ Mutual inductance $(M) = \dfrac{80 \times 350}{36860}$

$$= 0.760 \text{ H.}$$

....

3.23 A solenoid 60 cm long and 24 cm in radius is wound with 1500 turns. Calculate:
(a) the inductance
(b) the energy stored in the magnetic field when a current of 5 A flows in the solenoid.

Solution

Given: $l = 60 \text{ cm} = 0.6 \text{ m}, N = 1500 \text{ turns}, A = \pi\,(0.24)^2 \text{ m}^2$
$\mu = \mu_o\mu_r = 4\pi \times 10^{-7} \times 1, I = 5 \text{ A}.$

(a) *Inductance:*

We know $$L = \frac{\mu \cdot N^2 \cdot A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times (1500)^2 \times \pi\,(0.24)^2}{0.6}$$

$$= 0.8534 \text{ H}$$

(b) *Energy stored:*

We have $$W = \frac{1}{2} LI^2$$

$$= \frac{1}{2}(0.8534)\,(5)^2 = 10.67 \text{ J.}$$

....

3.54 Calculate the self-inductance of an air-cored solenoid, 40 cm long, having an area of cross-section 20 cm^2 and 800 turns.

Hints: $$L = \frac{\mu_0 \cdot N^2 \cdot A}{l}$$

[here we assume $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}].

$$\therefore \quad L = \frac{4\pi \times 10^{-7} \times 800^2 \times 20 \times 10^{-4}}{40 \times 10^{-2}} = 4.022 \times 10^{-3} \text{ H}$$

....

4-6 EXAMPLES

Example 4-1 (Section 4-3)

The cast steel core in Fig. E-4-1 is assumed to have constant permeability of 1.1×10^{-3} henrys per meter. The coil has 1200 turns. Effective dimensions are: $A_s = 0.003$ m^2, $l_s = 0.5$ m, $A_g = 0.0034$ m^2, $l_g = 0.0004$ m. The flux in the air gap is 0.003 weber. (a) Find the current in the coil. (b) Find the energy stored in the air gap. (c) Find the energy stored in the steel. (d) Find the self-inductance.

Solution

(a) Solve the magnetic circuit problem using a table.

Part	ϕ(weber)	A(m^2)	B(weber/m^2)	H(At/m)	l(m)	Hl(At)
steel core	0.003	0.003	1.0	910	0.5	455
air gap	0.003	0.0034	0.884	704,000	0.0004	281

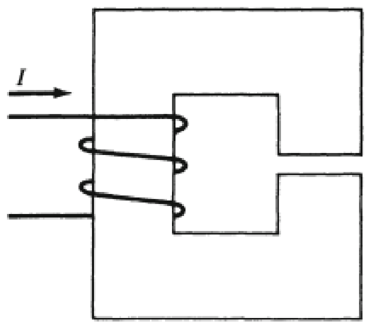


Fig. E-4-1.

The mmf required in the coil is $NI = \Sigma Hl = 736$ At. The current in the coil is $I = 736/1200 = 0.613$ amp.

(b) Use Eq. 4-14 with $B_{k_1} = 0$, and restrict it to just one portion of the magnetic circuit. For the air gap

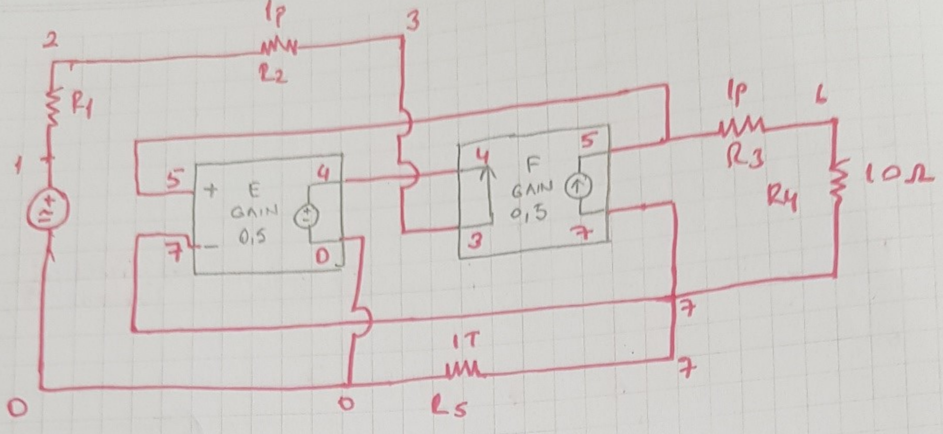
$$W_g = (l_g A_g)\,\tfrac{1}{2}\,(H_g B_g)$$
$$= 0.0004 \times 0.0034 \times \tfrac{1}{2} \times 704{,}000 \times 0.884$$
$$= 0.422 \text{ joules}$$

(c) For the steel portion, the stored energy is

$$W_s = (l_s A_s)\,\tfrac{1}{2}\,(H_s B_s)$$
$$= 0.5 \times 0.003 \times \tfrac{1}{2} \times 910 \times 1$$
$$= 0.683 \text{ joules}$$

