Solved Problems

1. A 220 volts, 1500 rpm, 10 Amps separately excited dc motor has an armature resistance of 10 Ω. It is fed from a single phase fully controlled bridge rectifier with an ac source voltage of 230 volts, at 50 Hz. Assuming continuous load current, compute
   i. The motor speed at firing angle of 30 degrees and torque of 5 NM
   ii. Developed torque at the firing angle of 45 degrees and speed of 1000 RPM

Solution:

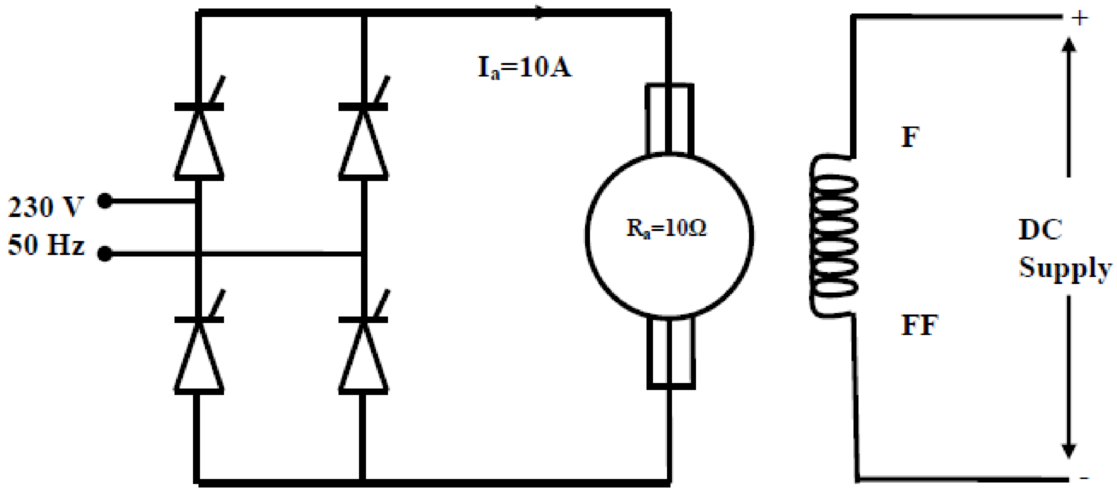
V=200 Volts

N = 1500 rpm

I_a = 10 A

R_a= 10 Ω

Source Voltage V_s=230 volts



Under operating Conditions of separately excited DC motor

$$V_a = E_b + I_a R_a$$
$$V_a = K_b \varphi \omega_m + I_a R_a$$
$$\text{Let } K_b \varphi = K_m$$

$$\therefore V_a = K_m \omega_m + I_a R_a$$
$$220 = K_m \left[\frac{2\pi \, x \, 1500}{60} \right] + (10x10)$$
$$K_m = K_b \varphi = 0.7639 \text{ Volts Seconds/Radians}$$

i. For a torque of 5 Nm, motor armature current is

$$T = K_m I_a$$

$$I_a = \frac{T}{K_m} = \frac{5}{0.7639} = 6.545A$$

The equation giving the operation of converter motor is

$$V_a = E_b + I_a R_a = \frac{2V_m}{\pi}\cos\alpha$$

$$\frac{2V_m}{\pi}\cos\alpha = K_m\omega_m + I_a R_a$$

$$\frac{2x\sqrt{2}\ x\ 230}{\pi}\cos\alpha = K_m\omega_m + I_a R_a$$

$$\frac{2x\sqrt{2}\ x\ 230}{\pi}\cos 30^o = 0.7639\ x\ \omega_m + 6.545\ x\ 10$$

$$179.33 = 0.7639\ x\ \omega_m + 65.45$$

$$\omega_m = 149.07 rad/\sec$$

$$\text{Motor Speed in RPM} = \text{N} = \frac{149.07\ x\ 60}{2\pi}$$

N= 1423.58 rpm

ii. For $\alpha = 45^0$

$$\frac{2V_m}{\pi}\cos\alpha = K_m\omega_m + I_a R_a$$

$$\frac{2\ \text{x}\ \sqrt{2}\ \text{x}\ 230}{\pi}\cos\alpha = 0.7639\ \text{x}\ \frac{2\pi\ \text{x}\ 1000}{60} + (I_a\ \text{x}\ 10)$$

$$146.4 = 79.99 + 10I_a$$

I_a=6.641 Amps

Motor developed torque

$T = K_m I_a = 0.7639$ x 6.641

T= 5.07 Nm

2. A separately excited DC motor rated at 10KW, 240 V, 1000 rpm is supplied from a fully controlled two pulse bridge converter. The converter is supplied at 250 V, 50 Hz supply. An extra inductance is connected in the load circuit to make the conduction continuous. Determine the speed, power factor and efficiency of operation for thyristors firing angles of 0 and 60 degrees assuming the armature resistance of 0.4 Ω and an efficiency of 87% at rated conditions. Assume constant torque load

Solution:

The input current to the motor at rated conditions

$$= \frac{10x10^3}{0.87} = 11.494x10^3 W$$

The supply current to the motor is

$$= \frac{11.494x10^3}{240} = 47.89A$$

Neglecting the field copper loss the armature current = 47.89A

The back EMF at the rated conditions is

$$= 240 - 47.89x0.4 = 220.843 Volts$$

At $\alpha = 0$, the converter voltage is

$$V_a = \frac{2V_m}{\pi}\cos\alpha = \frac{2x\sqrt{2}x250}{\pi}\cos 0^0 = 225 Volts$$

As the load torque is constant the armature current is same. Therefore the back EMF is

$$= 225 - 47.89x0.4 = 200.844 Volts$$

We know that $E_b = K\omega$

$$\omega = \frac{1000x2\pi}{60} = 104.719 rad/\sec$$

$$\therefore K = \frac{E_b}{\omega} = \frac{220.844}{104.719} = 2.11 Volts.\sec/rad$$

At $\alpha = 0^0$

$$\text{Speed} = \frac{E_b}{K} = \frac{200.844}{2.108} = 95.235 rad/\sec$$

$$\text{N}=\frac{95.235x60}{2\pi} = 909.43RPM$$

Displacement factor DF = $Cos\varphi = \cos 0^0 = 1$

Power Factor PF = $\frac{2x\sqrt{2}}{\pi}\cos\alpha = \frac{2x\sqrt{2}}{\pi}\cos 0^0 = 0.9$

Input = 225 x 47.89 = 10775.25 W

Output varies linearly with speed

$$\therefore output \text{ at } 909.4 \text{ rpm} = 10\text{kw}_{(\text{rated})} x \frac{909.4}{1000} = 9.094KW$$

$$\therefore \eta = \frac{O/P}{i/p} = \frac{9.094}{10.7752} = 84.4\%$$

At $\alpha = 60$, the converter voltage is

$$V_a = \frac{2V_m}{\pi}\cos\alpha = \frac{2x\sqrt{2}x250}{\pi}\cos 60^0 = 112.5Volts$$

As the load torque is constant the armature current is same. Therefore the back EMF is

$$= 112.5 - 47.89x0.4 = 93.344Volts$$

We know that $E_b = K\omega$

$$\omega = \frac{1000x2\pi}{60} = 104.719 rad / \sec$$

$$\therefore K = \frac{E_b}{\omega} = \frac{220.844}{104.719} = 2.11 Volts.\sec / rad$$

At $\alpha = 60^0$

$$\text{Speed} = \frac{E_b}{K} = \frac{93.344}{2.108} = 44.28 rad / \sec$$

$$\text{N} = \frac{44.28x60}{2\pi} = 422.8RPM$$

Displacement factor DF = $Cos\varphi = \cos 60^0 = 0.5$

Power Factor PF = $\frac{2x\sqrt{2}}{\pi}\cos\alpha = \frac{2x\sqrt{2}}{\pi}\cos 60^0 = 0.45$

Input = 112.5 x 47.89 = 5.387 KW

Output varies linearly with speed

$$\therefore output \text{ at 909.4 rpm} = 10\text{kw}_{\text{(rated)}} x \frac{422.8}{1000} = 4.227KW$$

$$\therefore \eta = \frac{O / P}{i / p} = \frac{4.227}{5.387} = 78.46\%$$

3. A 200 volts, 875 rpm, 150 A separately excited DC motor has an armature resistance of 0.06 Ω. It is fed from a single phase fully controlled rectifier with an ac source of 220 Volts, 50Hz. Assuming continuous conduction calculate

i. Firing angle for rated motor torque and 750 rpm

ii. Motor speed for α=160 degrees and rated torque

At rated Conditions

$$E_{b1} = V - I_a R_a = 200 - 150x0.06 = 191 Volts$$

$$N_1 = 875 rpm$$

$$\omega_1 = \frac{875x2\pi}{60} = 91.629 rad/\sec$$

We know that

$$E_{b1} = K\omega_1$$

$$\therefore K = \frac{191}{91.629} = 2.08 volts.\sec/rad$$

I. E_{b2} at 750 rpm

$$\omega_2 = \frac{750x2\pi}{60} = 78.54 rad/\sec$$

$$\therefore E_{b2} = 2.08x78.54 = 163.37 volts$$

$$\therefore V_a = E_{b2} + I_a R_a = 163.37 + (150x0.06) = 172.7 Volts$$

We know that

$$V_a = \frac{2V_m}{\pi}\cos\alpha$$

$$172.7 = \frac{2x\sqrt{2}x220}{\pi}\cos\alpha$$

$$\alpha = 29.3^0$$

II. At $\alpha = 160^0$ N =? at rated torque

$$V_a = \frac{2V_m}{\pi}\cos\alpha = \frac{2x\sqrt{2}x220}{\pi}\cos 160^0 = -186.12 Volts$$

We know that $V_a = E_b + I_a R_a$

$$-186.12 = E_b + (150x0.06)$$

$$E_b = -195.12 Volts$$

$$\therefore \omega = \frac{-195.12}{2.08} = -93.81$$

$$N = \frac{-93.81x60}{2\pi} = -895.79 rpm$$

4. A 220 volts, 1500 rpm, 10 Amps separately excited dc motor has an armature resistance of 0.5 Ω is fed from a three phase fully controlled rectifier. Available AC source has a line voltage of 400 volts, 50 Hz. A star-delta connected transformer is used to feed the armature so that motor terminal voltage equals rated voltage when converter firing angle is zero. Calculate transformer turns ratio. Determine the value of firing angle when

i. Motor is running at 1200 rpm and rated torque
ii. When motor is running at (-800 rpm) and twice the rated torque. Assume continuous conduction

For 3 phase controlled rectifier the average output voltage is given by

$$V_a = \frac{3V_m}{\pi}\cos\alpha$$

Given that V_a =220 Volts

$$\therefore 220 = \frac{3V_m}{\pi}\cos 0^\circ$$

$$\Rightarrow V_m = 230.4 Volts$$

At 1500 rpm

$E_{b1} = V_a - I_a R_a = 220 - (10x0.5) = 215 volts$

We know that

$\therefore E_{b1} = K\omega_1$

$$\omega_1 = \frac{1500x2\pi}{60} = 157.08$$

$$K = \frac{215}{157.08} = 1.37 volt.\sec/rad$$

At 1200 rpm

$\therefore E_{b2} = K\omega_2$

$$\omega_2 = \frac{1200x2\pi}{60} = 125.66 rad/\sec$$

$K = 1.37 volt.\sec/rad$

$E_{b2} = 1.37x125.66 = 172.2 Volts$

Average output voltage is

$$V_a = E_{b2} + I_a R_a = 172.2 + (10x0.5) = 177 Volts$$

$$\because V_a = \frac{3V_m}{\pi} Cos\alpha$$

$$\Rightarrow Cos\alpha = \frac{V_a \pi}{3V_m} = \frac{177x\pi}{3x230.4} = 0.8$$

$$\Rightarrow \alpha = 36.4^0$$

At -800 rpm, α = ? T = 2 x I_{rated}

$$\omega = \frac{-800x2\pi}{60} = -83.77 rad/\sec$$

$\therefore E_b = 1.37x - 83.77 = 114.77 volts$

$Current = 2xRated = 20A$

5. The speed of a separately excited DC motor is controlled by a chopper. The DC supply voltage is 120 V, armature circuit resistance is 0.5 Ω, armature circuit inductance is 20 mH, and back emf constant is 0.05 V/RPM. The motor drives a constant torque load requiring an average current of 20A. Assuming the motor current to be continuous, determine the range of speed control and the range of duty cycle.

Given Data:

V_s=120 volts, R_a=0.5 ohms, L_a=20mH, K=0.05 V/RPM. I_a=20A

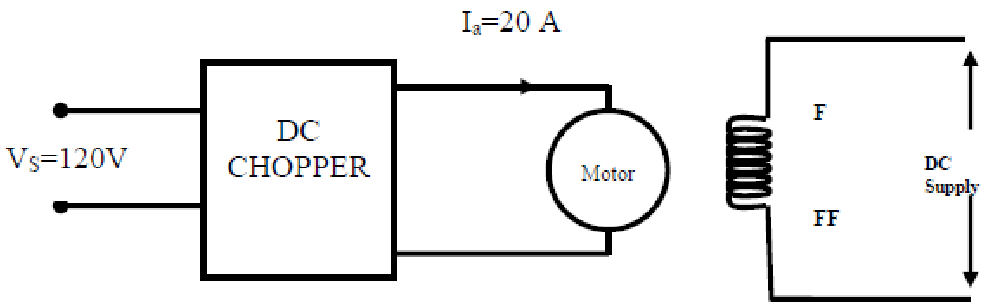
Constant Load, Separately excited DC motor

Find

- ✓ The range of Speed Control
- ✓ The range of duty cycle

Assume Continuous current mode

Solution



(i) Range of Duty cycle

Average output voltage of the motor

$$V_a = E_b + I_a R_a$$

$$\alpha V_s = E_b + I_a R_a \quad \begin{bmatrix} \because V_a = \alpha V_s \\ E_b = KN \end{bmatrix}$$

$$\alpha V_s = KN + I_a R_a$$

As motor drives a constant load, T is constant and I_a is 20A and minimum possible speed is **ZERO**

$$\alpha x 120 = (0.05)x0 + (20x0.05)$$

$$120\alpha = 10$$

$$\alpha = \frac{10}{120} = 0.08$$

Maximum possible speed corresponds to $\alpha = 1$, i.e. when 120 volts is directly applied to the motor. Therefore the range of duty cycle is

$$0.08 \leq \alpha \leq 1$$

(ii) The range of Speed

$\alpha V_s = KN + I_a R_a$

Minimum speed N=0

Maximum speed at $\alpha = 1$

$$1x120 = 0.05xN + (20x0.5)$$
$$120 = 0.05N + 10$$
$$N = \frac{120 - 10}{0.05} = 2200rpm$$

The range of speed control is $0 \leq N \leq 2200RPM$

6. A 230 volts, 960 rpm, 200 Amps separately excited DC motor has an armature resistance of 0.02 Ω. The motor is fed from a dc source of 230 volts through a chopper. Assuming continuous conduction

a) Calculate the duty ratio of chopper for monitoring operation at rated torque and 350 rpm

b) If maximum duty ratio of chopper is limited to 0.95 and maximum permissible motor speed obtainable without field weakening

Given Data

V_s=230 volts, N=960 rpm, I_a=200 amps, R_a=0.02 ohms separately excited DC motor, chopper drive for both motoring and braking operation, Assume continuous conduction

Find

(i) $\alpha = ?$ at rated Torque and Speed =350rpm.

