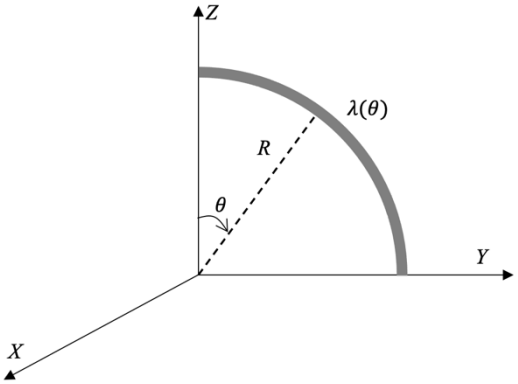
YTÜ - Fen-Edebiyat Fakültesi, Sınav Soru ve Cevap Kağıdı				NOT TABLOSU		
				1. S	2. S	TOPLAM
Adı Soyadı						
Öğrenci Numarası		Grup No	1, 2			
Bölümü				Sınav Tarihi	25.11.2020	
Dersin Adı	KLASİK EMT I I. ARA SINAV-1	Sınav Süresi	40 dak.	Sınav Yeri		
Dersi veren Öğretim Üyesinin Adı Soyadı	Prof. Dr. Çetin TAŞSEVEN, Prof. Dr. Zeynel YALÇIN			İmza		

YÖK nun 2547 sayılı Kanunun *Öğrenci Disiplin Yönetmeliğinin* 9. Maddesi olan *"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"* fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.

QUESTIONS

1.a) Quarter of a circle of radius R on zy plane has non uniform linear charge density given by $\lambda(\theta) = \lambda_0 Cos(\theta)$, where λ_0 is a constant. Find the electric field vector $\vec{E}$ on the x-axis.



1.b) Electrostatic potential of a charge distribution in spherical coordinates given as $V(r,\phi) = -Aln(r) - BSin(\phi)$, where A and B are constants.

i) Find the corresponding electrostatic field vector $\vec{E}(r,\theta,\phi)$

ii) Is this a legitimate electric field?

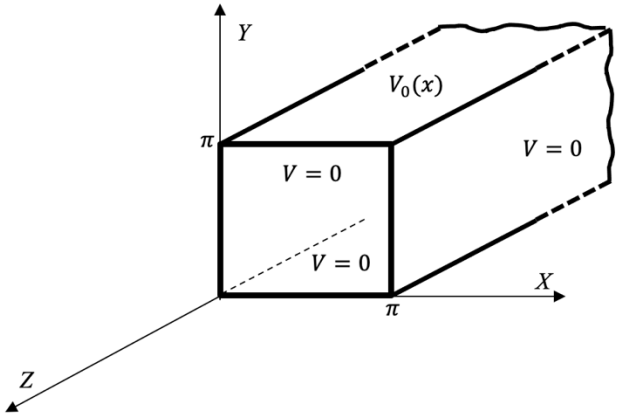
iii) Find the charge q inside a sphere of radius R with the center at the origin.

YTÜ - Fen-Edebiyat Fakültesi, Sınav Soru ve Cevap Kağıdı				NOT TABLOSU		
				1. S	2. S	TOPLAM
Adı Soyadı						
Öğrenci Numarası		Grup No	1, 2			
Bölümü				Sınav Tarihi	25.11.2020	
Dersin Adı	KLASİK EMT I I. ARA SINAV-1	Sınav Süresi	40 dak.	Sınav Yeri		
Dersi veren Öğretim Üyesinin Adı Soyadı	Prof. Dr. Çetin TAŞSEVEN, Prof. Dr. Zeynel YALÇIN			İmza		

YÖK nun 2547 sayılı Kanunun *Öğrenci Disiplin Yönetmeliğinin* 9. Maddesi olan *"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"* fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.

QUESTIONS

Q 2. A square pipe with cross sectional dimensions $\pi x \pi$, running parallel to the z−axis (from $-\infty$ to ∞), has three grounded metal sides, at $y = 0$, $x = 0$, and $x = \pi$. The fourth side, at $y = \pi$, is maintained at a specific potential $V_0(x) = Sinx + Sin2x$. Find the potential inside the pipe that satisfies the boundary conditions below:

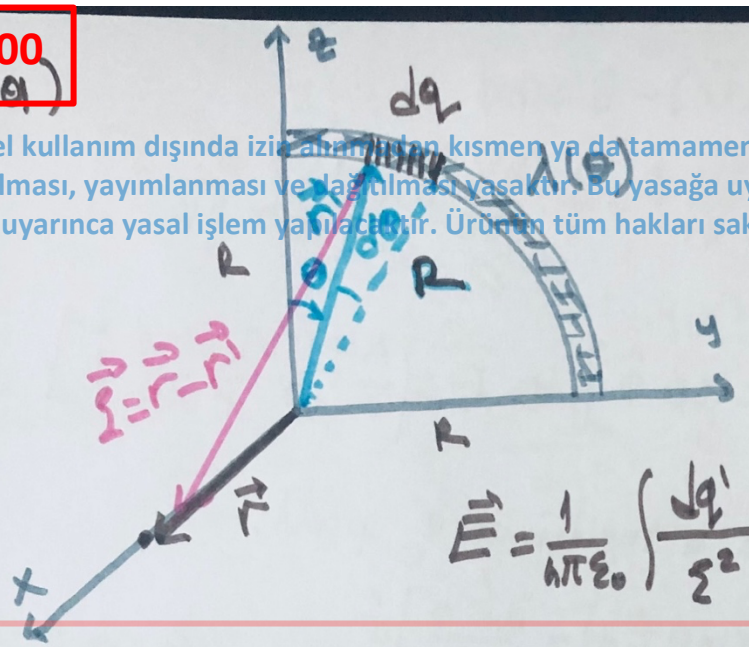


i. $V(0, y) = 0$; *ii.* $V(\pi, y) = 0$;

iii. $V(x, 0) = 0$; *iv.* $V(x, \pi) = V_0(x) = Sinx + Sin2x$

25/100

1-a)



$$dq = \lambda\, dl$$

$$= \lambda_0 \cos\theta\, R\, d\theta$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{\xi^2}\,\hat{\xi}$$

$$\boxed{\vec{\xi} = \vec{r}-\vec{r}' = x\hat{x} - (R\sin\theta\,\hat{y} + R\cos\theta\,\hat{z})}$$ 4

$$\hat{\xi} = \frac{\vec{\xi}}{\xi} = \frac{\vec{\xi}}{(x^2+R^2)^{1/2}}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq\,\vec{\xi}}{\xi^3} = \boxed{\frac{1}{4\pi\varepsilon_0}\int_0^{\pi/2} \frac{\lambda_0 \cos\theta\, R\, d\theta}{(x^2+R^2)^{3/2}}\left(x\hat{x} - R\sin\theta\,\hat{y} - R\cos\theta\,\hat{z}\right)}$$ 5

$$= \frac{\lambda_0 R}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}}\left\{ x\int_0^{\pi/2}\cos\theta\, d\theta\,\hat{x} - R\underbrace{\int_0^{\pi/2}\cos\theta\sin\theta\, d\theta}_{\substack{\cos\theta = u \\ -\sin\theta\, d\theta = du}}\,\hat{y} - R\underbrace{\int_0^{\pi/2}\cos^2\theta\, d\theta}_{\cos^2\theta = \frac{1+\cos 2\theta}{2}}\,\hat{z}\right\}$$

$$= \frac{\lambda_0 R}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}}\left\{ x\sin\theta\Big|_0^{\pi/2}\hat{x} + R\,\frac{\cos^2\theta}{2}\Big|_0^{\pi/2}\hat{y} - R\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)\Big|_0^{\pi/2}\hat{z}\right\}$$

3 4 5

$$\boxed{= \frac{\lambda_0 R}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}}\left\{ x\hat{x} - \frac{R}{2}\hat{y} - \frac{R}{4}\pi\,\hat{z}\right\}}$$ 4

1-b

25/100

$$V(r,\phi) = -A\ln(r) - B\sin\phi$$

$$\vec{E} = -\vec{\nabla}V(r,\phi) = -\left[\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\cancel{\frac{\partial V}{\partial \theta}}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}\right]$$

0

3 3

$$= +\left[+\frac{A}{r}\hat{r} + \frac{B}{r\sin\theta}\cos\phi\,\hat{\phi}\right] \Rightarrow \boxed{\vec{E} = \left[\frac{A}{r}\hat{r} + \frac{B\cos\phi}{r\sin\theta}\hat{\phi}\right]}$$

if $\vec{\nabla}\times\vec{E} = 0$ it is a logitimate field.

6

$$\vec{\nabla}\times\vec{E} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, E_\phi) - \cancel{\frac{\partial E_\theta}{\partial\phi}}\right]\hat{r}$$

0

$$\times\ \frac{B\sin\phi}{r\sin\theta} = \frac{B\sin\phi}{\sin\theta}$$

$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\cancel{\frac{\partial E_r}{\partial\phi}} - \cancel{\frac{\partial}{\partial r}(r E_\phi)}\right]\hat{\theta}$$

0 0

$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(r\cancel{E_\theta}) - \cancel{\frac{\partial E_r}{\partial\theta}}\right]\hat{\phi}$$