(d) The self-inductance is given by

$$L = \lambda/i = N\phi/i = (1200 \times 0.003)/0.613 = 5.87 \text{ henrys}$$



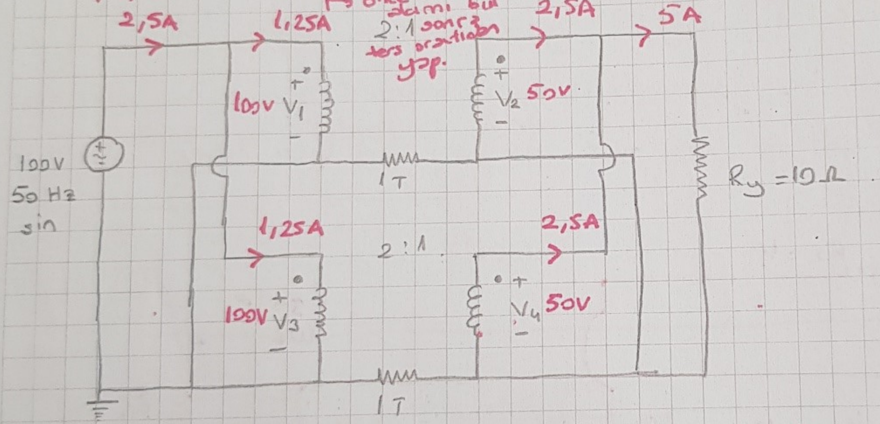
yükseltici

Trafo : (indirici)

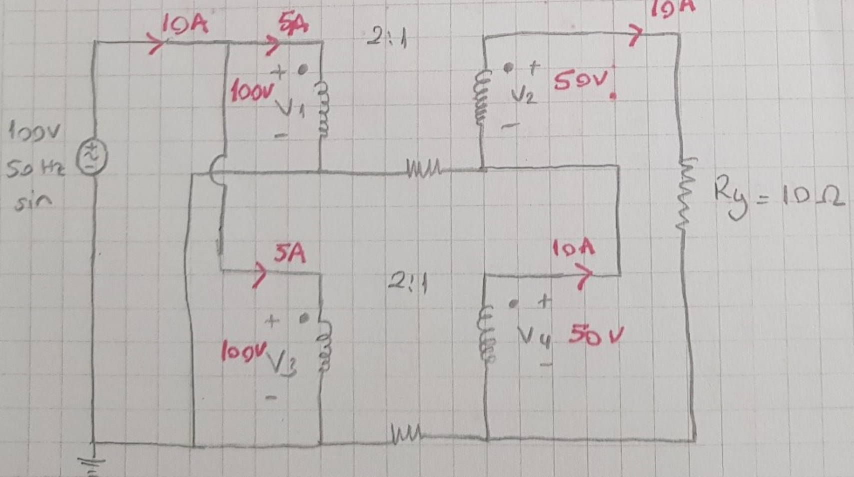
* p/p (paralel-paralel)

→ önce karşıya geç akımı bul sonra ters orantıdan yap.

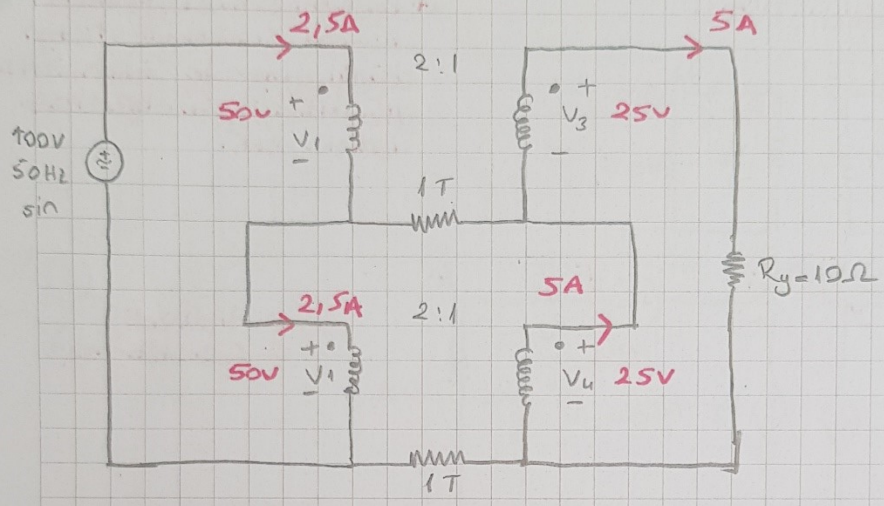
→ paralel olduğu için paylaşır.



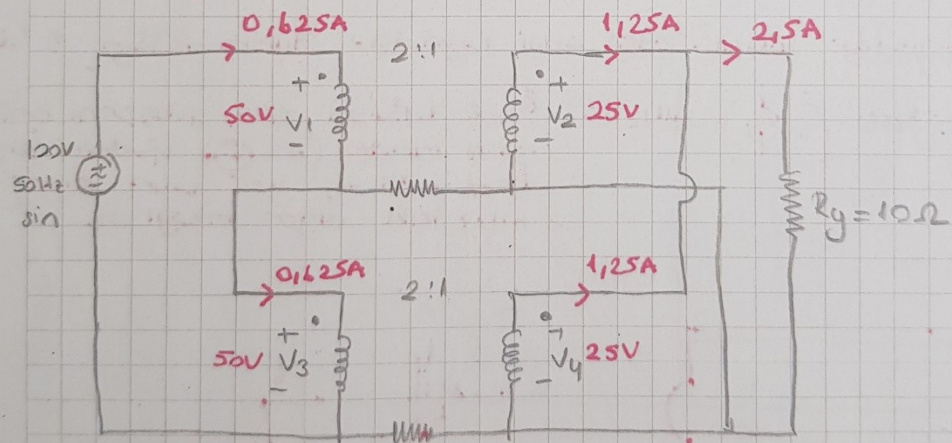
* p/s (paralel - seri)