(ii) If $\alpha = 0.95$ and current is twice rated calculate speed

Solution

(i) At rated operation

$$E_1 = V_a - I_a R_a$$
$$\Rightarrow 230 - (200x0.02) = 226volts$$
$$E \text{ at 350 rpm (ie) } E_2 = ?$$

From rated condition

$$E_1 = K\omega_1$$
$$220 = Kx\omega_1$$
$$\omega_1 = \frac{960x2\pi}{60} = 100.53 rad/\sec$$
$$\therefore K = \frac{226}{100.53} = 2.24 Volts.\sec/rad$$

E_2 at 350 rpm is given by

$$\omega_2 = \frac{350x2\pi}{60} = 36.651$$
$$\therefore E_2 = 36.65x2.24 = 82.1 Volts$$

Motor terminal voltage at 350 rpm is

$$V_{350rpm} = 82.1 + (200x0.02) = 86.1 Volts$$
$$\alpha = \frac{V_{350\text{ rpm}}}{\mathrm{V}_{960\text{ rpm}}} = \frac{86.1}{230} = 0.37$$

(ii) Maximum available

$$V_a = \alpha V_s$$
$$= 0.95\text{x}230 = 218.5 \text{ Volts}$$
$$\therefore E = V_a + I_a R_a = 218.5 + (200x0.02) = 222.5 Volts$$

Speed at 222.5 volts E_b is

$$E_b = K\omega$$
$$\omega = \frac{222.5}{2.24} = 99.330 rad/\sec$$
$$N = \frac{99.330x60}{2\pi} = 948.53 rpm$$

7. A DC series motor is fed from a 600 volts source through a chopper. The DC motor has the following parameters armature resistance is equal to 0.04 Ω, field resistance is equal to 0.06 Ω, constant $k = 4 \text{ x } 10^{-3} Nm / Amp^2$. The average armature current of 300 Amps is ripple free. For a chopper duty cycle of 60% determine

i. Input power drawn from the source.
ii. Motor speed and
iii. Motor torque.

Given Data

V_s=600 volts, I_a=300 amps, R_a=0.04 ohms, R_f=0.06 ohms, K=$4x10^{-3}\, Nm / amp^2$ $\delta = 0.6$

DC SERIES motor.

Solution

a. Power input to the motor = P = $V_a I_a$

$$V_a = \delta V_s = 0.6x600 = 360 Volts$$
$$\therefore P = 360x300 = 108KW$$

b. For a DC series motor

$$E_a = K_a \phi \omega_m$$
$$= \mathrm{KI_a} \omega_m \left[\because \phi = I_a\right]$$
$$= 4x10^{-3} x300x\omega_m$$
$$\therefore V_a = E + I_a(R_a + R_s) = KI_a\omega_m + I_a(R_a + R_s)$$
$$\Rightarrow 0.6x600 = 4x10^{-3}x300x\omega_m + 300(0.04 + 0.06)$$
$$\omega_m = \frac{360 - 30}{1.2} = 27.5 rad/\sec (or) 2626 rpm$$
$$Motor \text{ Torque T} = \mathrm{Ka}\phi \mathrm{I_a} = KI_a^{\,2}$$
$$= 4\mathrm{x}10^{-3} x300^2$$
$$= 360 \text{ N-M}$$

8. A 230 V, 1100 rpm, 220 Amps separately excited DC motor has an armature resistance of 0.02 Ω. The motor is fed from a chopper, which provides both motoring and braking operations. Calculate

   i. The duty ratio of chopper for motoring operation at rated torque and 400 rpm

   ii. The maximum permissible motor speed obtainable without field weakening, if the maximum duty ratio of the chopper is limited to 0.9 and the maximum permissible motor current is twice the rated current.

Given Data

V_s=230 volts, N=1100 rpm, I_a=220 amps, R_a=0.02 ohms separately excited DC motor, chopper drive for both motoring and braking operation, Assume continuous conduction

Find

(i) $\alpha = ?$ at rated Torque and Speed =400rpm.

(ii) If $\alpha = 0.9$ and current is twice rated calculate speed

Solution

(i) At rated operation

$$E_1 = V_a - I_a R_a$$
$$\Rightarrow 230 - (220x0.02) = 225.6volts$$
$$E \text{ at 400 rpm (ie) } E_2 = ?$$

From rated condition

$$E_1 = K\omega_1$$
$$\omega_1 = \frac{1110x2\pi}{60} = 115.192 rad/\sec$$
$$\therefore K = \frac{225.6}{115.192} = 1.95 Volts.\sec/rad$$

E_2 at 400 rpm is given by

$$\omega_2 = \frac{400x2\pi}{60} = 41.887 rad/\sec$$
$$\therefore E_2 = 41.887x1.95 = 81.68Volts$$

Motor terminal voltage at 400 rpm is

$$V_{400rpm} = 81.68 + (220x0.02) = 86.1Volts$$
$$\alpha = \frac{V_{400\text{ rpm}}}{V_{1100\text{ rpm}}} = \frac{86.1}{230} = 0.37$$

(ii) Maximum available

$$V_a = \alpha V_s$$
$$= 0.9x230 = 207 \text{ Volts}$$
$$\therefore E = V_a + I_a R_a = 207 + (2x220x0.02) = 215.8Volts$$

Speed at 222.5 volts E_b is

$$E_b = K\omega$$
$$\omega = \frac{215.8}{1.95} = 110.667 rad/\sec$$
$$N = \frac{110.667x60}{2\pi} = 1056.78rpm$$

9. A DC chopper is used to control the speed of a separately excited dc motor. The DC voltage is 220 V, R_a= 0.2 Ω and motor constant $K_e\phi$=0.08 V/rpm. The motor drives a constant load requiring an average armature current of 25 A. Determine

   iv. The range of speed control

   v. The range of duty cycle. Assume continuous conduction

Given Data:

V_s=220 volts, R_a=0.2 ohms, L_a=20mH, K=0.08 V/RPM. I_a=25A

Constant Load, Separately excited DC motor

Find

- ✓ The range of Speed Control
- ✓ The range of duty cycle

Assume Continuous current mode

Solution

(i) Range of Duty cycle

Average output voltage of the motor

$$V_a = E_b + I_a R_a$$

$$\alpha V_s = E_b + I_a R_a \quad \begin{bmatrix} \because V_a = \alpha V_s \\ E_b = KN \end{bmatrix}$$

$$\alpha V_s = KN + I_a R_a$$

As motor drives a constant load, T is constant and I_a is 25A and minimum possible speed is **ZERO**

$$\alpha x 220 = (0.08)x0 + (25x0.2)$$

$$220\alpha = 10$$

$$\alpha = \frac{10}{220} = 0.04$$

Maximum possible speed corresponds to $\alpha = 1$, i.e. when 120 volts is directly applied to the motor. Therefore the range of duty cycle is

$$0.04 \le \alpha \le 1$$

(ii) The range of Speed

$$\alpha V_s = KN + I_a R_a$$

Minimum speed N=0

Maximum speed at $\alpha = 1$

$$1x220 = 0.08xN + (25x0.2)$$

$$220 = 0.08N + 5$$

$$N = \frac{220-5}{0.08} = 2687.5 rpm$$

The range of speed control is $0 \le N \le 2687.5 RPM$

Example 14.1 Finding the Voltage and Current of a Separately Excited Motor

A 15-hp, 220-V, 2000-rpm separately excited dc motor controls a load requiring a torque of $T_L = 45$ N · m at a speed of 1200 rpm. The field circuit resistance is $R_f = 147\ \Omega$, the armature circuit resistance is $R_a = 0.25\ \Omega$, and the voltage constant of the motor is $K_v = 0.7032$ V/A rad/s. The field voltage is $V_f = 220$ V. The viscous friction and no-load losses are negligible. The armature current may be assumed continuous and ripple free. Determine (a) the back emf E_g, (b) the required armature voltage V_a, and (c) the rated armature current of the motor.

Solution

$R_f = 147\ \Omega$, $R_a = 0.25\ \Omega$, $K_v = K_t = 0.7032$ V/A rad/s, $V_f = 220$ V, $T_d = T_L = 45$ N · m, $\omega = 1200\ \pi/30 = 125.66$ rad/s, and $I_f = 220/147 = 1.497$ A.

a. From Eq. (14.4), $I_a = 45/(0.7032 \times 1.497) = 42.75$ A. From Eq. (14.2), $E_g = 0.7032 \times 125.66 \times 1.497 = 132.28$ V.

b. From Eq. (14.3), $V_a = 0.25 \times 42.75 + 132.28 = 142.97$ V.

c. Because 1 hp is equal to 746 W, $I_{rated} = 15 \times 746/220 = 50.87$ A.

Example 14.2 Determining the Effects of Gear Ratio on the Effective Motor Torque and Inertia

The parameters of the gearbox shown in Figure 14.7 are $B_1 = 0.025$ Nm/rad/s, $\omega_1 = 210$ rad/s, $B_m = 0.045$ kg-m^2, $J_m = 0.32$ kg-m^2, $T_2 = 20$ Nm, and $\omega_2 = 21$ rad/s. Determine (a) the gear ratio $GR = N_2/N_1$, (b) the effective motor torque T_1, (c) the effective inertia J, and (d) the effective friction coefficient B.

Solution

$B_1 = 0.025$ Nm/rad/s, $\omega_1 = 210$ rad/s, $B_m = 0.045$ kg-m^2, $J_m = 0.32$ kg-m^2, $T_2 = 20$ Nm, and $\omega_2 = 21$ rad/s.

a. Using Eq. (14.14), $GR = \dfrac{N_2}{N_1} = \dfrac{\omega_1}{\omega_2} = \dfrac{210}{21} = 10$

b. Using Eq. (14.15), $T_1 = \dfrac{T_2}{GR^2} = \dfrac{20}{10^2} = 0.2$ Nm

c. Using Eq. (14.16), $J = J_m + \dfrac{J_1}{GR^2} = 0.32 + \dfrac{0.25}{10^2} = 0.323$ kg-m^2

d. Using Eq. (14.17), $B = B_m + \dfrac{B_1}{GR^2} = 0.045 + \dfrac{0.025}{10^2} = 0.045$ Nm/rad/s

Example 14.3 Finding the Performance Parameters of a Single-Phase Semiconverter Drive

The speed of a separately excited motor is controlled by a single-phase semiconverter in Figure 14.12a. The field current, which is also controlled by a semiconverter, is set to the maximum possible value. The ac supply voltage to the armature and field converters is one phase, 208 V, 60 Hz. The armature resistance is $R_a = 0.25\ \Omega$, the field resistance is $R_f = 147\ \Omega$, and the motor voltage constant is $K_v = 0.7032$ V/A rad/s. The load torque is $T_L = 45$ N · m at 1000 rpm.

The viscous friction and no-load losses are negligible. The inductances of the armature and field circuits are sufficient enough to make the armature and field currents continuous and ripple free. Determine (a) the field current I_f; (b) the delay angle of the converter in the armature circuit α_a, and (c) the input power factor of the armature circuit converter.

Solution

$V_s = 208$ V, $V_m = \sqrt{2} \times 208 = 294.16$ V, $R_a = 0.25\ \Omega$, $R_f = 147\ \Omega$, $T_d = T_L = 45$ N · m, K_v= 0.7032 V/A rad/s, and $\omega = 1000\ \pi/30 = 104.72$ rad/s.

a. From Eq. (14.19), the maximum field voltage (and current) is obtained for a delay angle of $\alpha_f = 0$ and

$$V_f = \frac{2V_m}{\pi} = \frac{2 \times 294.16}{\pi} = 187.27 \text{ V}$$

The field current is

$$I_f = \frac{V_f}{R_f} = \frac{187.27}{147} = 1.274 \text{ A}$$

b. From Eq. (14.4),

$$I_a = \frac{T_d}{K_v I_f} = \frac{45}{0.7032 \times 1.274} = 50.23 \text{ A}$$

From Eq. (14.2),

$$E_g = K_v \omega I_f = 0.7032 \times 104.72 \times 1.274 = 93.82 \text{ V}$$

From Eq. (14.3), the armature voltage is

$$V_a = 93.82 + I_a R_a = 93.82 + 50.23 \times 0.25 = 93.82 + 12.56 = 106.38 \text{ V}$$

From Eq. (14.18), $V_a = 106.38 = (294.16/\pi) \times (1 + \cos \alpha_a)$ and this gives the delay angle as $\alpha_a = 82.2°$.

c. If the armature current is constant and ripple free, the output power is $P_o = V_a I_a = 106.38 \times 50.23 = 5343.5$ W. If the losses in the armature converter are neglected, the power from the supply is $P_a = P_o = 5343.5$ W. The rms input current of the armature converter, as shown in Figure 14.12, is

$$I_{sa} = \left(\frac{2}{2\pi}\int_{\alpha_a}^{\pi} I_a^2\, d\theta\right)^{1/2} = I_a\left(\frac{\pi - \alpha_a}{\pi}\right)^{1/2}$$

$$= 50.23\left(\frac{180 - 82.2}{180}\right)^{1/2} = 37.03 \text{ A}$$

and the input volt–ampere (VA) rating is $\text{VI} = V_s I_{sa} = 208 \times 37.03 = 7702.24$. Assuming negligible harmonics, the input PF is approximately

$$\text{PF} = \frac{P_o}{\text{VI}} = \frac{5343.5}{7702.24} = 0.694 \text{ (lagging)}$$

$$\text{PF} = \frac{\sqrt{2}\,(1 + \cos 82.2°)}{[\pi(\pi - 82.2°)]^{1/2}} = 0.694 \text{ (lagging)}$$

The input power factor is given by [12]

$$\text{PF} = \frac{\sqrt{2}(1 + \cos \alpha)}{\sqrt{\pi(\pi + \cos \alpha)}}$$

Example 14.4 Finding the Performance Parameters of a Single-Phase Full-Converter Drive

The speed of a separately excited dc motor is controlled by a single-phase full-wave converter in Figure 14.13a. The field circuit is also controlled by a full converter and the field current is set to the maximum possible value. The ac supply voltage to the armature and field converters is one phase, 440 V, 60 Hz. The armature resistance is $R_a = 0.25\ \Omega$, the field circuit resistance is $R_f = 175\ \Omega$, and the motor voltage constant is $K_v = 1.4$ V/A rad/s. The armature current corresponding to the load demand is $I_a = 45$ A. The viscous friction and no-load losses are negligible. The inductances of the armature and field circuits are sufficient to make the armature and field currents continuous and ripple free. If the delay angle of the armature converter is $\alpha_a = 60°$ and the armature current is $I_a = 45$ A, determine (a) the torque developed by the motor T_d, (b) the speed ω, and (c) the input PF of the drive.