0

$$= \frac{1}{r\sin\theta}\left[\underbrace{\frac{\partial}{\partial\theta}\cancel{\sin\theta}\,\frac{B\cos\phi}{r\cancel{\sin\theta}}}_{0}\hat{r} - \underbrace{\frac{\partial}{\partial r}\cancel{r}\,\frac{B\cos\phi}{r\sin\theta}}_{0}\hat{\phi}\right] = 0$$

$$q = \int\rho(r')\,d\tau' \qquad \vec{\nabla}\cdot\vec{E} = \frac{\rho(r)}{\varepsilon_0} \Rightarrow \rho(r) = \varepsilon_0\,\vec{\nabla}\cdot\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\underbrace{r^2 E_r}_{Ar} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\cancel{E_\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}E_\phi$$

0

4

$$= \frac{A}{r^2} - \frac{B\sin\phi}{r^2\sin\theta} \Rightarrow$$

4

$$q = \int\varepsilon_0\left[\frac{A}{r^2} - \frac{B\sin\phi}{r^2\sin\theta}\right]r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$q = \varepsilon_0\int_0^R\int_0^\pi\int_0^{2\pi}\left(A\sin\theta\,dr\,d\theta\,d\phi - \cancel{B\sin\phi\,dr\,d\theta\,d\phi}\right) \qquad \int_0^{2\pi}\sin\phi\,d\phi = 0$$

0

5

$$q = 4\pi\varepsilon_0 A R$$

2 BCs I) $x=0$ $V=0$ II) $x=\pi$ $V=0$

50/100

III) $y=0$ $V=0$ IV) $y=\pi$ $V=\sin x+\sin 2x$

$\nabla^2 V(x,y,z)=0$ $V(x,y,z)=X(x)\,Y(y)$

$$\frac{1}{X}\frac{d^2X}{dx^2}+\frac{1}{Y}\frac{d^2Y}{dy^2}=0$$

$=-k^2$ $=+k^2$

5 $X(x)=A\sin kx+B\cos kx$ $Y(y)=Ce^{ky}+De^{-ky}$

I. BC $\Rightarrow$ 3 $B=0$ II. BC 3 $k\pi=n\pi$ $\Rightarrow k=n=1,2,3,\dots$

III. BC $\Rightarrow Ce^0+De^0=0 \Rightarrow$ 4 $C=-D$

$$V(x,y)=A\sin nx\left(e^{ny}-e^{-ny}\right)=\sum_{n=1}^{\infty}C_n\sin nx\left(e^{ny}-e^{-ny}\right)$$ 4

or $$V(x,y)=\sum_{n=1}^{\infty}C_n\sin nx\sinh ny$$

IV BC 3 $$\sum_{n=1}^{\infty}C_n\sin nx\sinh n\pi=\sin x+\sin 2x$$

$$\sum_{n=1}^{\infty}C_n\sinh n\pi\int_0^{\pi}\sin nx\sin n'x\,dx=\int\sin x\sin n'x\,dx+\int\sin 2x\sin n'x\,dx$$

$C_{n'}$ $n=n'\Rightarrow\frac{\pi}{2}$ 3 solution exists only for $n'=1$ 3 exists only for $n'=2$

$n=n'=1$

$C_{n'}\sinh(\pi)\frac{\pi}{2}=\frac{\pi}{2}+0\Rightarrow$ 5 $C_1=\frac{1}{\sinh\pi}$

$n=n'=2$

$C_2\sinh(2\pi)\frac{\pi}{2}=0+\frac{\pi}{2}$ 5 $C_2=\frac{1}{\sinh 2\pi}$

5 $V(x,y)=\frac{1}{\sinh\pi}\sin x\sinh y+\frac{1}{\sinh 2\pi}\sin 2x\sinh 2y$

	YTÜ - Fen-Edebiyat Fakültesi FİZİK Bölümü			NOT TABLOSU		
				1. S	2. S	TOPLAM
Adı Soyadı						
Öğrenci Numarası		Grup No	1, 2			
Bölümü				Sınav Tarihi	16.11.2020	
Dersin Adı	Classical Electromagnetic Theory 1 2dn Midterm	Sınav Süresi	100 dak.	Sınav Yeri		
Dersi veren Öğretim Üyesinin Adı Soyadı	Prof. Dr. Çetin TAŞSEVEN, Prof. Dr. Zeynel YALÇIN			İmza		

YÖK nun 2547 sayılı Kanunun *Öğrenci Disiplin Yönetmeliğinin* 9. Maddesi olan *"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"* fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.

QUESTIONS

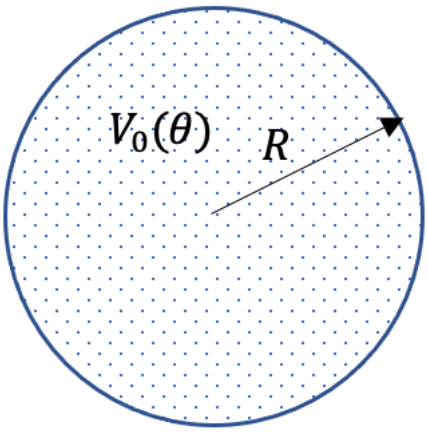
Q1. A sphere of radius R has the potential on its surface given by $V_0(\theta) = 2Cos^2\theta - Sin^2\theta$.Using the azimuthal symmetric solution of the Laplace equation in spherical coordinate:

a) find the potential $V(r,\theta)$ inside $r \leq R$ and outside $r \geq R$ the sphere,

b) find the surface charge density $\sigma(\theta)$ of the sphere,

c) find the total charge of the sphere, and

d) find the electric field inside $r \leq R$ and outside the sphere $r \geq R$.