* s/s (seri-seri)



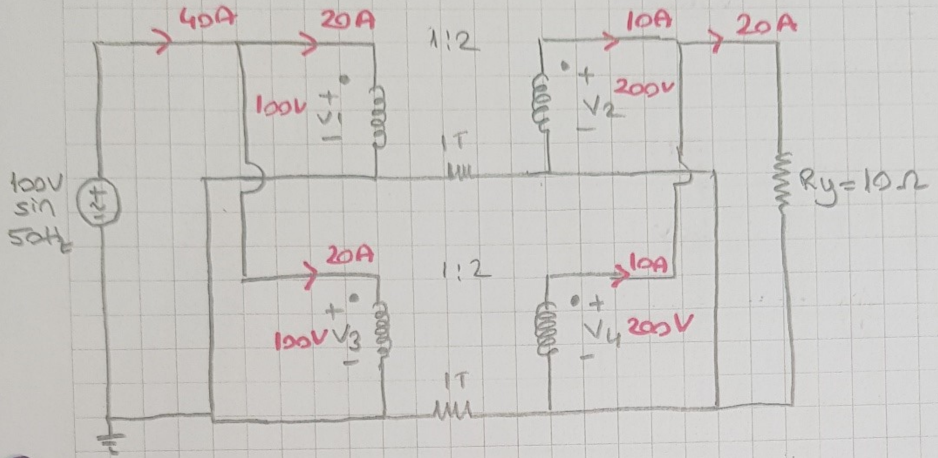
* s/p (seri-paralel)



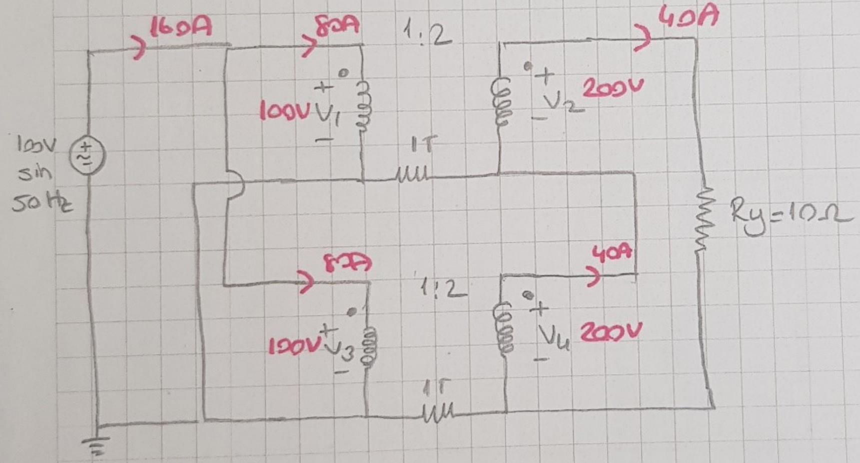
* Trafoların primer, sekonder, kaynak ve yük akımlarını bulunuz.

Trafo: (yükseltici)