Solution

$V_s = 440$ V, $V_m = \sqrt{2} \times 440 = 622.25$ V, $R_a = 0.25\ \Omega$, $R_f = 175\ \Omega$, $\alpha_a = 60°$, and $K_v = 1.4$ V/A rad/s.

a. From Eq. (14.21), the maximum field voltage (and current) would be obtained for a delay angle of $\alpha_f = 0$ and

$$V_f = \frac{2V_m}{\pi} = \frac{2 \times 622.25}{\pi} = 396.14 \text{ V}$$

The field current is

$$I_f = \frac{V_f}{R_f} = \frac{396.14}{175} = 2.26 \text{ A}$$

From Eq. (14.4), the developed torque is

$$T_d = T_L = K_v I_f I_a = 1.4 \times 2.26 \times 45 = 142.4 \text{ N}\cdot\text{m}$$

From Eq. (14.20), the armature voltage is

$$V_a = \frac{2V_m}{\pi}\cos 60° = \frac{2 \times 622.25}{\pi}\cos 60° = 198.07 \text{ V}$$

The back emf is

$$E_g = V_a - I_a R_a = 198.07 - 45 \times 0.25 = 186.82 \text{ V}$$

b. From Eq. (14.2), the speed is

$$\omega = \frac{E_g}{K_v I_f} = \frac{186.82}{1.4 \times 2.26} = 59.05 \text{ rad/s or } 564 \text{ rpm}$$

c. Assuming lossless converters, the total input power from the supply is

$$P_i = V_a I_a + V_f I_f = 198.07 \times 45 + 396.14 \times 2.26 = 9808.4 \text{ W}$$

The input current of the armature converter for a highly inductive load is shown in Figure 14.13b and its rms value is $I_{sa} = I_a = 45$ A. The rms value of the input current of field converter is $I_{sf} = I_f = 2.26$ A. The effective rms supply current can be found from

$$I_s = (I_{sa}^2 + I_{sf}^2)^{1/2}$$
$$= (45^2 + 2.26^2)^{1/2} = 45.06 \text{ A}$$

and the input VA rating, VI $= V_s I_s = 440 \times 45.06 = 19{,}826.4$. Neglecting the ripples, the input power factor is approximately

$$\text{PF} = \frac{P_i}{\text{VI}} = \frac{9808.4}{19{,}826.4} = 0.495 \text{ (lagging)}$$

From Eq. (10.7),

$$\text{PF} = \left(\frac{2\sqrt{2}}{\pi}\right)\cos\alpha_a = \left(\frac{2\sqrt{2}}{\pi}\right)\cos 60° = 0.45 \text{ (lagging)}$$

Example 14.5 Finding the Delay Angle and Feedback Power in Regenerative Braking

If the polarity of the motor back emf in Example 14.4 is reversed by reversing the polarity of the field current, determine (a) the delay angle of the armature circuit converter, α_a, to maintain the armature current constant at the same value of $I_a = 45$ A; and (b) the power fed back to the supply due to regenerative braking of the motor.

Solution

a. From part (a) of Example 14.4, the back emf at the time of polarity reversal is $E_g = 186.82$ V and after polarity reversal $E_g = -186.82$ V. From Eq. (14.3),

$$V_a = E_g + I_a R_a = -186.82 + 45 \times 0.25 = -175.57 \text{ V}$$

From Eq. (14.20),

$$V_a = \frac{2V_m}{\pi} \cos \alpha_a = \frac{2 \times 622.25}{\pi} \cos \alpha_a = -175.57 \text{ V}$$

and this yields the delay angle of the armature converter as $\alpha_a = 116.31°$.

b. The power fed back to the supply is $P_a = V_a I_a = 175.57 \times 45 = 7900.7$ W.

Note: The speed and back emf of the motor decrease with time. If the armature current is to be maintained constant at $I_a = 45$ A during regeneration, the delay angle of the armature converter has to be reduced. This would require a closed-loop control to maintain the armature current constant and to adjust the delay angle continuously.

Example 14.6 Finding the Performance Parameters of a Three-Phase Full-Converter Drive

The speed of a 20-hp, 300-V, 1800-rpm separately excited dc motor is controlled by a three-phase full-converter drive. The field current is also controlled by a three-phase full converter and is set to the maximum possible value. The ac input is a three-phase, Y-connected, 208-V, 60-Hz supply. The armature resistance is $R_a = 0.25\ \Omega$, the field resistance is $R_f = 245\ \Omega$, and the motor voltage constant is $K_v = 1.2$ V/A rad/s. The armature and field currents can be assumed to be continuous and ripple free. The viscous friction is negligible. Determine (a) the delay angle of the armature converter α_a, if the motor supplies the rated power at the rated speed; (b) the no-load speed if the delay angles are the same as in (a) and the armature current at no load is 10% of the rated value; and (c) the speed regulation.

Solution

$R_a = 0.25\ \Omega$, $R_f = 245\ \Omega$, $K_v = 1.2$ V/A rad/s, $V_L = 208$ V, and $\omega = 1800\ \pi/30 = 188.5$ rad/s. The phase voltage is $V_p = V_L/\sqrt{3} = 208/\sqrt{3} = 120$ V and $V_m = 120 \times \sqrt{2} = 169.7$ V. Because 1 hp is equal to 746 W, the rated armature current is $I_{\text{rated}} = 20 \times 746/300 = 49.73$ A; for maximum possible field current, $\alpha_f = 0$. From Eq. (14.28),

$$V_f = 3\sqrt{3} \times \frac{169.7}{\pi} = 280.7 \text{ V}$$

$$I_f = \frac{V_f}{R_f} = \frac{280.7}{245} = 1.146 \text{ A}$$

a. $I_a = I_{\text{rated}} = 49.73$ A and

$$E_g = K_v I_f \omega = 1.2 \times 1.146 \times 188.5 = 259.2 \text{ V}$$

$$V_a = 259.2 + I_a R_a = 259.2 + 49.73 \times 0.25 = 271.63 \text{ V}$$

From Eq. (14.27),

$$V_a = 271.63 = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_a = \frac{3\sqrt{3} \times 169.7}{\pi} \cos \alpha_a$$

and this gives the delay angle as $\alpha_a = 14.59°$.

b. $I_a = 10\%$ of $49.73 = 4.973$ A and

$$E_{go} = V_a - R_a I_a = 271.63 - 0.25 \times 4.973 = 270.39 \text{ V}$$

From Eq. (14.4), the no-load speed is

$$\omega_0 = \frac{E_{go}}{K_v I_f} = \frac{270.39}{1.2 \times 1.146} = 196.62 \text{ rad/s} \quad \text{or} \quad 196.62 \times \frac{30}{\pi} = 1877.58 \text{ rpm}$$

c. The speed regulation is defined as

$$\frac{\text{no-load speed} - \text{full-load speed}}{\text{full-load speed}} = \frac{1877.58 - 1800}{1800} = 0.043 \quad \text{or} \quad 4.3\%$$

Example 14.7 Finding the Performance of a Three-Phase Full-Converter Drive with Field Control

The speed of a 20-hp, 300-V, 900-rpm separately excited dc motor is controlled by a three-phase full converter. The field circuit is also controlled by a three-phase full converter. The ac input to the armature and field converters is three-phase, Y-connected, 208 V, 60 Hz. The armature resistance is $R_a = 0.25\ \Omega$, the field circuit resistance is $R_f = 145\ \Omega$, and the motor voltage constant is $K_v = 1.2$ V/A rad/s. The viscous friction and no-load losses can be considered negligible. The armature and field currents are continuous and ripple free. (a) If the field converter is operated at the maximum field current and the developed torque is $T_d = 116$ N · m at 900 rpm, determine the delay angle of the armature converter α_a. (b) If the field circuit converter is set for the maximum field current, the developed torque is $T_d = 116$ N · m, and the delay angle of the armature converter is $\alpha_a = 0$, determine the speed of the motor. (c) For the same load demand as in (b), determine the delay angle of the field converter if the speed has to be increased to 1800 rpm.

Solution

$R_a = 0.25\ \Omega$, $R_f = 145\ \Omega$, $K_v = 1.2$ V/A rad/s, and $V_L = 208$ V. The phase voltage is $V_p = 208/\sqrt{3} = 120$ V and $V_m = \sqrt{2} \times 120 = 169.7$ V.

a. $T_d = 116$ N · m and $\omega = 900\ \pi/30 = 94.25$ rad/s. For maximum field current, $\alpha_f = 0$. From Eq. (14.28),

$$V_f = \frac{3 \times \sqrt{3} \times 169.7}{\pi} = 280.7 \text{ V}$$

$$I_f = \frac{280.7}{145} = 1.936 \text{ A}$$

From Eq. (14.4),

$$I_a = \frac{T_d}{K_v I_f} = \frac{116}{1.2 \times 1.936} = 49.93 \text{ A}$$

$$E_g = K_v I_f \omega = 1.2 \times 1.936 \times 94.25 = 218.96 \text{ V}$$

$$V_a = E_g + I_a R_a = 218.96 + 49.93 \times 0.25 = 231.44 \text{ V}$$

From Eq. (14.27),

$$V_a = 231.44 = \frac{3 \times \sqrt{3} \times 169.7}{\pi} \cos \alpha_a$$

which gives the delay angle as $\alpha_a = 34.46°$.

b. $\alpha_a = 0$ and

$$V_a = \frac{3 \times \sqrt{3} \times 169.7}{\pi} = 280.7 \text{ V}$$

$$E_g = 280.7 - 49.93 \times 0.25 = 268.22 \text{ V}$$

and the speed

$$\omega = \frac{E_g}{K_v I_f} = \frac{268.22}{1.2 \times 1.936} = 115.45 \text{ rad/s} \quad \text{or} \quad 1102.5 \text{ rpm}$$

c. $\omega = 1800\ \pi/30 = 188.5$ rad/s

$$E_g = 268.22 \text{ V} = 1.2 \times 188.5 \times I_f \quad \text{or} \quad I_f = 1.186 \text{ A}$$

$$V_f = 1.186 \times 145 = 171.97 \text{ V}$$

From Eq. (14.28),

$$V_f = 171.97 = \frac{3 \times \sqrt{3} \times 169.7}{\pi} \cos \alpha_f$$

which gives the delay angle as $\alpha_f = 52.2°$.

Example 14.8 Finding the Performance Parameters of a Dc–dc Converter Drive

A dc separately excited motor is powered by a dc–dc converter (as shown in Figure 14.15a), from a 600-V dc source. The armature resistance is $R_a = 0.05\ \Omega$. The back emf constant of the motor is $K_v = 1.527$ V/A rad/s. The average armature current is $I_a = 250$ A. The field current is $I_f = 2.5$ A. The armature current is continuous and has negligible ripple. If the duty cycle of the dc–dc converter is 60%, determine (a) the input power from the source, (b) the equivalent input resistance of the dc–dc converter drive, (c) the motor speed, and (d) the developed torque.

Solution

$V_s = 600$ V, $I_a = 250$ A, and $k = 0.6$. The total armature circuit resistance is $R_m = R_a = 0.05\ \Omega$.

a. From Eq. (14.33),

$$P_i = kV_sI_a = 0.6 \times 600 \times 250 = 90\ \text{kW}$$

b. From Eq. (14.35) $R_{eq} = 600/(250 \times 0.6) = 4\ \Omega$.

c. From Eq. (14.32), $V_a = 0.6 \times 600 = 360$ V. The back emf is

$$E_g = V_a - R_mI_m = 360 - 0.05 \times 250 = 347.5\ \text{V}$$

From Eq. (14.2), the motor speed is

$$\omega = \frac{347.5}{1.527 \times 2.5} = 91.03\ \text{rad/s} \quad \text{or} \quad 91.03 \times \frac{30}{\pi} = 869.3\ \text{rpm}$$

d. From Eq. (14.4),

$$T_d = 1.527 \times 250 \times 2.5 = 954.38\ \text{N} \cdot \text{m}$$

Example 14.9 Finding the Performance of a Dc–dc Converter-Fed Drive in Regenerative Braking

A dc–dc converter is used in regenerative braking of a dc series motor similar to the arrangement shown in Figure 14.16a. The dc supply voltage is 600 V. The armature resistance is $R_a = 0.02\ \Omega$ and the field resistance is $R_f = 0.03\ \Omega$. The back emf constant is $K_v = 15.27$ mV/A rad/s. The average armature current is maintained constant at $I_a = 250$ A. The armature current is continuous and has negligible ripple. If the duty cycle of the dc–dc converter is 60%, determine (a) the average voltage across the dc–dc converter V_{ch}; (b) the power regenerated to the dc supply P_g; (c) the equivalent load resistance of the motor acting as a generator, R_{eq}; (d) the minimum permissible braking speed ω_{min}; (e) the maximum permissible braking speed ω_{max}; and (f) the motor speed.

Solution

$V_s = 600$ V, $I_a = 250$ A, $K_v = 0.01527$ V/A rad/s, $k = 0.6$. For a series motor $R_m = R_a + R_f = 0.02 + 0.03 = 0.05\ \Omega$.

a. From Eq. (14.37), $V_{ch} = (1 - 0.6) \times 600 = 240$ V.

b. From Eq. (14.38), $P_g = 250 \times 600 \times (1 - 0.6) = 60$ kW.

c. From Eq. (14.40), $R_{eq} = (600/250)(1 - 0.6) + 0.05 = 1.01\ \Omega$.

d. From Eq. (14.42), the minimum permissible braking speed,

$$\omega_{min} = \frac{0.05}{0.01527} = 3.274 \text{ rad/s} \quad \text{or} \quad 3.274 \times \frac{30}{\pi} = 31.26 \text{ rpm}$$

e. From Eq. (14.43), the maximum permissible braking speed,

$$\omega_{max} = \frac{600}{0.01527 \times 250} + \frac{0.05}{0.01527} = 160.445 \text{ rad/s} \quad \text{or} \quad 1532.14 \text{ rpm}$$

f. From Eq. (14.39), $E_g = 240 + 0.05 \times 250 = 252.5$ V and the motor speed,

$$\omega = \frac{252.5}{0.01527 \times 250} = 66.14 \text{ rad/s or } 631.6 \text{ rpm}$$

Note: The motor speed would decrease with time. To maintain the armature current at the same level, the effective load resistance of the series generator should be adjusted by varying the duty cycle of the dc–dc converter.

Example 14.10 Finding the Performance of a Dc–dc Converter-Fed Drive in Rheostatic Braking

A dc–dc converter is used in rheostatic braking of a dc separately excited motor, as shown in Figure 14.17a. The armature resistance is $R_a = 0.05\ \Omega$. The braking resistor is $R_b = 5\ \Omega$. The back emf constant is $K_v = 1.527$ V/A rad/s. The average armature current is maintained constant at $I_a = 150$ A. The armature current is continuous and has negligible ripple. The field current is $I_f = 1.5$ A. If the duty cycle of the dc–dc converter is 40%, determine (a) the average voltage across the dc–dc converter V_{ch}; (b) the power dissipated in the braking resistor P_b; (c) the equivalent load resistance of the motor acting as a generator, R_{eq}; (d) the motor speed; and (e) the peak dc–dc converter voltage V_p.

Solution

$I_a = 150$ A, $K_v = 1.527$ V/A rad/s, $k = 0.4$, and $R_m = R_a = 0.05\ \Omega$.

a. From Eq. (14.45), $V_{ch} = V_b = 5 \times 150 \times (1 - 0.4) = 450$ V.

b. From Eq. (14.47), $P_b = 150 \times 150 \times 5 \times (1 - 0.4) = 67.5$ kW.

c. From Eq. (14.46), $R_{eq} = 5 \times (1 - 0.4) + 0.05 = 3.05\ \Omega$.

d. The generated emf $E_g = 450 + 0.05 \times 150 = 457.5$ V and the braking speed,

$$\omega = \frac{E_g}{K_v I_f} = \frac{457.5}{1.527 \times 1.5} = 199.74 \text{ rad/s} \qquad \text{or} \qquad 1907.4 \text{ rpm}$$

e. The peak dc–dc converter voltage $V_p = I_a R_b = 150 \times 5 = 750$ V.

Example 14.11 Finding the Peak Load Current Ripple of Two Multiphase Dc–dc Converters

Two dc–dc converters control a dc separately excited motor and they are phase shifted in operation by $\pi/2$. The supply voltage to the dc–dc converter drive is $V_s = 220$ V, the total armature circuit resistance is $R_m = 4\ \Omega$, the total armature circuit inductance is $L_m = 15$ mH, and the frequency of each dc–dc converter is $f = 350$ Hz. Calculate the maximum peak-to-peak load ripple current.

Solution

The effective chopping frequency is $f_e = 2 \times 350 = 700$ Hz, $R_m = 4\ \Omega$, $L_m = 15$ mH, $u = 2$, and $V_s = 220$ V. $4ufL_m = 4 \times 2 \times 350 \times 15 \times 10^{-3} = 42$. Because $42 \gg 4$, approximate Eq. (14.49) can be used, and this gives the maximum peak-to-peak load ripple current, $\Delta I_{max} = 220/42 = 5.24$ A.