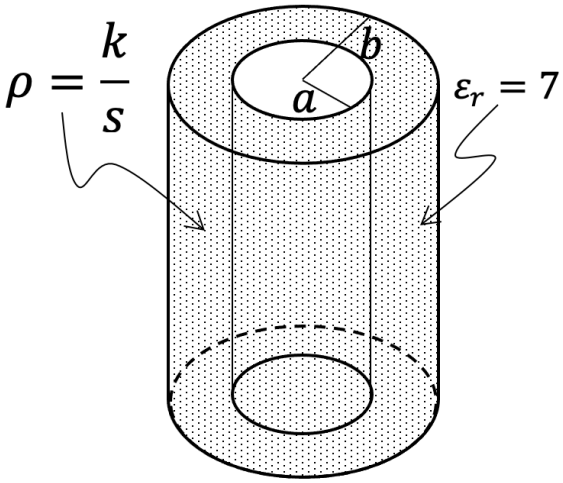


Q2. A long insulating cylindrical shell of inner radius a and outer radius b with relative permittivity $\varepsilon_r = 7$ has non uniform charge density given by $\rho = \frac{k}{s}$, where k is a constant.

a) Find the electric displacement $\vec{D}$ and electric field $\vec{E}$ at the regions of $s < a$, $a < s < b$ and $b < s$.

b) Find the bound charge densities (ρ_b, σ_b^{inner}, σ_b^{outer}, σ_b^{top}, σ_b^{bottom})

c) Taking the potential zero at the central axis $V(s = 0) = 0$ find the potential on the outer surface of the material.



d) $V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$

* $r=0$. $B_l = 0$ (2) $V(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l \quad r \leq R$

* $r \to \infty$ $A_l = 0$ (2) $V(r,\theta) = \sum_{l=0}^{\infty} B_l r^{-(l+1)} P_l \quad r \geq R$

* $r=R$ $V_{r\leq R} = V_{r \geq R} \Rightarrow$ (2) $\boxed{A_l R^l = B_l R^{-(l+1)}}$

* at $r=R$ $V_0(R,\theta) = 2\cos^2\theta - \sin^2\theta = 3\cos^2\theta - 1$
$= 2P_2(\cos\theta)$

$3\cos^2\theta - 1 = \sum A_l R^l P_l$ (2)

$\int P_{l'} 2 P_2 \, d(\cos\theta) = \sum A_l R^l \int P_l P_{l'} \, d(\cos\theta)$

if $l' \neq 2$ no solution

$2\int P_2 P_{l'} \, d(\cos\theta) = A_{l'} R^l \int P_l P_{l'} \, d(\cos\theta)$

$l = l' = 2$

$2 \frac{2}{2\times 2+1} = A_{l'} R^2 \frac{2}{2\times 2+1} \Rightarrow \boxed{A_2 = 2R^{-2}}$ (3)

$2R^{-2} R^2 = B_2 R^{-(3)}$

$\boxed{B_2 = 2R^3}$ (3)

$V(r,\theta) = 2R^{-2} r^2 P_2 = 2\left(\frac{r}{R}\right)^2 (3\cos^2\theta - 1) \quad r \leq R$ (3)

$V(r,\theta) = 2R^3 r^{-(3)} P_2 = \left(\frac{R}{r}\right)^3 (3\cos^2\theta - 1) \quad r \geq R$ (3)

b)

$$\sigma(\theta) = -\varepsilon_0 \left(\frac{\partial V_{out}}{\partial n} - \frac{\partial V_{in}}{\partial n} \right) \Bigg|_{r=R}$$

1 2

$$= -\varepsilon_0 \left(\frac{R^3(-3)}{r^4} - \frac{2r}{R^2} \right) \Bigg|_{r=R} (3\cos^2\theta - 1)$$

$$= -\varepsilon_0 \left(-\frac{3R^3}{R^4} - \frac{2R}{R^2} \right) (3\cos^2\theta - 1)$$

4

$$\boxed{\sigma(\theta) = \frac{5\varepsilon_0}{R}(3\cos^2\theta - 1)}$$

c)

$$Q = \int \sigma(\theta)\, da = \int_0^{\pi}\int_0^{2\pi} \frac{5\varepsilon_0}{R}(3\cos^2\theta - 1) R^2 \sin\theta\, d\theta\, d\phi$$