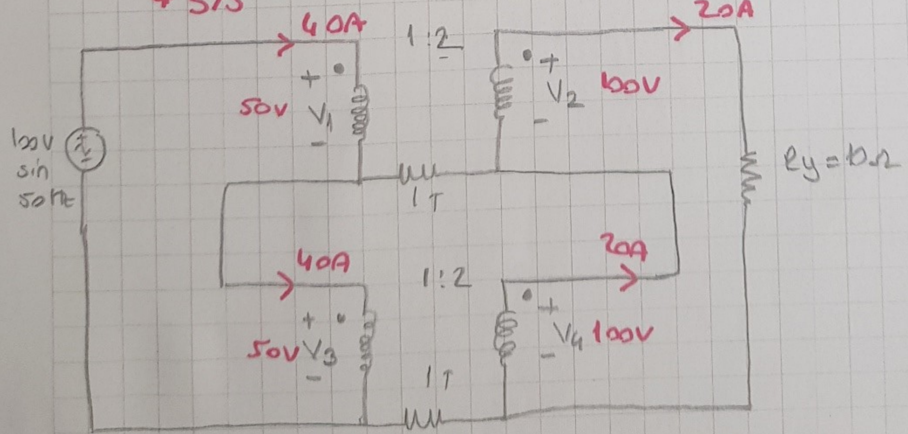
* p/p



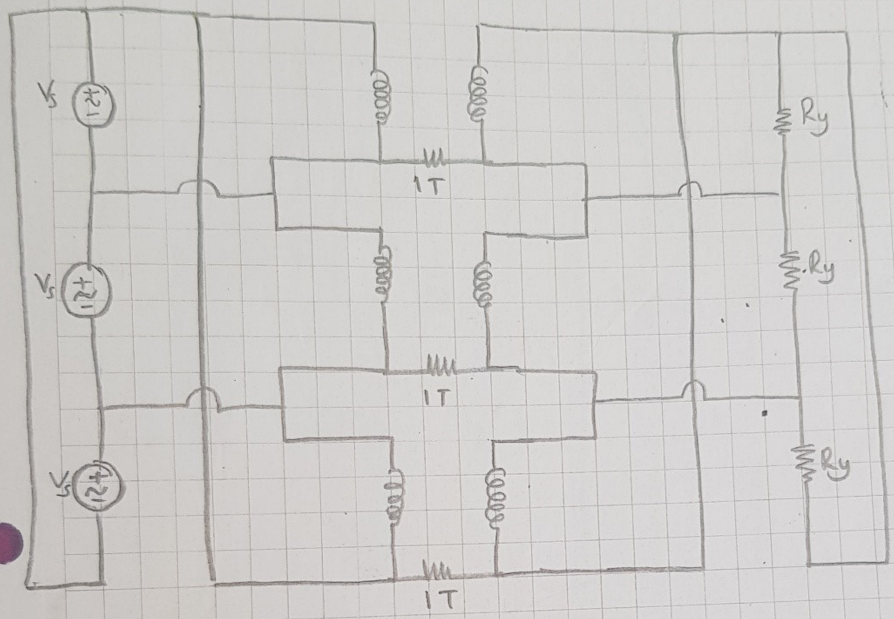
* p/s



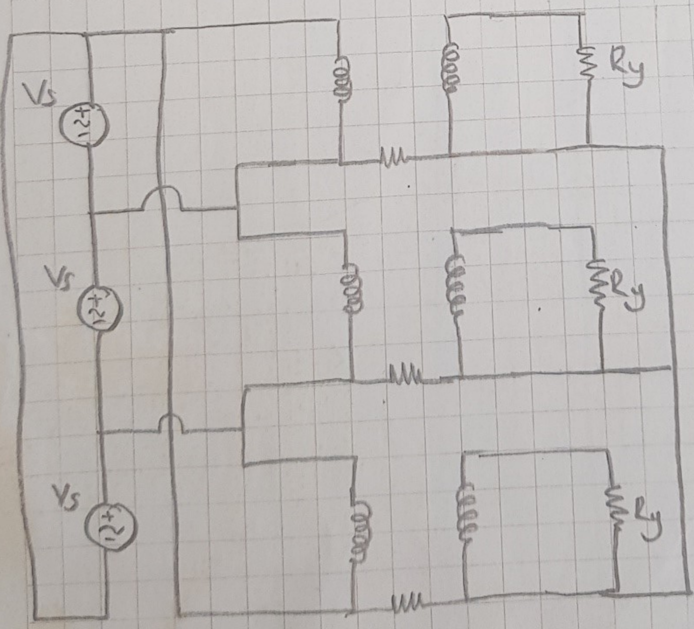
* s/s



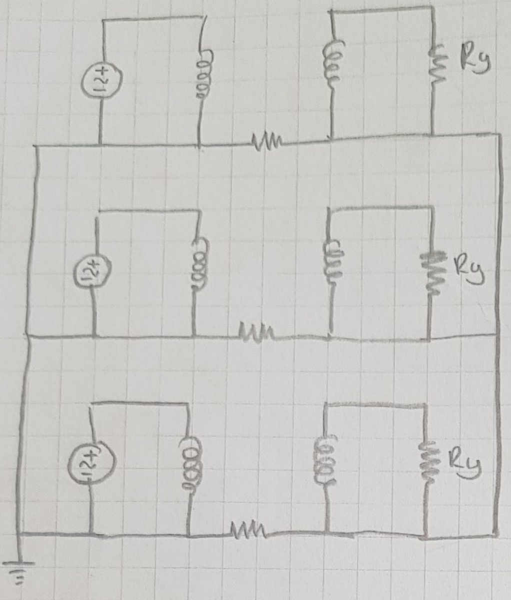
Δ – Δ :



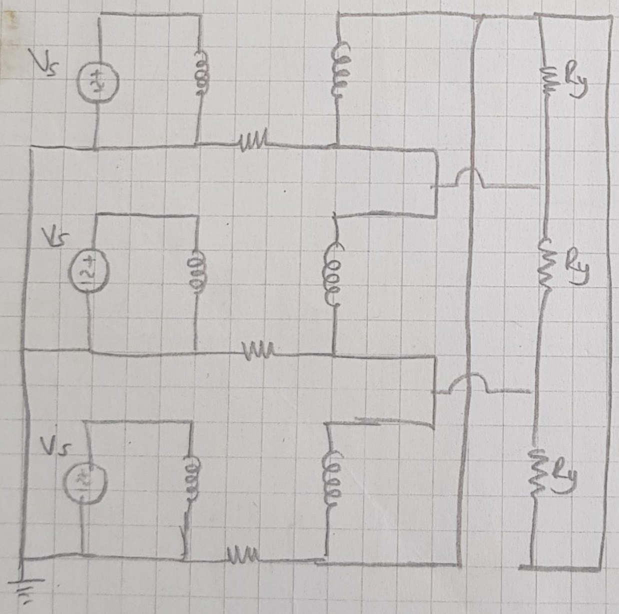
Δ – Y :

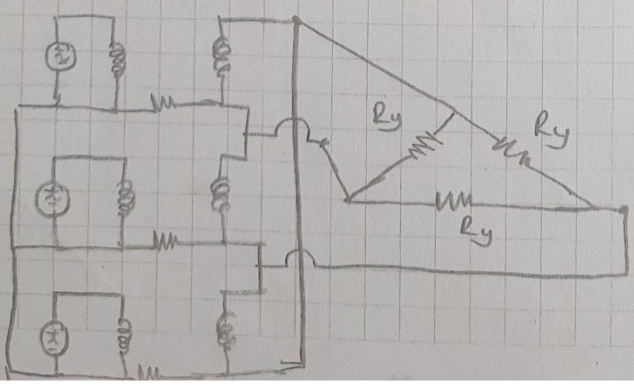


Y-Y:



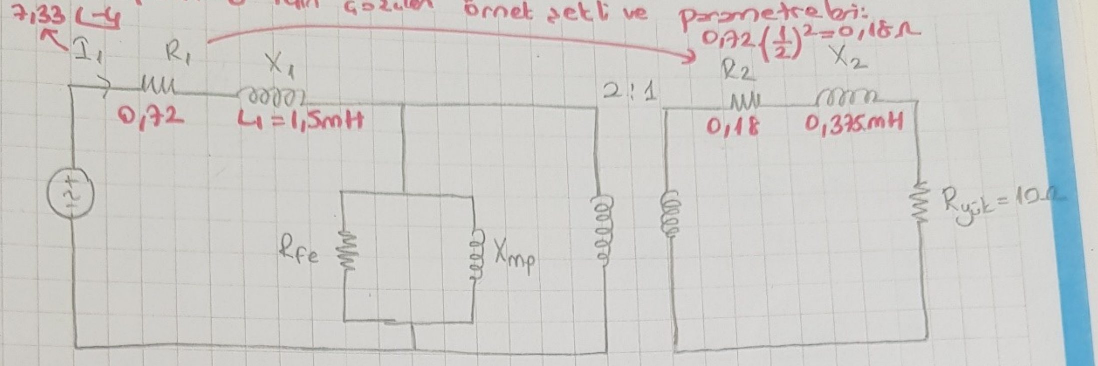
Y-Δ:





* Makalede sf: 11-6 için çizilen örnek şekli ve parametreleri:

$0{,}72\left(\frac{1}{2}\right)^2 = 0{,}18\,\Omega$



$V_S = 220{,}59\ \angle 0°$

$I_1 = 7{,}33\ \angle -4°$

$V_{yük} = 103{,}3°\ \angle 0°$

$I_2 = 14{,}3\ \angle -1°$

$V_{kısadevre} = 12{,}5\ \angle 0{,}95°$ → açısı 0° olarak yanlış yazıldı.

$I_{kısa} = 7{,}3\ \angle -32°$

Çözüm:

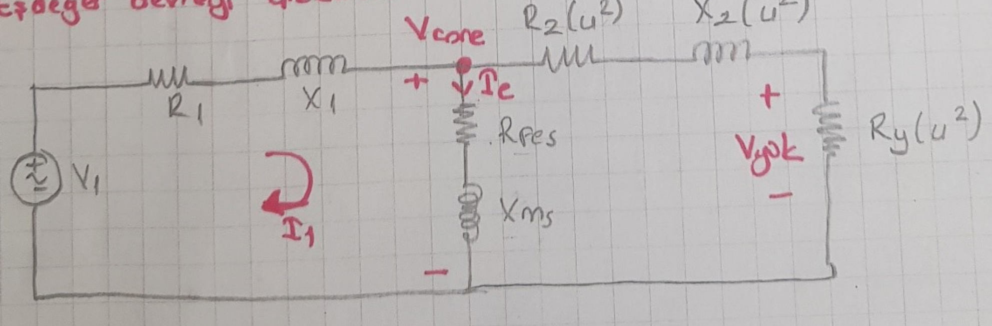
$$Z_{bakır} = \frac{V_{kısadevre}}{I_{kısa}} = \frac{12{,}5\ \angle 0{,}95}{7{,}3\ \angle -32°} = 1{,}72\ \angle 32{,}95°$$

$1{,}72 \div 2 = 0{,}86$ $\qquad 0{,}86\ \angle 32{,}95°$

$$0{,}86\,(\cos 32{,}95 + j\sin 32{,}95) = \underbrace{0{,}72}_{R} + j\,\underbrace{0{,}47}_{X_L}$$

$$X_L = 2\pi f L \qquad L = \frac{0{,}47}{2 \cdot 3{,}14 \cdot 50} = 1{,}5\ mH$$

Eşdeğer devreyi çizelim:



u=2 (Transformasyon oranı)

$$V_1 . I_1 . \cos(\varphi_V - \varphi_I) = R_1 . I_1^2 + P_{Fes} + R_2 . u^2 . \times \left(\frac{I_2}{u}\right)^2 + R_y . u^2 . \left(\frac{I_2}{u}\right)^2$$

$$P_{Fes} = V_1 . I_1 . \cos(\varphi_{V_1} - \varphi_I) - R_1 . I_1^2 - R_2 . I_2^2 - R_y . I_2^2$$

$$V_1 . I_1 . \sin(\varphi_{V_1} - I_{I_1}) = X_1 . I_1^2 + Q_{ms} + X_L . u^2 . \frac{I_2^2}{u^2} + X_y . u^2 . \frac{I_2^2}{u^2}$$

$$\underline{Q_{ms}} = V_1 . I_1 . \sin(\varphi_{V_1} - \varphi_{I_1}) - X_1 . I_1^2 - X_2 . I_2^2 - X_y . I_2^2$$