Example 14.12 Finding the Line Harmonic Current of Two Multiphase Dc–dc Converters with an Input Filter

A dc separately excited motor is controlled by two multiphase dc–dc converters. The average armature current is $I_a = 100$ A. A simple LC-input filter with $L_e = 0.3$ mH and $C_e = 4500\ \mu$F is used. Each dc–dc converter is operated at a frequency of $f = 350$ Hz. Determine the rms fundamental component of the dc–dc converter-generated harmonic current in the supply.

Solution

$I_a = 100$ A, $u = 2$, $L_e = 0.3$ mH, $C_e = 4500\ \mu$F, and $f_0 = 1/(2\pi\sqrt{L_e C_e}) = 136.98$ Hz. The effective dc–dc converter frequency is $f_e = 2 \times 350 = 700$ Hz. From the results of Example 5.9, the rms value of the fundamental component of the dc–dc converter current is $I_{1h} = 45.02$ A. From Eq. (14.50), the fundamental component of the dc–dc converter-generated harmonic current is

$$I_{1s} = \frac{45.02}{1 + (2 \times 350/136.98)^2} = 1.66 \text{ A}$$

Example 14.13 Finding the Speed and Torque Response of a Converter-Fed Drive

A 50-kW, 240-V, 1700-rpm separately excited dc motor is controlled by a converter, as shown in the block diagram Figure 14.29. The field current is maintained constant at $I_f = 1.4$ A and the machine back emf constant is $K_v = 0.91$ V/A rad/s. The armature resistance is $R_m = 0.1\ \Omega$ and the viscous friction constant is $B = 0.3$ N · m/rad/s. The amplification of the speed sensor is $K_1 = 95$ mV/rad/s and the gain of the power controller is $K_2 = 100$. (a) Determine the rated torque of the motor. (b) Determine the reference voltage V_r to drive the motor at the rated speed. (c) If the reference voltage is kept unchanged, determine the speed at which the motor develops the rated torque. (d) If the load torque is increased by 10% of the rated value, determine the motor speed. (e) If the reference voltage is reduced by 10%, determine the motor speed. (f) If the load torque is increased by 10% of the rated value and the reference voltage is reduced by 10%, determine the motor speed. (g) If there was no feedback in an open-loop control, determine the speed regulation for a reference voltage of $V_r = 2.31$ V. (h) Determine the speed regulation with a closed-loop control.

Solution

$I_f = 1.4$ A, $K_v = 0.91$ V/A rad/s, $K_1 = 95$ mV/rad/s, $K_2 = 100$, $R_m = 0.1\ \Omega$, $B = 0.3$ N · m/rad/s, and $\omega_{\text{rated}} = 1700\ \pi/30 = 178.02$ rad/s.

a. The rated torque is $T_L = 50{,}000/178.02 = 280.87$ N · m.

b. Because $V_a = K_2 V_r$, for open-loop control Eq. (14.70) gives

$$\frac{\omega}{V_a} = \frac{\omega}{K_2 V_r} = \frac{K_v I_f}{R_m B + (K_v I_f)^2} = \frac{0.91 \times 1.4}{0.1 \times 0.3 + (0.91 \times 1.4)^2} = 0.7707$$

At rated speed,

$$V_a = \frac{\omega}{0.7707} = \frac{178.02}{0.7707} = 230.98 \text{ V}$$

and feedback voltage,

$$V_b = K_1 \omega = 95 \times 10^{-3} \times 178.02 = 16.912 \text{ V}$$

With closed-loop control, $(V_r - V_b)K_2 = V_a$ or $(V_r - 16.912) \times 100 = 230.98$, which gives the reference voltage, $V_r = 19.222$ V.

c. For $V_r = 19.222$ V and $\Delta T_L = 280.87$ N · m, Eq. (14.95) gives

$$\Delta\omega = -\frac{0.1 \times 280.86}{0.1 \times 0.3 + (0.91 \times 1.4)^2 + 95 \times 10^{-3} \times 100 \times 0.91 \times 1.4}$$

$$= -2.04 \text{ rad/s}$$

The speed at rated torque,

$$\omega = 178.02 - 2.04 = 175.98 \text{ rad/s} \quad \text{or} \quad 1680.5 \text{ rpm}$$

d. $\Delta T_L = 1.1 \times 280.87 = 308.96$ N · m and Eq. (14.95) gives

$$\Delta\omega = -\frac{0.1 \times 308.96}{0.1 \times 0.3 + (0.91 \times 1.4)^2 + 95 \times 10^{-3} \times 100 \times 0.91 \times 1.4}$$

$$= -2.246 \text{ rad/s}$$

The motor speed

$$\omega = 178.02 - 2.246 = 175.774 \text{ rad/s} \quad \text{or} \quad 1678.5 \text{ rpm}$$

e. $\Delta V_r = -0.1 \times 19.222 = -1.9222$ V and Eq. (14.94) gives the change in speed,

$$\Delta\omega = -\frac{100 \times 0.91 \times 1.4 \times 1.9222}{0.1 \times 0.3 + (0.91 \times 1.4)^2 + 95 \times 10^{-3} \times 100 \times 0.91 \times 1.4}$$

$$= -17.8 \text{ rad/s}$$

The motor speed is

$$\omega = 178.02 - 17.8 = 160.22 \text{ rad/s} \quad \text{or} \quad 1530 \text{ rpm}$$

f. The motor speed can be obtained by using superposition:

$$\omega = 178.02 - 2.246 - 17.8 = 158 \text{ rad/s} \quad \text{or} \quad 1508.5 \text{ rpm}$$

g. $\Delta V_r = 2.31$ V and Eq. (14.70) gives

$$\Delta\omega = \frac{100 \times 0.91 \times 1.4 \times 2.31}{0.1 \times 0.3 + (0.91 \times 1.4)^2} = 178.02 \text{ rad/s} \quad \text{or} \quad 1700 \text{ rpm}$$

and the no-load speed is $\omega = 178.02$ rad/s or 1700 rpm. For full load, $\Delta T_L = 280.87$ N · m, Eq. (14.71) gives

$$\Delta\omega = -\frac{0.1 \times 280.87}{0.1 \times 0.3 + (0.91 \times 1.4)^2} = -16.99 \text{ rad/s}$$

and the full-load speed

$$\omega = 178.02 - 16.99 = 161.03 \text{ rad/s} \quad \text{or} \quad 1537.7 \text{ rpm}$$

The speed regulation with open-loop control is

$$\frac{1700 - 1537.7}{1537.7} = 10.55\%$$

h. Using the speed from (c), the speed regulation with closed-loop control is

$$\frac{1700 - 1680.5}{1680.5} = 1.16\%$$

Example 14.14 Determining the Optimized Gains and Time Constants of the Current and Speed Loop Controllers

The motor parameters of a converter-fed dc motor are 220 V, 6.4 A, 1570 rpm, $R_m = 6.5\,\Omega$, $J = 0.06\,\text{kg-m}^2$, $L_m = 67\,\text{mH}$, $B = 0.087\,\text{N-rn/rad/s}$, $K_b = 1.24\,\text{V/rad/s}$. The converter is supplied from a Y-connected 230 V, three-phase ac source at 60 Hz. The converter can be assumed linear

and its maximum control input voltage is $V_{cm} = \pm 10\,\text{V}$. The tachogenerator has the transfer function $G\omega(s) = 0.074/(1 + 0.002s)$. The speed reference voltage has a maximum of 10 V. The maximum permissible motor current is 20 A. Determine (a) the gain K_r and time constant τ_r of the converter, (b) the current feedback gain H_c, (c) the motor time constant τ_1, τ_2, and τ_m, (d) the gain K_c and the time constant τ_c of the current controller, (e) the gain K_i and the time constant τ_i of the simplified current loop, and (f) the optimized gain K_s and the time constant τ_s of the speed controller.

Solution

$V_{dc} = 220\,\text{V}$, $I_{dc} = 6.4\,\text{A}$, $N = 1570$ rpm, $R_m = 6.5\,\Omega$, $J = 0.06\,\text{kg-m}^2$, $L_m = 67\,\text{mH}$, $B = 0.087$ N-m/rad/s, $K_b = 1.24\,\text{V/rad/s}$, $V_L = 220\,\text{V}$, $f_s = 60\,\text{Hz}$, $V_{cm} = \pm 10\,\text{V}\ 20\,\text{A}$, $I_{a(\max)} = 20\,\text{A}$, $K_\omega = 0.074$, $t_\omega = 0.002\,\text{s}$.

a. The phase voltage is $V_s = \dfrac{V_L}{\sqrt{3}} = \dfrac{220}{\sqrt{3}} = 127.02\,\text{V}$

The maximum dc voltage is $V_{dc(\max)} = K_r V_{cm} = 29.71 \times 10 = 297.09\,\text{V}$

The converter control voltage is $V_c = \dfrac{V_{dc}}{V_{dc(\max)}} V_{cm} = \dfrac{220 \times 110}{297.09} = 7.41\,\text{V}$

Using Eq. (14.88), $K_s = \dfrac{2.339 V_s}{V_{cm}} = \dfrac{2.339 \times 127.02}{10} = 29.71\,\text{V/V}$

Using Eq. (14.91a) $\tau_r = \dfrac{1}{12 f_s} = \dfrac{1}{12 \times 60} = 1.39\,\text{ms}$

b. The current feedback gain is $H_c = \dfrac{V_c}{I_{a(max)}} = \dfrac{7.41}{20} = 0.37\,\text{V/A}$

c. Using $B_t = B$ and Eq. (14.99), $K_m = \dfrac{B}{K_b^2 + R_m B} = \dfrac{0.087}{1.24^2 + 6.5 \times 0.087} = 0.04$

Using Eq. (14.100),

$$r_1 = -5.64; \quad \tau_1 = \frac{-1}{r_1} = 0.18\,\text{s}$$

$$r_2 = -92.83; \quad \tau_2 = \frac{-1}{r_2} = 0.01\,\text{s}$$

$$\tau_m = \frac{J}{B} = \frac{0.06}{0.087} = 0.69\,\text{s}$$

d. The time constant of the current controller is $\tau_c = \tau_2 = 0.01\,\text{s}$

Using Eq. (14.110), $K_c = \dfrac{\tau_1 \tau_c}{2\tau_r}\left(\dfrac{1}{K_m K_r H_c \tau_m}\right) = \dfrac{0.18 \times 0.01}{0.04 \times 29.71 \times 0.37 \times 0.69} = 2.19$

Using Eqs (14.112) to (14.115),

$$K_1 = \frac{K_m K_c K_r H_c \tau_m}{\tau_c} = \frac{0.4 \times 2.19 \times 29.71 \times 0.37 \times 0.69}{0.01} = 63.88$$

$$K_i = \frac{1}{H_c}\left(\frac{K_1}{1 + K_1}\right) = \frac{1}{0.37}\left(\frac{63.88}{1 + 63.88}\right) = 2.66$$

$$\tau_3 = \tau_1 + \tau_r = 0.18 + 0.00139 = 0.18\,\mathrm{s}$$

$$\tau_i = \frac{\tau_3}{1 + K_1} = \frac{0.18}{1 + 63.88} = 2.76\,\mathrm{ms}$$

e. Using Eqs (14.118) and (14.119),

$$\tau_e = \tau_i + \tau_\omega = 2.76 \times 10^{-3} + 2 \times 10^{-3} = 4.76\,\mathrm{ms}$$

$$K_\omega = \frac{K_i K_b H_\omega}{B\tau_m} = \frac{2.66 \times 1.24 \times 0.074}{0.087 \times 0.69} = 4.07$$

Using Eqs (14.121) and (14.122)

$$K_s = \frac{1}{2K_\omega \tau_e} = \frac{1}{2 \times 4.07 \times 4.76 \times 10^{-3}} = 25.85$$

$$\tau_s = 4\tau_e = 4 \times 4.76 \times 10^{-3} = 19.02\,\mathrm{ms}$$

Example 14.1: ***Single-phase, full-wave half-controlled rectifier***

A 120V ac single-phase, full-wave half-controlled rectifier supports a 100A constant load current at a delay trigger angle of 90°. Specify and characterise the input and output waveforms.

Solution

The output voltage waveform is defined by average and rms voltages of

$$\bar{V}_o = \frac{\sqrt{2}V_s}{\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times 120\text{V}}{\pi}(1+\cos\tfrac{1}{2}\pi) = 54.0\text{V}$$

and

$$V_{o,rms} = V_s\sqrt{\frac{\pi-\alpha+\tfrac{1}{2}\sin 2\alpha}{\pi}} = 120\text{V}\sqrt{\frac{\pi-\tfrac{1}{2}\pi+\tfrac{1}{2}\sin\pi}{\pi}} = 84.85\text{V}$$

The input fundamental and rms currents are

$$I_{s1} = \frac{2\sqrt{2}}{\pi}I_o\cos\tfrac{1}{2}\alpha = \frac{2\sqrt{2}}{\pi}100\text{A}\times\cos\tfrac{1}{4}\pi = 63.3\text{A}$$

and

$$I_s = I_o\sqrt{1-\alpha/\pi} = 100\text{A}\sqrt{1-\tfrac{1}{2}\pi/\pi} = 70.7\text{A}$$

The various input factors are

$$DPF = \cos\tfrac{1}{2}\alpha = \cos\tfrac{1}{4}\pi = 0.707$$

$$DF = \sqrt{\frac{4(1+\cos\alpha)}{\pi(\pi-\alpha)}} = \sqrt{\frac{4(1+\cos\tfrac{1}{2}\pi)}{\pi(\pi-\tfrac{1}{2}\pi)}} = 0.90$$

$$pf = DF\times DPF = 0.90\times 0.707 = 0.636$$

$$THD = \sqrt{\frac{I_{s1}^2}{I_s^2}-1} = \sqrt{1/DF^2-1} = \sqrt{\frac{1}{0.90^2}-1} = 0.484$$

♣

Example 14.2: ***Single-phase, half-wave controlled rectifier***

The ac supply of the half-wave controlled single-phase converter in figure 14.4a is $v = \sqrt{2}\, 240 \sin\omega t$. For the following loads

Load #1: $R = 10\Omega$, $\omega L = 0\ \Omega$
Load #2: $R = 0\ \Omega$, $\omega L = 10\Omega$
Load #3: $R = 7.1\Omega$, $\omega L = 7.1\Omega$

Determine in each load case, for a firing delay angle $\alpha = \pi/6$

- the conduction angle $\gamma=\beta - \alpha$, hence the current extinction angle β
- the dc output voltage and the average output current
- the rms load current and voltage, load current and voltage ripple factor, and power dissipated in the load
- the supply power factor

Solution

Load #1: $Z = R = 10\Omega$, $\omega L = 0\ \Omega$

From equation (14.43), $Z = 10\Omega$ and $\phi = 0°$.

From equation (14.48), $\beta = \pi$ for all α, thus for $\alpha = \pi/6$, $\gamma = \beta - \alpha = 5\pi/6$.