2

$$= 5\varepsilon_0 R \int_0^{2\pi} d\phi \left[\int_0^{\pi} 3\cos^2\theta \sin\theta\, d\theta - \int_0^{\pi} \sin\theta\, d\theta \right]$$

$\cos\theta = u$

$-\sin\theta\, d\theta = du$

3 3

$$= 10\pi\varepsilon_0 R \left[-\cos^3\theta + \cos\theta \right]_0^{\pi} \boxed{= [redacted] = 0}$$

2

d)

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} - \frac{1}{r\sin\theta}\cancel{\frac{\partial V}{\partial \phi}}^{\,0}\hat{\phi}$$

$r \leq R \quad \vec{E} = -\frac{2r}{R^2}(3\cos^2\theta - 1)\hat{r} - \frac{r}{R^2}(-6\cos\theta\sin\theta)\hat{\theta}$

2 3

$r \geq R \quad \vec{E} = +3\frac{R^3}{r^4}(3\cos^2\theta - 1)\hat{r} - \frac{R^3}{r^4}(-6\cos\theta\sin\theta)\hat{\theta}$

2 3

a) 24

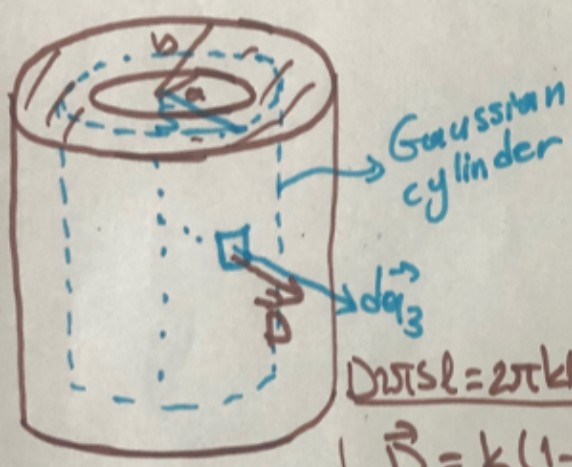
$s<a$; $q_f = 0$; $\vec{D} = 0$ (2) ; $\vec{E} = 0$ (2)

$a<s<b$; $q_f = \int_0^{2\pi}\int_a^{s}\int_0^{l} \frac{k}{s}\, s\, d\phi\, ds\, dz = 2\pi k l (s-a)$ (3)

$\oint \vec{D}\cdot d\vec{a} = q_f$

$\underbrace{\int \vec{D}\cdot d\vec{a}_1}_{0} + \underbrace{\int \vec{D}\cdot d\vec{a}_2}_{0} + \int \vec{D}\cdot d\vec{a}_3$

$= D\, 2\pi s l$



$D 2\pi s l = 2\pi k l (s-a) \Rightarrow \vec{D} = \frac{k(s-a)}{s}\hat{s}$

$\boxed{\vec{D} = k\left(1-\frac{a}{s}\right)\hat{s}}$ (4) $\quad \vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r}$

$\boxed{\vec{E} = \frac{k}{\varepsilon_0}\left(1-\frac{a}{s}\right)\hat{s}}$ (3)

$s>b$ $\quad q_f = \int_0^{2\pi}\int_a^{b}\int_0^{l} \frac{k}{s}\, s\, d\phi\, ds\, dz = 2\pi k l (b-a)$ (3)

$\oint \vec{D}\cdot d\vec{a} = \ldots = D 2\pi s l = 2\pi k l (b-a) \Rightarrow \boxed{\vec{D} = \frac{k(b-a)}{s}\hat{s}}$ (4)

$\boxed{\vec{E} = \frac{k(b-a)}{\varepsilon_0 s}\hat{s}}$ (3)

b) $\sigma_b = \vec{P}\cdot\hat{n}$ $\quad \rho_b = -\vec{\nabla}\cdot\vec{P}$ $\quad \vec{P} = \vec{D} - \varepsilon_0\vec{E}$

10

$$\vec{P} = k\left(1-\frac{a}{s}\right)\hat{s} - \varepsilon_0\frac{k}{7\varepsilon_0}\left(1-\frac{a}{s}\right)\hat{s} = \frac{6}{7}k\left(1-\frac{a}{s}\right)\hat{s} \quad (3)$$

$$\sigma_b^{s=a} = \frac{6}{7}k\left(1-\frac{a}{a}\right)\hat{s}\cdot(-\hat{s}) = 0 \quad (2)$$

$$\sigma_b^{s=b} = \frac{6}{7}k\left(1-\frac{a}{b}\right)\hat{s}\cdot\hat{s} = \frac{6}{7}k\left(1-\frac{a}{b}\right) \quad (3)$$

$$\sigma_b^{top} = \frac{6}{7}k\left(1-\frac{a}{s}\right)\hat{s}\cdot\hat{z} = 0 \quad (2)$$