$$\tan\varphi = \frac{Q_{ms}}{P_{Fes}} \Rightarrow \cos\varphi$$

$$-V_1 + (R_1 + X_1) . I_1 + V_C = 0$$

$$\boxed{V_C = V_1 - (R_1 + jX_1) . I_1}$$

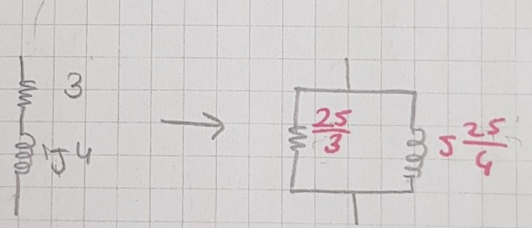
$$P_{Fes} = V_C . I_e . \cos\varphi \Rightarrow I_e = \frac{P_{Fes}}{V_C . \cos\varphi}$$

$$P_{Fes} = R_{Fes} . I_e^2 \Rightarrow R_{Fes} = \frac{P_{Fes}}{I_e^2}$$

$$X_{ms} = \frac{Q_{ms}}{I_e^2}$$

$$R_{Fep} = \frac{R_{Fes}^2 + X_{ms}^2}{R_{Fes}}$$

$$= \frac{3^2 + 4^2}{3} = \frac{25}{3}$$

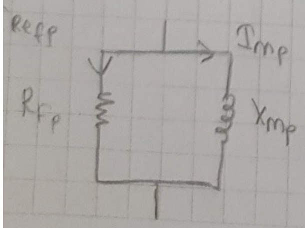


$$X_{mp} = \frac{R_{Fes}^2 + X_{ms}^2}{X_{ms}}$$

$$= \frac{3^2 + 4^2}{4} = \frac{25}{4}$$

$$\frac{\frac{25}{3} . j\frac{25}{4}}{\frac{25}{3} + j\frac{25}{4}} = \frac{\frac{25 \times j25}{3.4}}{\frac{100 + j75}{3.4}} = \frac{j\,25}{\begin{matrix}4 + j3 \\ 4 - j3\end{matrix}} = \frac{j25(4 - j3)}{25}$$

$$= \cancel{4 + j}\,3 + j\,4$$



$$I_{rfep} = \frac{V_C}{R_{fep}} \qquad I_{Xmp} = \frac{V_C}{X_{mp}}$$

3.46 A coil having a resistance of 10 Ω and inductance of 15 H is connected across a d.c. voltage of 150 V. Calculate: (i) The value of current at 0.4 sec after switching on the supply. (ii) With the current having reached the final value the time it would take for the current to reach a value of 9 A after switching off the supply.

Solution

It is given that

$$V(d.c) = 150 \text{ V}$$
$$R = 10\ \Omega$$
$$L = 15 \text{ H}$$

(i) ∴ The value of the current

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{150}{10}\left(1 - e^{-\frac{10}{15}\times 0.4}\right)$$

$$= 15\left(1 - e^{-\frac{4}{15}}\right)$$

$$= 3.51 \text{ A}$$

(ii) Let us assume that at $t = t_1$, $i = 9$ A

∴ $$9 = 15 \times e^{-\frac{t_1}{1.5}}$$ (for decaying)

$$e^{-\frac{t_1}{1.5}} = \frac{9}{15}$$

taking $\log_e$ in both sides,

$$-\frac{t_1}{1.5} = \log_e \frac{9}{15}$$

∴ $$t_1 = 0.7662 \text{ sec.}$$

....

3.47 For the network shown in Fig. 3.36

(a) Find the mathematical expression for the variation of the current in the inductor following the closure of the switch at $t = 0$ on to position 'a';

(b) The switch is closed on to position 'b' when t = 100 m/sec, calculate the new expression for the inductor current and also for the voltage across R;

(c) Plot the current waveforms for $t = 0$ to t = 200 m/sec.

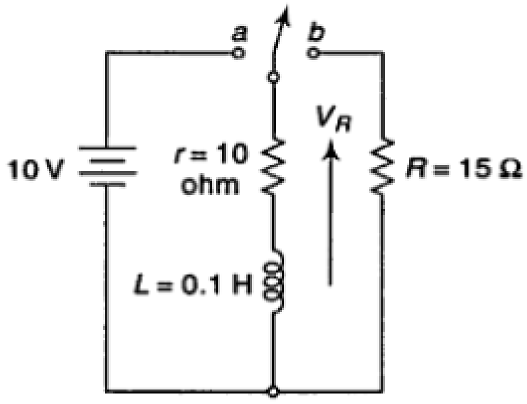


Fig. 3.36 *Network of Ex. 3.47*

Solution

(a) For the switch in position 'a', the time constant is

$$\Upsilon_a = \frac{L}{r} = \frac{0.1}{10} = 10 \text{ milli-sec(ms)}$$

$$\therefore \qquad i_a = \frac{V}{r}\left(1 - e^{-\frac{t}{\Upsilon_a}}\right) = \frac{10}{10}\left(1 - e^{-\frac{t}{10 \times 10^{-3}}}\right)$$

$$= \left(1 - e^{-\frac{t}{10^{-2}}}\right) \text{A.}$$

(b) For the switch in position 'b' the time constant

$$\Upsilon_b = \frac{L}{R + r} = \frac{0.1}{15 + 10} = 4 \text{ ms}$$

$$\therefore \qquad i_b = \frac{V}{R} e^{-\frac{t}{\Upsilon_b}} \qquad \text{(for decaying)}$$

$$= \frac{10}{10} e^{-\frac{t}{4 \times 10^{-3}}} = e^{-\frac{t}{4 \times 10^{-3}}} \text{ A.}$$

The current continues to flow in the same direction as before, therefore the voltage drop across R is negative to the direction of the arrow shown in Fig. 3.36. $v_R = i_b \cdot R = -15 \times e^{-t/4 \times 10^{-3}}$ V.