From equation (14.49)

$$V_o = \overline{I}_o R = \frac{\sqrt{2}V}{2\pi}(1+\cos\alpha)$$

$$= \frac{\sqrt{2}V}{2\pi}(1+\cos\pi/6) = 100.9\text{V}$$

The average load current is

$$\overline{I}_o = V_o / R = \frac{\sqrt{2}V}{2\pi R}(1+\cos\alpha) = 100.9\text{V}/10\Omega = 10.1\text{A}.$$

The rms load voltage is given by equation (14.50), that is

$$V_{rms} = V\left[\tfrac{1}{2\pi}\left\{(\pi-\alpha)+\tfrac{1}{2}\sin 2\alpha\right\}\right]^{\frac{1}{2}}$$

$$= 240\text{V}\times\left[\tfrac{1}{2\pi}\left\{(\pi-\pi/6)+\tfrac{1}{2}\sin\pi/3\right\}\right]^{\frac{1}{2}} = 167.2\text{V}$$

Since the load is purely resistive, the power delivered to the load is

$$P_o = I_{rms}^2 R = V_{rms}^2 / R = 167.2\text{V}^2 / 10\Omega = 2797.0\text{W}$$

$$I_{rms} = V_{rms} / R = 167.9\text{V} / 10\Omega = 16.8\text{A}$$

For a purely resistive load, the voltage and current factors are equal:

$$FF_i = FF_v = \frac{167.2\text{V}}{100.9\text{V}} = \frac{16.8\text{A}}{10.1\text{A}} = 1.68$$

$$RF_i = RF_v = \sqrt{FF^2 - 1} = 1.32$$

The power factor is

$$pf = \frac{2797\text{W}}{240\text{V}\times 16.7\text{A}} = 0.70$$

Alternatively, use of equation (14.52) gives

$$pf = \sqrt{\tfrac{1}{2} - \frac{\pi/6}{2\pi} + \frac{\sin\pi/6}{4\pi}} = 0.70$$

Load #2: $R = 0\ \Omega$, $Z = X = \omega L = 10\Omega$

From equation (14.43), $Z = X = 10\Omega$ and $\phi = ½\pi$. From equation (14.54), which is based on the equal area criterion, $\beta = 2\pi - \alpha$, thus for $\alpha = \pi/6$, $\beta = 11\pi/6$ whence the conduction period is $\gamma = \beta - \alpha = 5\pi/3$. From equation (14.55) the average output voltage is

$$V_o = 0\text{V}$$

The average load current is

$$\overline{I}_o = \frac{\sqrt{2}\ V}{\pi\ \omega L}\left[(\pi-\alpha)\cos\alpha + \sin\alpha\ \right]$$

$$= \frac{\sqrt{2}\ 240}{\pi\times 10}\times\left[(5\pi/6)\cos\pi/6 + \sin\pi/6\ \right] = 14.9\text{A}$$

Using equations (14.57) and (14.58), the load rms voltage and current are

$$V_{rms} = 240\text{V}\left[\frac{1}{\pi}\left\{\pi - \pi/6 + ½\sin\pi/3\right\}\right]^{½} = 236.5\text{V}$$

$$I_{rms} = \frac{240\text{V}}{10\Omega}\left[\frac{1}{\pi}\left\{\left(\pi-\pi/6\right)(2+\cos 2\alpha) + 3/2\sin\pi/3\right\}\right]^{½} = 37.9\text{A}$$

Since the load is purely inductive, the power delivered to the load is zero, as is the power factor, and the output voltage ripple factor is undefined. The output current ripple factor is

$$FF_i = \frac{I_{rms}}{\overline{I}_o} = \frac{37.9\text{A}}{14.9\text{A}} = 2.54 \quad \text{whence} \quad RF_i = \sqrt{2.54^2 - 1} = 2.34$$

Load #3: $R = 7.1\Omega$, $\omega L = 7.1\Omega$

From equation (14.43), $Z = 10\Omega$ and $\phi = ¼\pi$.

From figure 14.5a, for $\phi = ¼\pi$ and $\alpha = \pi/6$, $\gamma = \beta - \alpha = 195°$ whence $\beta = 225°$.

Iteration of equation (14.44) gives $\beta = 225.5° = 3.936$ rad. From equation (14.45)

$$V_o = \overline{I}_o R = \frac{\sqrt{2}V}{2\pi}(\cos\alpha - \cos\beta)$$

$$= \frac{\sqrt{2}\times 240}{2\pi}(\cos 30° - \cos 225°) = 85.0\text{V}$$

The average load current is

$$\overline{I}_o = V_o / R$$
$$= 85.0\text{V}/7.1\Omega = 12.0\text{A}$$

Alternatively, the average current can be extracted from figure 14.5b, which for $\phi = ¼\pi$ and $\alpha = \pi/6$ gives the normalised current as 0.35, thus

$$\overline{I}_o = \sqrt{2}V/Z \times 0.35$$

$$= \sqrt{2}\times 240\text{V}/10\Omega \times 0.35 = 11.9\text{A}$$

From equation (14.47), the rms current is

$$I_{rms} = \frac{V}{Z}\left[\frac{1}{2\pi}\left((\beta-\alpha) - \frac{\sin(\beta-\alpha)\cos(\alpha+\phi+\beta)}{\cos\phi}\right)\right]^{½}$$

$$= \frac{240\text{V}}{10\Omega}\times\left[\frac{1}{2\pi}\left((3.93-\pi/6) - \frac{\sin\left(3.93-\pi/6\right)\cos(\pi/6 + ¼\pi + 3.93)}{\cos ¼\pi}\right)\right]^{½} = 18.18\text{A}$$

The power delivered to the load resistor is

$$P_o = I_{rms}^2 R = 18.18\text{A}^2\times 7.1\Omega = 2346\text{W}$$

The load rms voltage, from equation (14.46), is

$$V_{rms} = V\left[1/2\pi\left\{(\beta-\alpha) - ½(\sin 2\beta - \sin 2\alpha)\right\}\right]^{½}$$

$$= 240\text{V}\left[1/2\pi\left\{(3.94 - \tfrac{1}{6}\pi) - ½\times(\sin(2\times 3.94) - \sin(2\times\tfrac{1}{6}\pi))\right\}\right]^{½} = 175.1\text{V}$$

The load current and voltage ripple factors are

$$FF_i = \frac{18.18\text{A}}{12.0\text{A}} = 1.515 \qquad RF_i = \sqrt{FF_i^2 - 1} = 1.138$$

$$FF_v = \frac{175.1\text{V}}{85\text{V}} = 2.06 \qquad RF_v = \sqrt{FF_v^2 - 1} = 1.8$$

The supply power factor is

$$pf = \frac{2346\text{W}}{240\text{V}\times 18.18\text{A}} = 0.54$$

♣

Example 14.3: ***Controlled full-wave converter – continuous and discontinuous conduction***

The fully controlled full-wave, single-phase converter in figure 14.7a has an ac source of 240V rms, 50Hz, and a 10Ω-50mH series load. If the delay angle is 45°, determine

i. the average output voltage and current, hence thyristor mean current
ii. the rms load voltage and current, hence thyristor rms current and load ripple factors
iii. the power absorbed by the load and the supply power factor

If the delay angle is increased to 75° determine

iv. the load current in the time domain
v. numerically solve the load current equation for β, the current extinction angle
vi. the load average current and voltage
vii. the load rms voltage and current hence load ripple factors and power dissipated
viii. the supply power factor

Solution

The load natural power factor angle is given by

$$\phi = \tan^{-1} \omega L / R = \tan^{-1}(2\pi 50 \times 50\text{mH} / 10\Omega) = 57.5° = 1 \text{ rad}$$

Continuous conduction

Since $\alpha < \phi$ $(45° < 57.5°)$, continuous load current flows, which is given by equation (14.85).

$$i(\omega t) = \frac{\sqrt{2} \times 240\text{V}}{18.62\Omega} \left[\sin(\omega t - 1) - \frac{2 \times \sin(1.31 - 1)}{1 - e^{-\pi/1.56}} e^{[(1.31 - \omega t)/1.56]}\right]$$
$$= 18.2 \times [\sin(\omega t - 1) - 1.62 \times e^{-\omega t/1.56}]$$

i. The average output current and voltage are given by equation (14.84)

$$V_o = \overline{I}_o R = \frac{2\sqrt{2}V}{\pi}\cos\alpha = \frac{2\sqrt{2}V}{\pi}\cos 45° = 152.8\text{V}$$
$$\overline{I}_o = V_o / R = 152.8\text{V} / 10\Omega = 15.3\text{A}$$

Each thyristor conducts for 180°, hence thyristor mean current is ½ of 15.3A = 7.65A.

ii. The rms load current is determined by harmonic analysis. The voltage harmonics (peak magnitude are given by equation (14.89)

$$V_n = \frac{\sqrt{2}V}{2\pi} \times \left(\frac{1}{(n-1)^2} + \frac{1}{(n+1)^2} - \frac{2\cos 2\alpha}{(n-1)(n+1)}\right) \quad \text{for} \quad n = 2, 4, 6, ..$$

and the corresponding current is given from equation (14.91)

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{\sqrt{R^2 + (n\omega L)^2}}$$

harmonic n	V_n	$Z_n = \sqrt{R^2 + (n\omega L)^2}$	$I_n = \frac{V_n}{Z_n}$	$½I_n^2$
0	***(152.79)***	10.00	15.28	***(233.44)***
2	55.65	32.97	1.69	1.42
4	8.16	63.62	0.13	0.01
6	3.03	94.78	0.07	0.00
			$I_o^2 + \sum ½I_n^2 =$	234.4

The dc output voltage component is given by equation (14.84).

From the calculations in the table, the rms load current is

$$I_{rms} = \sqrt{I_o^2 + ½\sum I_n^2} = \sqrt{234.4} = 15.3\text{A}$$

Since each thyristor conducts for 180°, the thyristor rms current is $1/\sqrt{2}$ of 15.3A = 10.8A

The rms load voltage is given by equation (14.87), that is 240V.

$$FF_i = \frac{I_{rms}}{\overline{I}_o} = \frac{15.3\text{A}}{15.3\text{A}} = 1.0 \qquad RF_i = \sqrt{FF_i^2 - 1} = \sqrt{1.00^2 - 1} = 0.0$$

$$FF_v = \frac{V_{rms}}{V_o} = \frac{240\text{V}}{152.8\text{V}} = 1.57 \qquad RF_v = \sqrt{FF_v^2 - 1} = \sqrt{1.57^2 - 1} = 1.21$$

iii. The power absorbed by the load is

$$P_L = I_{rms}^2 R = 15.3\text{A}^2 \times 10\Omega = 2344\text{W}$$

The supply power factor is

$$pf = \frac{P_L}{V_{rms} I_{rms}} = \frac{2344\text{W}}{240\text{V} \times 15.3\text{A}} = 0.64$$

Discontinuous conduction

iv. When the delay angle is increased from 45° to 75° (1.31 rad), discontinuous load current flows since the natural power factor angle $\phi = \tan^{-1}\omega L / R = \tan^{-1}(2\pi 50 \times 50\text{mH} / 10\Omega) = 57.5^\circ \equiv 1\text{rad}$ is exceeded. The load current is given by equation (14.79)

$$i(\omega t) = \frac{\sqrt{2}\times 240\text{V}}{18.62\Omega}\,[\sin(\omega t - 1) - \sin(1.31 - 1)\, e^{\{(1.31 - \omega t)/1.56\}}]$$
$$= 18.2 \times [\sin(\omega t - 1) - 0.71 \times e^{-\omega t/1.56}]$$

v. Solving the equation in part iv for $\omega t = \beta$ and zero current, that is

$$0 = \sin(\beta - 1) - 0.71 \times e^{-\beta/1.56}$$

gives β = 4.09 rad or 234.3°.

vi. The average load voltage from equation (14.80) is

$$V_o = \frac{\sqrt{2}\times 240\text{V}}{\pi}(\cos 75^\circ - \cos 234.5^\circ) = 90.8\text{V}$$
$$\overline{I}_o = \frac{V_o}{R} = \frac{90.8V}{10\Omega} = 9.08\text{A}$$

vii. The rms load voltage is given by equation (14.81)

$$V_{rms} = 240\text{V} \times \left[\tfrac{1}{\pi}\{(4.09 - 1.31) - ½(\sin 8.18 - \sin 2.62)\}\right]^{½} = 216.46\text{V}$$

The rms current from equation (14.82) is

$$I_{rms} = \frac{240\text{V}}{18.62\Omega} \times \left[\frac{1}{\pi}\left((4.09 - 1.31) - \frac{\sin(4.09 - 1.31)\times\cos(1.31 + 1 + 4.09)}{\cos 1}\right)\right]^{½} = 13.55\text{A}$$

The load voltage and current form and ripple factors are

$$FF_i = \frac{I_{rms}}{\overline{I}_o} = \frac{13.55\text{A}}{9.08\text{A}} = 1.49 \qquad RF_i = \sqrt{FF_i^2 - 1} = \sqrt{1.49^2 - 1} = 1.11$$
$$FF_v = \frac{V_{rms}}{V_o} = \frac{216.46\text{V}}{90.8\text{V}} = 2.38 \qquad RF_v = \sqrt{FF_v^2 - 1} = \sqrt{2.38^2 - 1} = 2.16$$

The power dissipated in the 10Ω load resistor is

$$P = I_{rms}^2 R = 13.55^2 \times 10\Omega = 1836\text{W}$$

viii. The supply power factor is

$$pf = \frac{P_L}{V_{rms} I_{rms}} = \frac{1836\text{W}}{240\text{V}\times 13.55\text{A}} = 0.56$$

Example 14.4: ***Single-phase, controlled converter – continuous conduction and back emf***

The fully controlled full-wave converter in figure 14.7a has a source of 240V rms, 50Hz, and a 10Ω, 50mH, 50V emf opposing series load (a dc motor). The delay angle is 45°.

Determine

i. the average output voltage and current
ii. the rms load voltage and the rms voltage across the *R-L* part of the load
iii. the power absorbed by the 50V load back emf
iv. the rms load current hence power dissipated in the resistive part of the load
v. the load efficiency, that is percentage of energy into the back emf and power factor
vi. the load voltage and current form and ripple factors

Solution

From example 14.3, continuous conduction is possible since $\alpha < \phi \; (45° < 57.5°)$.

i. The average output voltage is given by equation (14.114)

$$V_o = \frac{2\sqrt{2}V}{\pi}\cos\alpha$$
$$= \frac{2\sqrt{2}\times 240}{\pi}\times\cos 45° = 152.8\text{V}$$

The average current, from equation (14.115) is

$$\bar{I}_o = \frac{V_o - E}{R} = \frac{152.8\text{V} - 50\text{V}}{10\Omega} = 10.28\text{A}$$

ii. From equation (14.87) the rms load voltage is 240V. The rms voltage across the *R-L* part of the load is

$$V_{RLrms} = \sqrt{V_{rms}^2 - E^2}$$
$$= \sqrt{240\text{V}^2 - 50\text{V}^2} = 234.7\text{V}$$

iii. The power absorbed by the 50V back emf load is

$$P = \bar{I}_o E = 10.28\text{A}\times 50\text{V} = 514\text{W}$$

iv. The *R-L* load voltage harmonics (which are even) are given by equations (14.118) and (14.119):

$$V_{oR\text{-}L} = \frac{2\sqrt{2}V}{\pi}\times\cos\alpha \; - \; E$$

$$V_n = \frac{\sqrt{2}\,V}{2\pi}\times\left(\frac{1}{(n-1)^2}+\frac{1}{(n+1)^2}-\frac{2\cos 2\alpha}{(n-1)(n+1)}\right) \qquad \text{for} \quad n = 2, 4, 6, ..$$

The harmonic currents and voltages are shown in the table to follow.

harmonic n	V_n	$Z_n = \sqrt{R^2 + (n\omega L)^2}$ (Ω)	$I_n = \frac{V_n}{Z_n}$ (A)	$\frac{1}{2}I_n^2$
0	***102.79***	10.00	10.28	***105.66***
2	60.02	32.97	1.82	1.66
4	8.16	63.62	0.13	0.01
6	3.26	94.78	0.04	0.00
			$I_o^2 + \sum \frac{1}{2}I_n^2 =$	107.33

From the table the rms load current is given by

$$I_{rms} = \sqrt{I_o^2 + \frac{1}{2}\sum I_n^2} = \sqrt{107.33} = 10.36\text{A}$$

The power absorbed by the 10Ω load resistor is

$$P_L = I_{rms}^2 R = 10.36\text{A}^2 \times 10\Omega = 1073.3\text{W}$$

v. The load efficiency, that is, percentage energy into the back emf *E* is

$$\eta = \frac{514\text{W}}{514\text{W} + 1073.3\text{W}} \times 100\% = 32.4\%$$

The power factor is

$$pf = \frac{P_L}{V_{rms} I_{rms}} = \frac{514\text{W} + 1073.3\text{W}}{240\text{V} \times 10.36\text{A}} = 0.64$$

vi. The output performance factors are

$$FF_i = \frac{I_{rms}}{\overline{I}_o} = \frac{10.36\text{A}}{10.28\text{A}} = 1.008 \qquad RF_i = \sqrt{FF_i^2 - 1} = \sqrt{1.008^2 - 1} = 0.125$$

$$FF_v = \frac{V_{rms}}{\overline{V}_o} = \frac{240\text{V}}{152.8\text{V}} = 1.57 \qquad RF_v = \sqrt{FF_v^2 - 1} = \sqrt{1.57^2 - 1} = 1.211$$

Note that the voltage form factor (hence voltage ripple factor) agrees with that obtained by substitution into equation (14.121), 1.57.