$$\sigma_b^{bottom} = \frac{6}{7}k\left(1-\frac{a}{s}\right)\hat{s}\cdot(-\hat{z}) = 0 \quad (2)$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = -\frac{1}{s}\frac{\partial}{\partial s}\left[s\left(\frac{6}{7}k\left(1-\frac{a}{s}\right)\right)\right] = -\frac{6}{7}\frac{k}{s} \quad (4)$$

c) 10

$$V(s=0)=0 \Rightarrow \quad V(s=b) - \underbrace{V(s=a)}_{0} = -\int_0^a \underbrace{\vec{E}(s<a)}_{0}\cdot d\vec{s} \quad (2)$$

$$-\int_a^b \vec{E}(a<s<b)\cdot d\vec{s}$$

$$V(s=b) = -\int_a^b \vec{E}(s)\cdot d\vec{s} = -\int_a^b \frac{k}{7\varepsilon_0}\left(1-\frac{a}{s}\right)ds = -\frac{k}{7\varepsilon_0}(b-a) + \frac{ka}{7\varepsilon_0}\ln\frac{b}{a}$$

(2) (4)

	YTÜ - Fen-Edebiyat Fakültesi FİZİK Bölümü			NOT TABLOSU				
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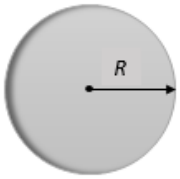
Adı Soyadı								
Öğrenci Numarası		Grup No	1, 2					
Bölümü				Sınav Tarihi		30.12.2020		
Dersin Adı	Klasik EMT 3. ARA SINAVI			Sınav Süresi	120 dak.	Sınav Yeri		
Dersi veren Öğretim Üyesinin Adı Soyadı	Prof. Dr. Çetin TAŞSEVEN, Prof. Dr. Zeynel YALÇIN					İmza		

YÖK nun 2547 sayılı Kanunun *Öğrenci Disiplin Yönetmeliğinin* 9. Maddesi olan *"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"* fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.

QUESTIONS

Q 1. A spherical linear dielectric material with electric susceptibility χ_e and radius R has a total charge of Q with non uniform charge density given by $\rho = \frac{k}{r}$ where k is a constant.

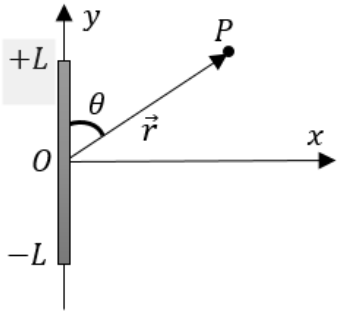


a) Find the electric displacement $\vec{D}$ inside $(r \leq R)$ and outside $(r \geq R)$ of the sphere.

b) Using $\vec{D}$, find the electric field $\vec{E}$ inside $(r \leq R)$ and outside $(r \geq R)$ of the sphere.

c) Find the energy of the sphere using $W = \frac{1}{2}\int \vec{D} \cdot \vec{E}\, d\tau$.

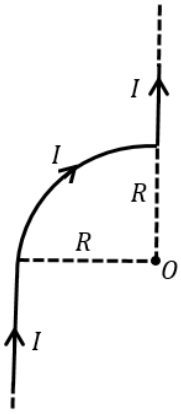
Q 2. A thin rod of length $2L$ with linear charge density $\lambda = \alpha y$, where $\alpha > 0$ is a constant, is placed on y-axis as shown in the figure



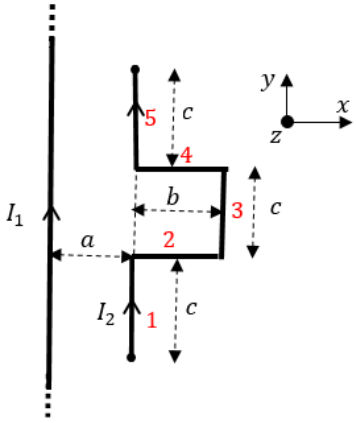
a) Find the monopole and dipole moments of the rod.

b) Find the approximate potential at point P. $V \cong V_{monopol} + V_{dipol}$

Q 3. A very long wire carrying a steady current I is bent into a shape given in the figure. Using Biot-Savart law find the magnetic field vector formed by the wire at the origin $\vec{B}_O$.



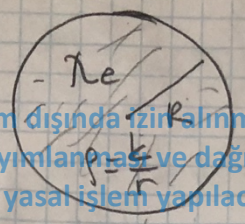
Q **4.** A wire carrying a steady current I_2 consists of five straight parts (1, 2, 3, 4, and 5) and is place next to a very long wire carrying a steady current I_1 as shown in the figure.



a) Find the forces acting on each parts of wire I_2 due to long wire I_1 in terms of unit vectors.

b) Find the total force acting on wire I_2 in terms of unit vectors.