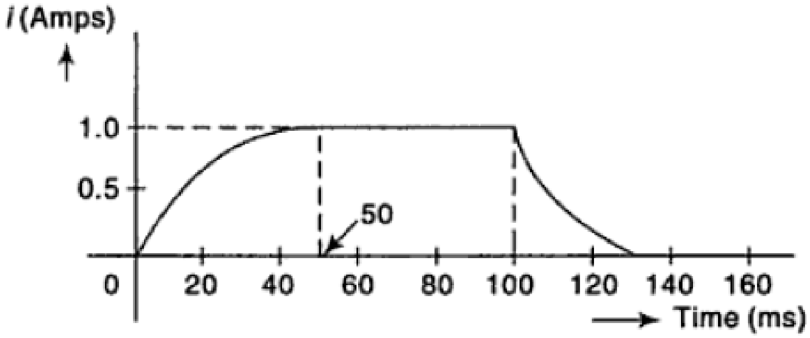


Fig. 3.37 *Current profile*

It will be noted that in the first switched period, five times the time constant is 50 m/sec. The transient has virtually finished at the end of this time and it would not have mattered whether the second switching took place then or later. During the second period the transient took only 25 m/sec.

(c) The profile current waveform has been plotted in Fig. 3.37.

....

3.48 For the network shown in Fig. 3.36 (Ex. No. 3.47) the switch is closed on the position '*a*'. Next, it is closed on to position '*b*' when τ = 10 ms. Again, find the expression of current and hence draw the current wave forms.

Solution

For the switch in position '*a*', the time constant Υ is 10 m/sec as in Ex. No. 3.47, and the current expression as is before. However, the switch is moved to position '*b*' while the transient is proceeding. When t = 10 m/sec.

$$i = \left(1 - e^{-\frac{t}{10\times10^{-3}}}\right) = \left(1.0 - e^{-\frac{10\times10^{-3}}{10\times10^{-3}}}\right)$$

$= (1 - e^{-1}) = 0.632$ A.

i.e., the second transient commences with an initial current in R of 0.632 A.

∴ The current decay is, $i_b = 0.632 \times \frac{t}{4\times10^{-3}}$ A. which is shown in Fig. 3.38.

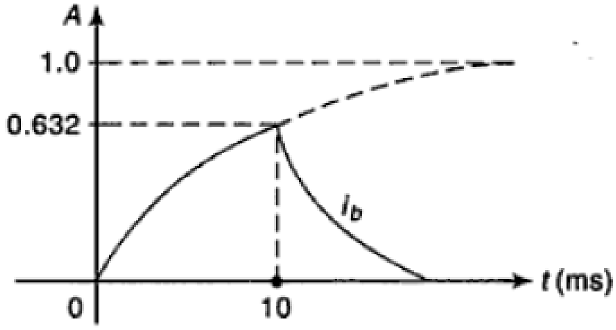


Fig. 3.38

....

3.49 A d.c. voltage of 150 V is applied to a coil whose resistance is 10 Ω and inductance is 15 H. Find: (i) the value of the current 0.3 sec after switching on the supply; (ii) with the current having reached the final value, how much time it would take for the current to reach a value of 6 A after switching off the supply.

Solution

(a) It is given that

$V = 150$ V, $R = 10$ Ω, $L = 15$ H

∴ The value of the current 0.3 sec after switching on is

$$i = \frac{150}{10}\left(1 - e^{-\frac{10}{15}\times 0.3}\right) = 2.72 \text{ A.}$$

(b) After switching off the supply, the current will be decaying and is given by

$$i = \frac{V}{R}e^{-\frac{R}{L}t} \quad \therefore \quad 6 = \frac{150}{10}e^{-\frac{10}{15}\times t}$$

$$\therefore \qquad t = 1\ .375 \text{ sec.}$$

3.50 A coil of resistance 24 Ω and having inductor 36 H is suddenly connected to a d.c. of 60 V supply. Determine

(a) the initial rate of change of current $\left(\dfrac{di}{dt}\right)$

(b) the time-constant

(c) the current after 3 sec.

(d) the enrgy stored in the magnetic field at $t = 3$ sec.

(e) the energy lost as heat energy at $t = 3$ sec.

Solution

It is given that: $V = 60$ V, $R = 24$ Ω, $L = 36$ H

(a) Initial rate of change of current:

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right)$$

$$\therefore \qquad \frac{di}{dt} = -\frac{V}{R}\cdot\left(-\frac{R}{L}\right)\cdot e^{-\frac{R}{L}\cdot t}$$

$$= \frac{V}{L}e^{-\frac{R}{L}\cdot t};$$

When $t = 0$,

$$\frac{di}{dt} = \frac{V}{L}\cdot e^{o} = \frac{V}{L} = \frac{60}{36} = 1.67 \text{ A/sec;}$$

(b) Time constant (Υ):

$$\Upsilon = \frac{L}{R} = \frac{36}{24} = 1.5 \text{ sec.}$$

(c) Current; the current at $t = 3$ sec is

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right) = \frac{60}{24}\left(1 - e^{-\frac{24}{36}\times 3}\right) = 2.16 \text{ A.}$$

(d) Energy stored:

at $t = 3$ sec, the energy stored in the magnetic field is $\frac{1}{2}Li^2$

$$= \frac{1}{2} \times 36 \times (2.16)^2 = 84 \text{ J.}$$

(e) Energy lost as heat energy:

at $t = 3$ sec, the energy lost as heat energy is

$$i^2 \times R = (2.16)^2 \times 24 \cong 112 \text{ J.}$$

3.51 If the vertical component of the earth's magnetic field be 4.0×10^{-5} Wb m^{-2}, then what will be the induced potential difference produced between the rails of a meter-gauge running north-south when a train is running on them with a speed of 36 km h^{-1}?