The harmonic currents and voltages are shown in the table to follow.

harmonic n	V_n	$Z_n = \sqrt{R^2 + (n\omega L)^2}$ (Ω)	$I_n = \frac{V_n}{Z_n}$ (A)	$\frac{1}{2}I_n^2$
0	***102.79***	10.00	10.28	***105.66***
2	60.02	32.97	1.82	1.66
4	8.16	63.62	0.13	0.01
6	3.26	94.78	0.04	0.00
			$I_o^2 + \sum \frac{1}{2}I_n^2 =$	107.33

From the table the rms load current is given by

$$I_{rms} = \sqrt{I_o^2 + \frac{1}{2}\sum I_n^2} = \sqrt{107.33} = 10.36\text{A}$$

The power absorbed by the 10Ω load resistor is

$$P_L = I_{rms}^2 R = 10.36\text{A}^2 \times 10\Omega = 1073.3\text{W}$$

v. The load efficiency, that is, percentage energy into the back emf *E* is

$$\eta = \frac{514\text{W}}{514\text{W} + 1073.3\text{W}} \times 100\% = 32.4\%$$

The power factor is

$$pf = \frac{P_L}{V_{rms} I_{rms}} = \frac{514\text{W} + 1073.3\text{W}}{240\text{V} \times 10.36\text{A}} = 0.64$$

vi. The output performance factors are

$$FF_i = \frac{I_{rms}}{\overline{I}_o} = \frac{10.36\text{A}}{10.28\text{A}} = 1.008 \qquad RF_i = \sqrt{FF_i^2 - 1} = \sqrt{1.008^2 - 1} = 0.125$$

$$FF_v = \frac{V_{rms}}{\overline{V}_o} = \frac{240\text{V}}{152.8\text{V}} = 1.57 \qquad RF_v = \sqrt{FF_v^2 - 1} = \sqrt{1.57^2 - 1} = 1.211$$

Note that the voltage form factor (hence voltage ripple factor) agrees with that obtained by substitution into equation (14.121), 1.57.

Example 14.5: ***Controlled converter – constant load current, back emf, and overlap***

The fully controlled single-phase full-wave converter in figure 14.7a has a source of 230V rms, 50Hz, and a series load composed of ½Ω, infinite inductance, 150V emf non-opposing. If the average load current is to be 200A, calculate the delay angle assuming the converter is operating in the inversion mode, taking into account 1mH of commutation inductance.

Solution

The mean load current is

$$\bar{I}_o = \frac{V_o(\alpha) - E}{R}$$

$$200\text{A} = \frac{V_o(\alpha) - -150\text{V}}{½\Omega}$$

which implies a load voltage $V_o(\alpha) = -50\text{V}$.

The output voltage is given by equation (14.84) $V_o = \frac{2\sqrt{2}V}{\pi}\cos\alpha$. Commutation of current from one rectifier to the other takes a finite time. The effect of commutation inductance is to reduce the output voltage, thus according to equation (14.198), the output voltage becomes

$$V_o = \frac{\sqrt{2}V}{\pi / n}\sin{}^{\pi}\!/_{n}\cos\alpha - n\omega L_c I_o / 2\pi \quad \text{where } n = 2$$

$$-50\text{V} = \frac{\sqrt{2}\times 230\text{V}}{\pi / 2}\times\cos\alpha - 2\times 50\text{Hz}\times 1\text{mH}\times 200\text{A}$$

$$= 207\text{V}\times\cos\alpha - 20\text{V}$$

which yields $\alpha = 98.3°$. The commutation overlap causes the output voltage to reduce to zero volts and the overlap period γ is given by equation (14.199)

$$I_o = \frac{\sqrt{2}V}{2\pi f L_c}\left(\cos\alpha - \cos(\gamma + \alpha)\right)$$

$$200\text{A} = \frac{\sqrt{2}\,230\text{V}}{2\pi 50\text{Hz}\times 1\text{mH}}\left(\cos 93.8° - \cos(\gamma + 93.8°)\right)$$

This gives an overlap angle of $\gamma = 11.2°$.

♣

Example 14.6: ***Three-phase half-wave controlled rectifier, with resistive load***

Three phase half-wave controlled rectifier is connected to 415Vac three phase supply via a delta way 415/460V transformer. The rectifier load is 10Ω and delay angle is α=25°, then increased to α=60°. Calculate the PIV of the thyristors and the rectification efficiency for each delay angle.

Solution

i. The thyristor PIV is

$$PIV = \sqrt{2}\, V_{LL} = \sqrt{2} \times 460 = 650.54 \text{ V}$$

ii. Rectification efficiency at α=25°

$$V_{dc} = \frac{3\sqrt{3}}{2\pi}\sqrt{2}\,V\cos\alpha = \frac{3}{\sqrt{2}\pi}V_{LL}\cos\alpha = \frac{3}{\sqrt{2}\pi}460 \times \cos 25° = 281.5\,\text{V}$$

$$V_{rms} = \sqrt{2}\,V_{LL}\sqrt{\frac{1}{6}+\frac{\sqrt{3}}{8\,\pi}\cos 2\alpha} = \sqrt{2}\;460\sqrt{\frac{1}{6}+\frac{\sqrt{3}}{8\,\pi}\cos(2\times 25)} = 298.8 \text{ V}$$

The rectification efficiency with a 25° delay angle (and since the load is purely resistive) is

$$\eta = \frac{V_{dc}\;I_{dc}}{V_{rms}\;I_{rms}}\times 100 = \frac{V_{dc}^2/R}{V_{rms}^2/R}\times 100 = \frac{V_{dc}^2}{V_{rms}^2}\times 100 = 88.75\%$$

iii. Rectification efficiency at α=60°

$$V_{dc} = \frac{3\,\sqrt{2}\,V}{2\pi}\left[1+\cos\left(\frac{\pi}{6}+\alpha\right)\right] = \frac{3\left(\frac{\sqrt{2}}{\sqrt{3}}\right)\times 460}{2\pi}\left[1+\cos\left(\frac{\pi}{6}+\frac{\pi}{3}\right)\right] = 179.33 \text{ V}$$

$$V_{rms} = V_{LL}\sqrt{\frac{5}{24}-\frac{\alpha}{4\pi}+\frac{1}{8\,\pi}\sin(\pi/3+2\alpha)} = 230\,\text{V}$$

The rectification efficiency with a 60° delay angle is

$$\eta = \frac{V_{dc}\;I_{dc}}{V_{rms}\;I_{rms}}\times 100 = \frac{V_{dc}^2}{V_{rms}^2}\times 100 = 60.79\,\%$$

For a purely resistive load, the rectification efficiency is independent of load resistance and transformer turns ratio, but decreases with increasing delay angle.

Example 14.7: ***Three-phase half-wave rectifier with freewheel diode***

The half-wave three-phase rectifier in figure 14.12 has a three-phase 415V 50Hz source (240V phase), and a 10Ω resistor and infinite series inductance as a load. If the delay angle is 60º determine the load current and output voltage if:

i. the phase commutation inductance is zero
ii. the phase commutation reactance is ¼Ω

Solution

i. The output voltage, without any line commutation inductance and a 60º phase delay angle, is given by equation (14.142)

$$V_o = \overline{I}_o R = \frac{\sqrt{2}V}{2\pi/3}\left(1+\cos\left(\alpha+\pi/6\right)\right)$$

$$= \frac{\sqrt{2}\,240\text{V}}{2\pi/3}\left(1+\cos\left(60^\circ+\pi/6\right)\right) = 162\text{V}$$

The constant load current is therefore

$$I_o = \frac{V_o}{R} = \frac{162\text{V}}{10\Omega} = 16.2\text{A}$$

ii. When the current changes paths, any inductance will control the rate at which the commutation from one path to the next occurs. The voltage drops across the commutating inductors modifies the output voltage. Since the voltage across the freewheel diode is not associated with commutation inductance, the output voltage is not effected when the current swaps from a phase to the freewheel diode. But when the current transfers from the freewheel diode to a phase, while the commutation inductance current in the phase is building up to the constant load current level, the output remains clamped at the diode voltage level, viz. zero. The average voltage across the load during this overlap period is therefore reduced. The commutation current is defined by

$$\sqrt{2}V\sin\omega t = L_c\frac{di_c}{dt} = X_c\frac{di_c}{d\omega t}$$

$$i_c = \frac{\sqrt{2}V}{X_c}\left(\cos\left(\alpha+\frac{\pi}{6}\right)-\cos\omega t\right)$$

Solving for when the current rises to the load current I_o^γ gives

$$I_o^\gamma = \frac{\sqrt{2}V}{X_c}\left(\cos\left(\alpha+\frac{\pi}{6}\right)-\cos\left(\alpha+\frac{\pi}{6}+\gamma\right)\right)$$

but

$$I_o^\gamma = \frac{V_o^\gamma}{R} = \frac{\sqrt{2}V}{R\,2\pi/3}\left(1+\cos\left(\alpha+\pi/6+\gamma\right)\right)$$

$$\frac{\cos\left(\alpha+\frac{\pi}{6}\right)-\frac{X_c}{R\,2\pi/3}}{\frac{X_c}{R\,2\pi/3}+1} = \cos\left(\alpha+\frac{\pi}{6}+\gamma\right)$$

$$\gamma = \cos^{-1}\left(\frac{\cos\left(\alpha+\frac{\pi}{6}\right)-\frac{X_c}{R\,2\pi/3}}{\frac{X_c}{R\,2\pi/3}+1}\right)-\left(\alpha+\frac{\pi}{6}\right) = 0.68^\circ$$

The load current and voltage are therefore

$$I_o^\gamma = \frac{\sqrt{2}V}{X_c}\left(\cos\left(\alpha+\frac{\pi}{6}\right)-\cos\left(\alpha+\frac{\pi}{6}+\gamma\right)\right) = \frac{\sqrt{2}\,240V}{¼\Omega}\left(\cos 90^\circ-\cos 90.68^\circ\right) = 16.11\text{A}$$

$$V_o^\gamma = I_o^\gamma R = 16.11\text{A}\times 10\Omega = 161.1\text{V}$$

Example 14.8: ***Three-phase full-wave controlled rectifier with constant output current***

The full-wave three-phase controlled rectifier in figure 14.13a has a three-phase 415V 50Hz source (240V phase), and provides a 100A constant current load.

Determine:

i. the average and rms thyristor current
ii. the rms and fundamental line current
iii. the apparent fundamental power S_1

If 25kW is delivered to the dc load, calculate:

iv. the supply power factor
iv. the dc output voltage, load resistance, hence the converter phase delay angle
v. the real active and reactive Q_1 ac supply power
vi. the delay angle range if the ac supply varies by ±5% (with 25kW and 100A dc).

Solution

i. From equations (14.160) and (14.161) the thyristor average and rms currents are

$$\overline{I}_{Th} = \tfrac{1}{3}\overline{I}_o = \tfrac{1}{3}\times 100\text{A} = 33\tfrac{1}{3}\text{A}$$

$$I_{Th\,rms} = \sqrt{\tfrac{1}{3}}\,\overline{I}_o = \sqrt{\tfrac{1}{3}}\times 100\text{A} = 57.7\text{A}$$

ii. The rms and fundamental line currents are

$$I_{L\,rms} = \sqrt{\frac{2}{3}}\,I_{o\,rms} = \sqrt{\frac{2}{3}}\times 100\text{A} = 81.6\text{A}$$

$$I_{1rms} = \sqrt{3}\,\frac{\sqrt{2}}{\pi}\,\overline{I}_o = \sqrt{3}\,\frac{\sqrt{2}}{\pi}\times 100\text{A} = 78.0\text{A}$$

iii. The apparent power is

$$S_1 = \sqrt{3}\,VI_{1rms} = \sqrt{3}\times 415\text{V}\times 78\text{A} = 56.1\text{kVA}$$

iv. The supply power factor, from equation (14.171), is

$$pf = \frac{P_L}{\sqrt{3}\,V_{rms}I_{rms}} = \frac{25\text{kW}}{\sqrt{3}\times 415\text{V}\times 81.6\text{A}} = 0.426 \qquad \left(=\frac{3}{\pi}\cos\alpha\right)$$

v. The output voltage is

$$V_o = \text{power}\big/\overline{I}_o = 25\text{kW}\big/100\text{A} = 250\text{V dc}$$

The load resistance is

$$R_L = \frac{V_o}{I_o} = \frac{250\text{V}}{100\text{A}} = 2.5\Omega$$

Thyristor delay angle is given by equation (14.145), that is

$$V_o = 2.34V\cos\alpha$$

$$250\text{Vdc} = 2.34\times 415\text{V}\big/\sqrt{3}\times\cos\alpha$$

which yields a delay angle of α = 1.11rad = 63.5°

vi. For a constant output power at 100A dc, the output voltage must be maintained at 250V dc independent of the ac input voltage magnitude, thus for equation (14.145)

$$\alpha = \cos^{-1}\frac{250\text{Vdc}}{2.34\times(415\pm 5\%)\big/\sqrt{3}}$$

$$\check{\alpha} = \cos^{-1}\frac{250\text{Vdc}}{2.34\times(415-5\%)\big/\sqrt{3}} = 1.08\,\text{rad} = 61.9^\circ$$

$$\hat{\alpha} = \cos^{-1}\frac{250\text{Vdc}}{2.34\times(415+5\%)\big/\sqrt{3}} = 1.13\,\text{rad} = 64.9^\circ$$

Example 14.9: ***Converter average load voltage***

Derive a general expression for the average load voltage of a *p*-pulse controlled converter.

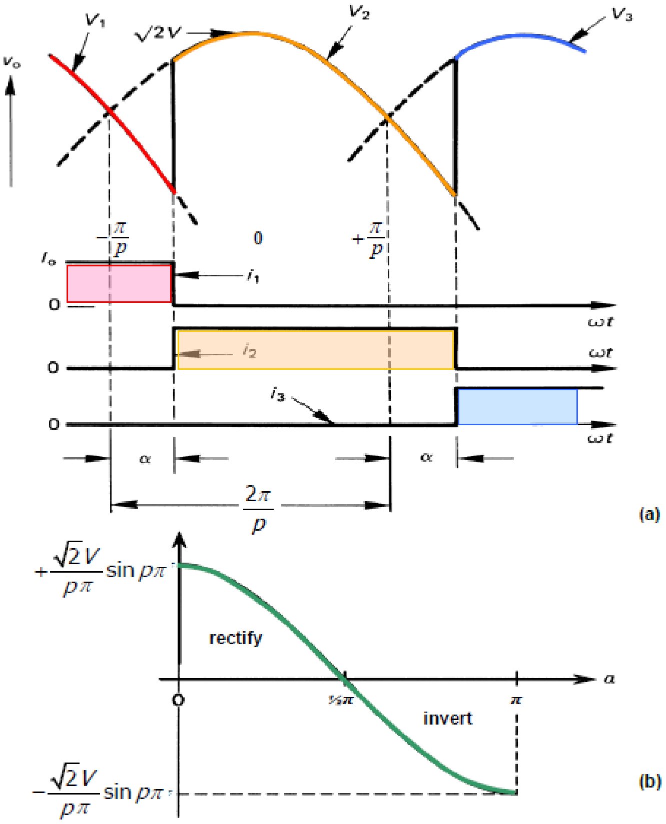


Figure 14.18. *A half-wave n-phase controlled converter:*
(a) output voltage and current waveform and (b) transfer function of voltage versus delay angle α.