1)



a) $\oint \vec{D} \cdot d\vec{a} = q_{free}$ $\quad r \leq R$

$\oint \vec{D} \cdot d\vec{a} = D 4\pi r^2$

$q_{free} = \int \rho \, d\tau = \int_0^r \frac{k}{r} 4\pi r^2 dr$

$q_{free} = 2\pi k r^2$

$D 4\pi r^2 = 2\pi k r^2 \Rightarrow \boxed{\vec{D} = \frac{k}{2} \hat{r}}$ $\quad r \leq R$ (4)

$r \geq R$ $\quad \oint \vec{D} \cdot d\vec{a} = D 4\pi r^2$ $\quad q_{free} = \int_0^R \frac{k}{r} 4\pi r^2 dr$

$q_{free} = 2\pi k R^2 \Rightarrow D 4\pi r^2 = 2\pi k R^2$

$\boxed{\vec{D} = \frac{k R^2}{2 r^2} \hat{r}}$ (4)

b) $\vec{E} = \frac{\vec{D}}{\varepsilon_0 (1+\chi_e)} \Rightarrow$ (2) $\vec{E} = \frac{k}{2\varepsilon_0 (1+\chi_e)} \hat{r}$ $\quad r \leq R$

$\chi_e = 0$ (2) $\vec{E} = \frac{k R^2}{2\varepsilon_0 r^2} \hat{r}$ $\quad r \geq R$

c) $W = \frac{1}{2} \int \vec{D} \cdot \vec{E} \, d\tau = \frac{1}{2} \int_0^R \left(\frac{k}{2}\right)^2 \frac{1}{\varepsilon_0 (1+\chi_e)} 4\pi r^2 dr$

$+ \frac{1}{2} \int_R^\infty \left(\frac{k R^2}{2 r^2}\right)^2 \frac{1}{\varepsilon_0} 4\pi r^2 dr$ (5)

$W = \frac{\pi k^2}{2\varepsilon_0 (1+\chi_e)} \frac{r^3}{3} \Big|_0^R + \frac{\pi k^2 R^4}{2\varepsilon_0} \left(-\frac{1}{r}\right)\Big|_R^\infty$ (6)

$W = \frac{\pi k^2 R^3}{2\varepsilon_0} \left(\frac{1}{3(1+\chi_e)} + 1\right) = \frac{\pi k^2 R^3 (4+3\chi_e)}{6\varepsilon_0 (1+\chi_e)}$ (2)

2) a) $q = \int \lambda(r')\,dl' = \int_{-L}^{L} \alpha y\,dy = \alpha \frac{y^2}{2}\Big|_{-L}^{L}$ (3)

$q = \frac{\alpha}{2}(L^2 - (-L^2)) = 0$ $\boxed{q = 0}$ (3)

(2)

$P_x = 0 \quad P_z = 0 \quad P_y = \int y\,\rho(\vec{r}')\,d\tau'$

$P_y = \int y\,\lambda(y)\,dy = \int_{-L}^{L} y\alpha y\,dy = \alpha \frac{y^3}{3}\Big|_{-L}^{L}$ (2) (3)

$P_y = \frac{\alpha}{3}\left(L^3 - (-L)^3\right) = \frac{2\alpha L^3}{3}$

$\boxed{\vec{P} = \frac{2\alpha L^3}{3}\hat{y}}$ (2)

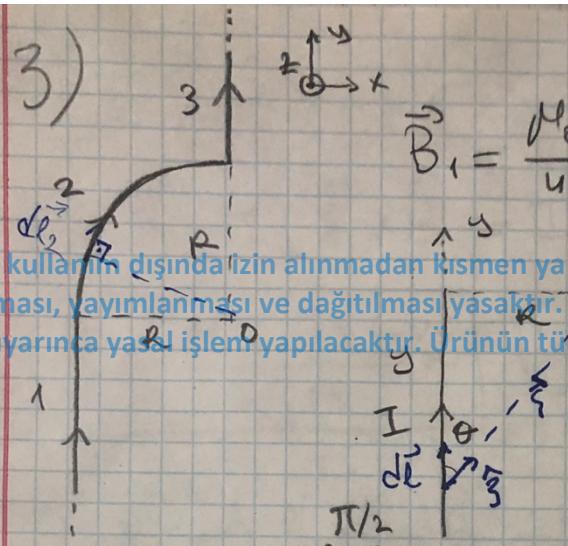
b) $V(\vec{r}) \cong \frac{1}{4\pi\varepsilon_0}\left[\cancel{\frac{q}{r}}^{\,0} + \frac{\vec{P}\cdot\hat{r}}{r^2} + \cdots\right]$

$\hat{r} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{(x^2+y^2+z^2)^{1/2}}$ (5) $\vec{P}\cdot\hat{r} = \frac{2\alpha L^3 y}{3(x^2+y^2+z^2)^{3/2}}$