Solution

When a train is on the rails, it cuts the magnetic flux lines of the vertical component of the earth's magnetic field. Hence, a potential dfifference is induced between the ends of its axle.

Distance between the rails = 1 m; speed of train (v) = 36 km/hr = 10 m/sec. Magnetic field $B_V = 4.0 \times 10^{-5}$ Wb/m. $\therefore$ The induced potential difference in $e = Bvl = (4.0 \times 10^{-5}) \times 10 \times 1 = 4.0 \times 10^{-9}$ V.

....

3.52 The current in the coil of a large electromagnet falls from 6 A to 2 A in 10 ms. The induced emf across the coil is 100 V. Find the self-inductance of the coil.

Solution

The self-induced emf is given by

$$e = -L\frac{di}{dt}$$

Here $di = 2 - 6 = -4$ A

$dt = 10$ ms $= 10^{-2}$ sec

and $e = 100$ V

$$\therefore \quad L = -e\frac{dt}{di} = -100 \times \frac{10^{-2}}{-4} = 0.25 \text{ H.}$$

....

3.53 The current (in ampere) in an inductor is given by $i = 5 + 16t$, where t is in seconds. The self-induced emf in it is 10 mV. Find (a) the self-inductance, and (b) the energy stored in the inductor and the power supplied to it at $t = 1$.

Solution

The induced emf in the inductor due to current change is

$$|e| = L\frac{di}{dt}$$

$$\therefore \qquad L = \frac{|e|}{di/dt}$$

Hence $\qquad i = 5 + 16t$, from this, we have

$$\frac{di}{dt} = 0 + 16 = 16 \text{ A sec}^{-1}, \text{ and } e = 10 \text{ mV} = 10 \times 10^{-3} \text{ V}$$

$$\therefore \qquad L = \frac{10 \times 10^{-3} V}{15 A \sec^{-1}} = 0.666 \times 10^{-3} \text{ H} = 0.666 \text{ mH}$$

(b) The current at $t = 1$ sec is

$$i = 5 + 16t = 5 + 16 \times 1$$
$$= 21 \text{ A}$$

$\therefore$ Energy stored in the inductor is

$$\frac{1}{2}Li^2 = \frac{1}{2} \times (0.666 \times 10^{-3}) \times (21)^2$$
$$= 137.8 \times 10^{-3} = 137.8 \text{ mJ}$$

Power supplied to the inductor at $t = 1$ sec is

$$P = li = (10 \times 10^{-3} \text{ V}) \times 21 = 0.21 \text{ W}.$$

....

3.55 A solenoid of inductance L and resistance R is connected to a battery. Prove that the time taken for the magnetic energy to reach 1/4 of its maximum value is $L/R \log_e(2)$.

Solution

The growth of current in an LR circuit is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}\cdot t}\right) \qquad \text{(i)}$$

where I_0 is the maximum current. The energy stored at time t is

$$u = \frac{1}{2}LI^2$$

We are required to find the time at which the energy stored is 1/4 the maximum value,

i.e., when $u = \frac{u_o}{4}$ where $u_o = \frac{1}{2}LI_0^2$.

i.e., $$\frac{1}{2}LI^2 = \frac{1}{4}\left(\frac{1}{2}LI_0^2\right) \quad \text{or} \quad I = \frac{I_0}{2}$$

∴ Using the equation 1, we have

$$\frac{I_0}{2} = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\frac{1}{2} = 1 - e^{-\frac{R}{L}\cdot t}$$

or $$e^{-\frac{R}{L}t} = \frac{1}{2}$$

or $$-\frac{R}{L}t = \log_e\left(\frac{1}{2}\right) - \log_e(2)$$

∴ $$t = \frac{L}{R}\log_e(2)$$

....

3.62 A coil has a resistance of 5 Ω and an inductance of 1 H. At $t = 0$ it is connected to a 2 V battery. Find (a) the rate of rise of current at $t = 0$; (b) the rate of rise of current when $i = 0.2$ Amps and (c) the stored energy when $i = 0$ and $i = 0.3$ A.

Solution

$$\Upsilon = \frac{L}{R} = \frac{1}{5} = 0.2 \text{ sec.}$$

(a) $$\frac{di}{dt} = \frac{E}{L}e^{-\frac{t}{\Upsilon}} = 2e^{-5t}$$

at $t = 0$, $\frac{di}{dt} = 2$ A/sec.

(b) $$i = \frac{E}{R}(1 - e^{-5t})$$
$$= 0.4(1 - e^{-5t})$$

time t_1 when $i = 0.2$ A is

$$0.2 = 0.4(1 - e^{-5t_1})$$

or $$t_1 = 0.1386 \text{ sec.}$$

$$\frac{di}{dt} = 2e^{-5(0.1386)}$$
$$= 1 \text{ A/sec}$$

(c) At $i = 0$, stored energy = 0
when $i = 0.3$ A, stored energy

$$= \frac{1}{2} L i^2 = \frac{1}{2} \times 1 \times (0.3)^2 = 0.045 \text{ J}.$$