Solution

Figure 14.18 defines the general output voltage waveform where *p* is the output pulse number per cycle of the ac supply. From the output voltage waveform

$$
\begin{aligned}
V_o &= \frac{1}{2\pi/p}\int_{-\pi/n+\alpha}^{\pi/n+\alpha}\sqrt{2}V\cos\omega t\; d\omega t \\
&= \frac{\sqrt{2}V}{2\pi/p}\left(\sin(\alpha+\pi/p)-\sin(\alpha-\pi/p)\right) \\
&= \frac{\sqrt{2}V}{2\pi/p}2\sin(\pi/p)\cos\alpha \\
V_o &= \frac{\sqrt{2}V}{\pi/p}\sin(\pi/p)\cos\alpha \\
&= \widehat{V}_o\cos\alpha \qquad \text{(V)}
\end{aligned}
$$

where

for p = 2 for the single-phase (n = 1) full-wave controlled converter in figure 14.7.
for p = 3 for the three-phase (n = 3) half-wave controlled converter in figure 14.11.
for p = 6 for the three-phase (n = 3) full-wave controlled converter in figure 14.13.

♣

Example 14.10: ***Converter overlap***

A three-phase full-wave converter is supplied from the 415 V ac, 50 Hz mains with phase source inductance of 0.1 mH. If the average load current is 100 A continuous, for phase delay angles of (i) 0º and (ii) 60º determine

i. the supply reactance voltage drop,

ii. mean output voltage (with and without commutation overlap), load resistance, and output power, and

iii. the overlap angle

Ignoring thyristor forward blocking recovery time requirements, determine the maximum allowable delay angle.

Solution

Using equations (14.197) and (14.198) with n = 6 and V = 415 V ac, the mean supply reactance voltage

$$\bar{v}_\gamma = \frac{n}{2\pi} 2\pi f\, L I_o = \frac{6}{2\pi} \times 2\pi 50 \times 10^{-4} \times 10^2$$
$$= 3\text{V}$$

(i) α = 0° - as for uncontrolled rectifiers. From equation (14.198), the maximum output voltage is

$$V_o^\gamma = \frac{\sqrt{2}V}{2\pi / n} \sin \tfrac{\pi}{n} \cos\alpha - nX_c I_o / 2\pi$$
$$= \frac{\sqrt{2} \times 415}{2\pi / 6} \sin \tfrac{\pi}{6} \times \cos 0 - 3\text{V} = 560.44\text{V - 3V} = 557.44\text{V}$$

where the mean output voltage without commutation inductance effects is 560.4V.
The power output for 100A is 560.4V×100A = 56.04kW and the load resistance is 560.4V/100A = 5.6Ω.

From equation (14.197)

$$V_o^\gamma = \frac{\sqrt{2}V}{2\pi / n} \sin \pi / n \left[\cos\alpha + \cos(\alpha + \gamma)\right]$$
$$557.44 = \frac{\sqrt{2} \times 415}{2\pi / 6} \times \sin \pi / 6 \times \left[1 + \cos\gamma\right]$$

that is $\gamma = 8.4°$

(ii) α = 60º

$$V_o^\gamma = \frac{\sqrt{2}V}{2\pi / n} \sin \tfrac{\pi}{n} \cos\alpha - nX_c I_o / 2\pi$$
$$= \frac{\sqrt{2} \times 415}{2\pi / 6} \sin \tfrac{\pi}{6} \times \cos 60° - 3\text{V} = 280.22\text{V - 3V} = 277.22\text{V}$$

where the mean output voltage without commutation inductance effects is 280.2V.

The power output for 100A is 280.2V×100A = 28.02kW and the load resistance is 280.2V/100A = 2.8Ω.

$$V_o^\gamma = \frac{\sqrt{2}V}{2\pi / n} \sin \pi / n \left[\cos\alpha + \cos(\alpha + \gamma)\right]$$
$$277.22 = \frac{\sqrt{2} \times 415}{2\pi / 6} \times ½ \times \left[\cos 60° + \cos(60° + \gamma)\right]$$

that is $\gamma = 0.71°$

Equation (14.202) gives the maximum allowable delay angle as

$$\hat{\alpha} = \cos^{-1}\left\{\frac{X I_o}{\sqrt{2}V \sin \pi / n} - 1\right\}$$
$$= \cos^{-1}\left\{\frac{2\pi 50 \times 10^{-4} \times 10^2}{\sqrt{2} \times 415 \times ½} - 1\right\}$$
$$= 171.56° \text{ and } V_o^\gamma = -557.41\text{V}$$

EXAMPLE 10.1

A single-phase full converter is used to control the speed of a 5 hp, 110 V, 1200 rpm, separately excited dc motor. The converter is connected to a single-phase 120 V, 60 Hz supply. The armature resistance is $R_a = 0.4\,\Omega$, and armature circuit inductance is $L_a = 5$ mH. The motor voltage constant is $K\Phi = 0.09$ V/rpm.

1. *Rectifier (or motoring) operation.* The dc machine operates as a motor, runs at 1000 rpm, and carries an armature current of 30 amperes. Assume that motor current is ripple-free.
   (a) Determine the firing angle α.
   (b) Determine the power to the motor.
   (c) Determine the supply power factor.
2. *Inverter operation (regenerating action).* The polarity of the motor back emf E_a is reversed, say by reversing the field excitation.
   (a) Determine the firing angle to keep the motor current at 30 amperes when the speed is 1000 rpm.
   (b) Determine the power fed back to the supply at 1000 rpm.

Solution

Refer to Fig. 10.21.

1. (a)

$$E_a = 0.09 \times 1000 = 90\text{ V}$$

$$V_0 = E_a + I_0 R_a = 90 + 30 \times 0.4 = 102\text{ V}$$

From Eq. 10.3,

$$102 = \frac{2\sqrt{2} \times 120}{\pi} \cos\alpha$$

$$\alpha = 19.2^\circ$$

(b)

$$P = I_0^2 R_a + E_a I_0 = V_0 I_0$$
$$= 102 \times 30$$
$$= 3060\text{ W}$$

(c) The supply current has a square waveform with amplitude 30 A ($=I_0$). The rms supply current is

$$I = 30\text{ A}$$

The supply volt–amperes are

$$S = VI = 120 \times 30 = 3600\text{ VA}$$

If losses in the converter are neglected, the power from the supply is the same as the power to the motor.

$$P_s = 3060\text{ W}$$

Thus, the supply power factor is

$$\text{PF} = \frac{P_s}{S} = \frac{3060}{3600} = 0.85$$

2. (a) At the time of polarity reversal, the back emf is

$$E_a = 90\text{ V}$$

From Eq. 10.4,

$$V_0 = E_a + I_0 R_a$$
$$= -90 + 30 \times 0.4$$
$$= -90 + 12$$
$$= -78\text{ V}$$

Now

$$V_0 = \frac{2\sqrt{2} \times 120}{\pi} \cos\alpha = -78\text{ V}$$

or $\alpha = 136.2^\circ$.

(b) Power from dc machine:

$$P_{dc} = 90 \times 30 = 2700\text{ W}$$

Power lost in R_a:

$$P_R = 30^2 \times 0.4 = 360\text{ W}$$

Power fed back to the ac supply:

$$P_s = 2700 - 360 = 2340\text{ W} \quad \blacksquare$$

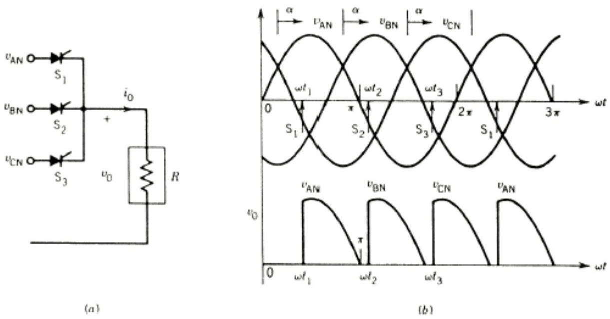


FIGURE 10.26 Three-phase half-wave controlled rectifier circuit with R-load. (*a*) Circuit. (*b*) Waveforms (R-load).

EXAMPLE 10.2

The load in Fig. 10.26*a* consists of a resistance and a very large inductance. The inductance is so large that the output current i_0 can be assumed to be continuous and ripple-free. For $\alpha = 60^\circ$,

(a) Draw the waveforms of v_0 and i_0.

(b) Determine the average value of the output voltage, if phase voltage $V_p = 120\,\text{V}$.

Solution

(a) The supply voltages and the firing instants of the thyristors are shown in Fig. E10.2*a*. The output current i_0 is constant and is shown in Fig. E10.2*b*.

During the interval $(30^\circ + \alpha) < \omega t < (30^\circ + \alpha + 120^\circ)$ thyristor S_1 conducts the load current and therefore during this interval $v_0 = v_{AN}$, as shown in Fig. E10.2*c*. Similarly, $v_0 = v_{BN}$ when S_2 conducts and $v_0 = v_{CN}$ when S_3 conducts. The output voltage waveform v_0 is shown in Fig. E10.2*c*.

(b)

$$V_0 = \frac{1}{2\pi/3} \int_{30^\circ+\alpha}^{30^\circ+\alpha+120^\circ} v_{AN}\, d(\omega t)$$

$$= \frac{3}{2\pi} \int_{30^\circ+\alpha}^{30^\circ+\alpha+120^\circ} \sqrt{2} V_p \sin \omega t\, d(\omega t)$$

$$= \frac{3\sqrt{6}}{2\pi} V_p \cos \alpha$$

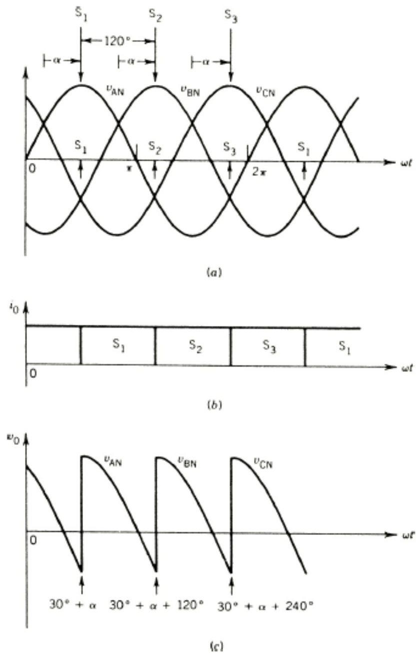


FIGURE E10.2

For $V_p = 120\,\text{V}$ and $\alpha = 60^\circ$,

$$V_0 = \frac{3\sqrt{6}}{2\pi} \times 120 \times \cos 60^\circ$$

$$= 70.2\,\text{V} \quad \blacksquare$$

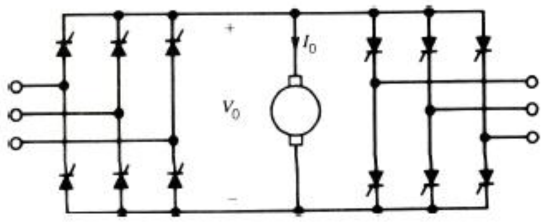


FIGURE 10.30 Three-phase dual converter.

Dual Converter[2]

Two full converters can be connected back-to-back to form a dual converter, as shown in Fig. 10.30. Both the voltage V_0 and the current I_0 can reverse in a dual converter.

EXAMPLE 10.3

A 3ϕ full converter is used to control the speed of a 100 hp, 600 V, 1800 rpm, separately excited dc motor. The converter is operated from a 3ϕ, 480 V, 60 Hz supply. The motor parameters are $R_a = 0.1\ \Omega$, $L_a = 5$ mH, $K\Phi = 0.3$ V/rpm ($E_a = K\Phi n$). The rated armature current is 130 A.

1. *Rectifier (or motoring) operation.* The machine operates as a motor, draws rated current, and runs at 1500 rpm. Assume that motor current is ripple-free.
   - **(a)** Determine the firing angle.
   - **(b)** Determine the supply power factor.

2. *Inverter operation.* The dc machine is operated in the regenerative braking mode. At 1000 rpm and rated motor current,
   - **(a)** Determine the firing angle.
   - **(b)** Determine the power fed back to the supply and the supply power factor.

Solution

1. (a)

$$V_p = \frac{480}{\sqrt{3}} = 277\text{ V}$$

$$E_a = 0.3 \times 1500 = 450\text{ V}$$

$$\begin{aligned} V_0 &= E_a + I_0 R_a \\ &= 450 + 130 \times 0.1 \\ &= 463\text{ V} \end{aligned}$$

From Eq. 10.10,

$$463 = \frac{3\sqrt{6} \times 277}{\pi} \cos\alpha$$

$$\alpha = 44.4^\circ$$

(b) Since ripple in the motor current is neglected, from Fig. 10.27 the supply current i_A is a square wave of magnitude 130 A and width 120°. The rms value of the supply current is

$$\begin{aligned} I_A &= \left(\frac{1}{\pi} \times 130^2 \times \frac{2\pi}{3}\right)^{1/2} \\ &= \sqrt{\frac{2}{3}} \times 130 \\ &= 106.1\text{ A} \end{aligned}$$

The supply volt–amperes are

$$\begin{aligned} S &= 3VI_A \\ &= 3 \times 277 \times 106.1 \\ &= 88{,}169.1\text{ VA} \end{aligned}$$

Assuming no losses in the converter, the power from the supply P_s is the same as the power input to the motor. Hence,

$$\begin{aligned} P_s &= V_0 I_0 \\ &= 463 \times 130 \\ &= 60{,}190\text{ W} \end{aligned}$$

Therefore, the supply power factor is

$$\mathrm{PF} = \frac{P_s}{S} = \frac{60{,}190}{88{,}169.1} = 0.68$$

2. (a)

$$E_a = 0.3 \times 1000 = 300\ \mathrm{V}$$

For inversion, the polarity of E_a is reversed

$$\begin{aligned} V_0 &= E_a + I_0 R_a \\ &= -300 + 130 \times 0.1 \\ &= -287\ \mathrm{V} \end{aligned}$$

Now,

$$V = \frac{3\sqrt{6} \times 277}{\pi} \cos\alpha = -287\ \mathrm{V}$$

$$\alpha = 116.3^\circ$$

(b) Power from the dc machine (operating as a generator):

$$P_{dc} = 300 \times 130 = 39{,}000\ \mathrm{W}$$

Power lost in R_a: $P_R = 130^2 \times 0.1 = 1690\ \mathrm{W}$

Power to source: $P_s = 39{,}000 - 1690 = 37{,}310\ \mathrm{W}$

Supply volt–amperes:

$$S = 88{,}169.1\ \mathrm{VA}$$

Supply power factor:

$$\mathrm{PF} = \frac{37{,}310}{88{,}169.1} = 0.423 \quad \blacksquare$$

Example 3.1

A separately-excited dc motor has the following ratings and constants:

2.625 hp., 120V, 1313 rpm, $R_a = 0.8\ \Omega$, $R_f = 100\ \Omega$, $K_b = 0.764$ V.s / rad, $L_a = 0.003$ H, $L_f = 2.2$ H

The dc supply voltage is variable from 0 to 120 V both to the field and armature, independently. Draw the torque–speed characteristics of the dc motor if the armature and field currents are not allowed to exceed their rated values. The rated flux is obtained when the field voltage is 120 V. Assume that the field voltage can be safely taken to a minimum of 12 V only.