$V(\vec{r}) \cong \frac{\alpha L^3 y}{6(x^2+y^2+z^2)^{3/2}} = \frac{\alpha L^3 y}{6(x^2+y^2)^{3/2}}$ or (5)

$V(\vec{r}) = \frac{\alpha L r\cos\theta}{6r^3} = \frac{\alpha L\cos\theta}{6r^2}$ polar coordinates

3)



$$\vec{B}_1 = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\hat{\xi}}{\xi^2}$$

$$d\vec{l}\times\hat{\xi} = dl\sin\theta\,(-\hat{k})$$

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi}\int \frac{dl\sin\theta}{\xi^2}(-\hat{k})$$

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi}\int_0^{\pi/2} \frac{\frac{R}{\sin^2\theta}\,d\theta\,\sin\theta}{\frac{R^2}{\sin^2\theta}}(-\hat{k})$$

$$dl = dy \qquad \cot\theta = -\frac{y}{R}$$

$$y = -R\cot\theta$$

$$dy = R\csc^2\theta\,d\theta = \frac{R}{\sin^2\theta}\,d\theta$$

$$\sin\theta = \frac{R}{\xi} \Rightarrow \boxed{\xi = \frac{R}{\sin\theta}}$$

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi R}(-\cos\theta)\Big|_0^{\pi/2}(-\hat{k})$$

$$-\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$\boxed{\vec{B}_1 = -\frac{\mu_0 I}{4\pi R}\hat{k}} \quad 8$$

$$\vec{B}_2 = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}_2\times\hat{\xi}}{\xi^2} = \frac{\mu_0 I}{4\pi}\int \frac{dl_2}{R^2}\sin 90\,(-\hat{k})$$

$$\vec{B}_2 = -\frac{\mu_0 I}{4\pi R^2}R\int_0^{\pi/2} d\theta\,\hat{k} = -\frac{\mu_0 I}{8R}\hat{k} \qquad dl_2 = R\,d\theta$$

$$\boxed{\vec{B}_2 = -\frac{\mu_0 I}{8R}\hat{k}} \quad 8 \qquad \vec{B}_3 = 0 \quad 4$$

$$\vec{B}_0 = -\frac{\mu_0 I}{4R}\left(\frac{1}{\pi} + \frac{1}{2}\right)\hat{k} \quad 5$$

4) $\vec{B} = \frac{\mu_0 I_1}{2\pi x}(-\hat{z})$

① $\vec{F_1} = I_2 \vec{l_1} \times \vec{B} = I_2 c \hat{y} \times \frac{\mu_0 I_1}{2\pi a}(-\hat{z})$

$$\boxed{\vec{F_1} = -\frac{\mu_0 I_1 I_2 c}{2\pi a}\hat{x}}$$ 4

$\hat{x}$ $\hat{y}$
$\hat{z}$

② $\vec{F_2} = I_2 \int d\vec{l_2} \times \vec{B} = I_2 \int_a^{a+b} dx\,\hat{x} \times \frac{\mu_0 I_1}{2\pi x}(-\hat{z})$

$$\boxed{\vec{F_2} = \frac{\mu_0 I_1 I_2}{2\pi} \ln\left|\frac{a+b}{a}\right| \hat{y}}$$ 5

③ $\vec{F_3} = I_2 \vec{l_3} \times \vec{B} = I_2 c \hat{y} \times \frac{\mu_0 I_1}{2\pi(a+b)}(-\hat{z})$

$$\boxed{\vec{F_3} = -\frac{\mu_0 I_1 I_2 c}{2\pi(a+b)}\hat{x}}$$ 4

$dx < 0$

④ $\vec{F_4} = I_2 \int d\vec{l_4} \times \vec{B} = I_2 \int_{a+b}^{a} dx(+\hat{x}) \times \frac{\mu_0 I_1}{2\pi x}(-\hat{z})$ 4

$$\vec{F_4} = \frac{\mu_0 I_1 I_2}{2\pi} \ln\left|\frac{a}{a+b}\right| \hat{y} = \boxed{-\frac{\mu_0 I_1 I_2}{2\pi} \ln\left|\frac{a+b}{a}\right| \hat{y}}$$

⑤ $\boxed{\vec{F_5} = \vec{F_1} = -\frac{\mu_0 I_1 I_2 c}{2\pi a}\hat{x}}$ 4

b) $\vec{F} = \vec{F_1} + \cancel{\vec{F_2}} + \vec{F_3} + \cancel{\vec{F_4}} + \vec{F_5} = -\frac{\mu_0 I_1 I_2 c}{\pi}\left(\frac{1}{a} + \frac{1}{2(a+b)}\right)\hat{x}$

$$\boxed{\vec{F} = -\frac{\mu_0 I_1 I_2 c}{\pi}\left(\frac{3a+2b}{2a(a+b)}\right)\hat{x}}$$ 4