Solution **(i)** Calculation of rated values:

$$\text{Rated speed, } \omega_{mr} = \frac{2\pi N}{60} = \frac{2\pi \times 1313}{60} = 137.56 \text{ rad/sec}$$

$$\text{Rated torque, } T_{er} = \frac{\text{Output power}}{\text{Rated speed}} = \frac{2.625 \times 745.6}{137.56} = 14.23 \text{ N·m}$$

$$\text{Rated armature current, } I_{ar} = \frac{\text{Rated torque}}{K_b} = \frac{14.23}{0.764} = 18.63 \text{ A}$$

$$\text{Rated field current, } I_{fr} = \frac{V_{fr}}{R_f} = \frac{120}{100} = 1.2 \text{ A}$$

(ii) Calculation of torque–speed characteristics:

Case (a) Constant-flux/torque region:

$$e_1 = V_{max} - I_{ar}R_a = 120 - 18.63 \times 0.8 = 105.1$$

$$\omega_{m1} = \frac{e_{m1}}{K_b} = \frac{105.1}{0.764} = 137.56 \text{ rad/sec.}$$

$$\omega_{m1n} = \frac{\omega_{m1}}{\omega_{mr}} = \frac{137.56}{137.56} = 1.0 \text{ p.u.}$$

Hence, constant rated torque is available from 0 to 1.0 p.u. speed.

Case (b) Field-weakening region:

For 1 p.u. armature current, the maximum induced emf is

$$e_n = \frac{e_1}{e_r} = \frac{105.1}{105.1} = 1.0 \text{ p.u.}$$

To maintain this induced emf in the field-weakening region,

$$\phi_{fn} = \frac{e_n}{\omega_{mn}} = \frac{1.0}{\omega_{mn}} \text{ p.u.}$$

If the range of field variation is known, the maximum speed can be computed as follows:

$$I_{f\,min} = \frac{V_{f(min)}}{R_f} = \frac{12}{100} = 0.12 \text{ A}$$

1.2 A of field current corresponds to rated field flux and hence 0.12 A corresponds to $0.1\phi_{fr}$, and hence

$$0.1 \text{p.u.} < \phi_{fn} < 1 \text{ p.u.}$$

$$\omega_{max} = \frac{1}{0.1} = 10 \text{ p.u.}$$

For various speeds between 1 and 10 p.u., the field flux is evaluated from the equation as

$$\phi_{fn} = \frac{1}{\omega_{mn}} \text{ in p.u.}$$

$$T_{en} = \phi_{fn} \text{ for } I_{an} = 1 \text{ p.u.}$$

The torque, power, and flux-vs.-speed plots are shown in Figure 3.2.

Example 3.2

Consider the dc motor given in Example 3.1, and draw the intermittent characteristics if the armature current is allowed to be 300% of rated value.

Solution **(i)** Constant-flux/torque region

$$I_{max} = 3I_{ar}$$
$$T_{em} = \text{maximum torque} = K_b I_{max} = 0.764 \times 3 \times 18.63 = 42.7 \text{ N·m.}$$

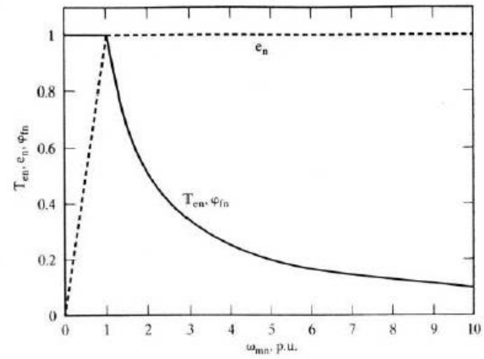


Figure 3.2 Continuous rating of the dc motor

$$T_{en} = \frac{T_{em}}{T_{er}} = \frac{42.7}{14.25} = 3 \text{ p.u.}$$

The maximum induced emf is

$$e_m = V_{max} - I_{max}R_a = 120 - (3 \times 18.63) \times 0.8 = 75.29 \text{ V}$$

Speed corresponding to this induced emf is

$$\omega_{ml} = \frac{e_m}{K_b} = \frac{75.29}{0.764} = 98.54 \text{ rad/sec}$$

$$\omega_{mln} = \frac{98.54}{137.56} = 0.716 \text{ p.u.}$$

Beyond this speed, field weakening is performed.

(ii) Field-weakening region:

$$I_{max} = 3I_{ar}$$
$$e_m = 75.29 \text{ V}$$
$$e_n = \frac{e_m}{105.1} = \frac{75.29}{105.1} = 0.716 \text{ p.u.}$$
$$\omega_{mn} = \frac{e_n}{\phi_{fn}} = \frac{0.716}{\phi_{fn}} \text{ p.u.}$$

The range of the normalized field flux is

$$0.1 < \phi_{fn} < 1$$

The maximum normalized speed is $\omega_{mn} = \dfrac{0.716}{\phi_{fn(min)}} = \dfrac{0.716}{0.1} = 7.16$ p.u.

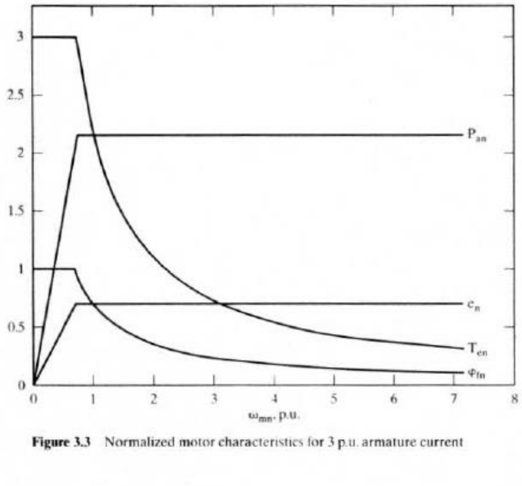


Figure 3.3 Normalized motor characteristics for 3 p.u. armature current

$$T_{en} = \phi_{fn} \text{ per rated current} = 3\phi_{fn} \text{ in the present case.}$$

The intermittent characteristics are drawn from the above equations and are shown in Figure 3.3.

Example 3.3

Consider a motor drive with $R_{an} = 0.1$ p.u., $\phi_{fn} = 1$ p.u., $V_n = 1.1$ p.u. and extreme load operating points $T_{el(min)} = 0.1$ p.u., $\omega_{mn(min)} = \omega_{mn1} = 0.1$ p.u., $T_{e2(max)} = 1$ p.u., and $\omega_{mn(max)} = \omega_{mn2} = 1$ p.u.

(i) Find the normalized control voltages to meet these operating points.

(ii) Compute the change in control voltages required for a simultaneous change of $\Delta T_{en} = 0.02$ p.u. and $\Delta\omega_{mn} = 0.01$ p.u. for both the extreme operating points. From this, calculate the resolution required for the control voltage.

Solution Assume that the controller is linearized.

$$\therefore \alpha = \cos^{-1}\left\{\frac{v_c}{V_{cm}}\right\} = \cos^{-1}\{V_{cn}\}$$

from which the electromagnetic torque is,

$$T_{en} = \left[\frac{1.35V_nV_{cn} - \Phi_{fn}\omega_{mn}}{R_{an}}\right]\Phi_{fn},\ \text{p.u.}$$

where V_{cn} is the normalized control voltage for a given steady-state operating point and is obtained as

$$V_{cn} = \frac{T_{en}R_{an} + \phi_{fn}\omega_{mn}}{1.35V_n}$$

Since $\phi_{fn} = 1$ p.u., the control voltage for minimum torque and speed is

$$V_{cn1} = \frac{T_{en1}R_{an} + \phi_{fn}\omega_{mn1}}{1.35V_n} = \frac{0.1 * 0.1 + 0.1}{1.35 * 1.1} = 0.074 \text{ p.u.}$$

Similarly for maximum torque and speed, the control voltage is

$$V_{cn2} = \frac{T_{en2}R_{an} + \phi_{fn}\omega_{mn2}}{1.35V_n} = \frac{1 * 0.1 + 1}{1.35 * 1.1} = 0.74 \text{ p.u.}$$

Incremental control voltage generates incremental torque and speed as

$$V_{cn} + \delta v_{cn} = \frac{R_{an}(T_{en} + \delta T_{en}) + \omega_{mn} + \delta\omega_{mn}}{1.35V_n}$$

For both changes, $\delta v_{cn} = \dfrac{R_{an}\Delta T_{en} + \delta\omega_{mn}}{1.35V_n}$

Dividing δV_{cn} by V_{cn} gives an expression in terms of steady-state operating points as

$$\frac{\delta v_{cn}}{V_{cn}} = \frac{R_{an}\delta T_{en} + \delta\omega_{mn}}{R_{an}T_{en} + \omega_{mn}}$$

$_n = 0.02$ p.u., $\delta\omega_{mn} = 0.01$ p.u., $T_{en1} = 0.1$ p.u., $\omega_{mn1} = 0.1$ p.u., $T_{en2} = 1$ p.u., $\omega_{mn2} = 1$ p.u.

For T_{en1}, ω_{mn1}: $\dfrac{\delta v_{cn}}{V_{cn}} = \dfrac{0.1 * 0.02 + 0.01}{0.1 * 0.1 + 0.1} = 0.109$

For T_{en2}, ω_{mn2}: $\dfrac{\delta v_{cn}}{V_{cn}} = \dfrac{0.1 * 0.02 + 0.01}{0.1 * 1 + 1} = 0.0109$

Therefore, the resolution required in control voltage is

$$\delta V_{cn} = 0.109 * V_{cn} = 0.109 * 0.074 = 0.008066 \text{ p.u.}$$

Example 3.4

A separately-excited dc motor has 0.05 p.u. resistance and is fed from a three-phase converter. The normalized voltage and field flux are 1 p.u. Draw the torque–speed characteristics in the first quadrant for constant delay angles of 0, 30, 45, and 60 degrees. Indicate the safe operating region if the maximum torque limit is 2.5 p.u.

Solution

$$T_{en} = \frac{[1.35V_n \cos\alpha - \Phi_{fn}\omega_{mn}]}{R_{an}}\Phi_{fn}, \text{ p.u.}$$

Substituting the given values yields

$$T_{en} = 20[1.35\cos\alpha - \omega_{mn}], \text{ p.u.}$$

The torque–speed characteristics for various angles of delay are shown in Figure 3.21. The safe operating region is shaded in the figure.

Example 3.5

The details and parameters of a separately-excited dc machine are

100 hp, 500 V, 1750 rpm, 153.7 A, $R_a = 0.088\ \Omega$, $L_a = 0.00183$ H, $K_b = 2.646$ V/(rad/sec)

The machine is supplied from a three-phase controlled converter whose ac input is from a three-phase 415 V, 60 Hz utility supply. Assume that the machine is operating at 100 rpm with a triggering angle delay of 65°. Find the maximum air gap torque ripple at this operating point.

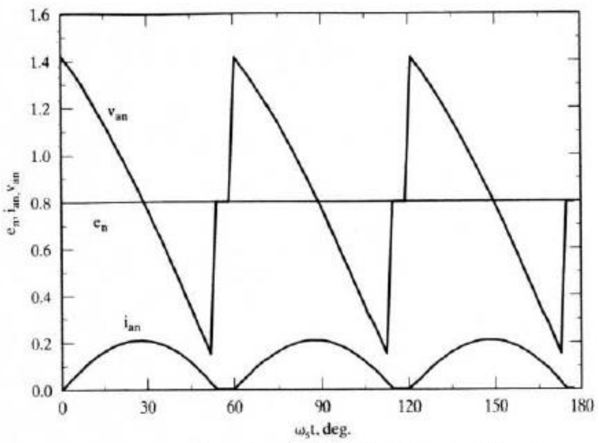


Figure 3.25 The armature current, applied voltage, and induced emf for a typical discontinuous conduction

Solution To find the current ripple, it is essential to determine whether the current is continuous for the given triggering delay by evaluating the critical triggering angle. It is found as follows:

Rotor speed, $\omega_m = \frac{2\pi N_r}{60} = \frac{2\pi * 100}{60} = 10.48$ rad/sec

Induced emf, $E = K_b\omega_m = 2.646 * 10.48 = 27.7$ V

Peak input voltage, $V_m = \sqrt{2} * 415 = 586.9$ V

Input angular frequency, $\omega_s = 2\pi f_s = 2\pi * 60 = 376.99$ rad/sec

Armature time constant, $T_a = \frac{L_a}{R_a} = \frac{0.00183}{0.088} = 0.0208$ sec

Machine impedance, $Z_a = \sqrt{R_a^2 + \omega_s^2 L_a^2} = 0.6955\Omega$

Machine impedance angle, $\beta = \tan^{-1}\left(\frac{\omega_s L_a}{R_a}\right) = 1.444$ rad

The critical triggering angle is

$$\alpha_c = \beta + \cos^{-1}\left\{\frac{E/V_m}{c_1}\cdot\frac{1}{\cos\beta}\cdot(1 - e^{-(\pi/3\tan\beta)})\right\} - \frac{\pi}{3} + \theta_1$$

where

$$a_1 = \frac{\sqrt{3}}{2} = 0.866$$

$$b_1 = \frac{1}{2} - e^{-\left(\frac{\pi}{3\tan\beta}\right)} = -0.375$$

$$c_1 = \sqrt{a_1^2 + b_1^2} = 0.9437$$

$$\theta_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right) = -0.4086 \text{ rad}$$

from which the critical angle is obtained as $\alpha_c = 1.5095$ rad $= 86.48°$. The triggering angle α is 65°, which is less than the critical triggering angle; therefore, the armature current is continuous. Having determined that the drive system is in continuous mode of conduction, we use the relevant equations to calculate the initial current, given by

$$i_{ai} = \frac{\left(\frac{V_m}{|Z_a|}\right)\left\{\sin\left(\frac{2\pi}{3} + \alpha - \beta\right) - \sin\left(\frac{\pi}{3} + \alpha - \beta\right)e^{-\left(\frac{\pi}{\omega_s T_a}\right)}\right\} - \frac{E}{R_a}\left(1 - e^{-\left(\frac{\pi}{\omega_s T_a}\right)}\right)}{1 - e^{-\frac{\pi}{\omega_s T_a}}} = 2308.1 \text{ A}$$

The peak armature current is found by having $\omega_s t = \pi/6$, i.e., at the midpoint of the cycle. This is usually the case, but the operating point can shift it beyond 30°; therefore, it is necessary to verify graphically or analytically where the maximum current occurs and then substitute that instant to get the peak armature current from the following equation:

$$i_a(t) = \left(\frac{V_m}{|Z_a|}\right)\{\sin(\omega_s t + \pi/3 + \alpha - \beta) - \sin(\pi/3 + \alpha - \beta)e^{-t/T_a}\} - \left(\frac{E}{R_a}\right)(1 - e^{-t/T_a})$$
$$+ i_{ai}e^{-t/T_a} = 2411.5 \text{ A}$$

The armature current ripple magnitude, $\Delta i_a = 2411.5 - 2308.1 = 103.4$ A

The ripple torque magnitude, $\Delta T_e = K_b \Delta i_a = 2.646 * 103.4 = 273.86$ N·m

Average air gap torque, $T_{e(av)} = I_{av}K_b \cong [(2411.5 + 2308.1) * 0.5] * 2.646 = 6244$ N·m

Note that the ripple current magnitude is less than 5% and therefore is approximated as a straight line between its minimum and maximum values in each part of its cycle.

Torque ripple as a percent of the operating average torque is

$$\Delta T_{en} = \frac{\Delta T_e}{T_{e(av)}} * 100 = \frac{273.86}{6244} * 100 = 4.4\%$$