FORMULA SHEET

SPHERICAL COORDINATES $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\phi})$: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + rsin\theta d\phi\hat{\phi}$; $d\tau = r^2 sin\theta dr d\theta d\phi$

$x = r\sin\theta\cos\phi$ $\quad$ $\hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$

$y = r\sin\theta\sin\phi$ $\quad$ $\hat{y} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}$

$z = r\cos\theta$ $\quad$ $\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$r = \sqrt{x^2 + y^2 + z^2}$ $\quad$ $\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$

$\theta = \cos^{-1}(z/r)$ $\quad$ $\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$

$\phi = \tan^{-1}(y/x)$ $\quad$ $\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$

1. $\nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\theta} + \frac{1}{rsin\theta}\frac{\partial t}{\partial\phi}\hat{\phi}$

2. $\nabla\cdot\vec{\mathrm{v}} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{rsin\theta}\frac{\partial}{\partial\theta}(sin\theta v_\theta) + \frac{1}{rsin\theta}\frac{\partial v_\phi}{\partial\phi}$

3. $\nabla\times\vec{\mathrm{v}} = \frac{1}{rsin\theta}\left[\frac{\partial}{\partial r}\left(sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(rv_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rv_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$

4. $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$

CYLINDRICAL COORDINATES $(\boldsymbol{s}, \boldsymbol{\phi}, \boldsymbol{z})$: $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$; $d\tau = sdsd\phi dz$

$x = s\cos\phi$ $\quad$ $\hat{x} = \cos\phi\,\hat{s} + \sin\phi\,\hat{\phi}$

$y = s\sin\phi$ $\quad$ $\hat{y} = -\sin\phi\,\hat{s} + \cos\phi\,\hat{\phi}$

$z = z$ $\quad$ $\hat{z} = \hat{z}$

$s = \sqrt{x^2 + y^2}$ $\quad$ $\hat{s} = \cos\phi\hat{x} + \sin\phi\,\hat{y}$

$\phi = \tan^{-1}(y/x)$ $\quad$ $\hat{\theta} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$

$z = z$ $\quad$ $\hat{z} = \hat{z}$

1. $\nabla t = \frac{\partial t}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z}$

2. $\nabla\cdot\vec{\mathrm{v}} = \frac{1}{s}\frac{\partial}{\partial s}(sv_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$

3. $\nabla\times\vec{\mathrm{v}} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}\left(sv_\phi\right) - \frac{\partial v_s}{\partial\phi}\right]\hat{z}$

4. $\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$

Divergence Theorem: $\int_{\mathcal{V}}(\nabla\cdot\vec{\mathrm{v}})d\tau = \oint_{\mathcal{S}}\vec{\mathrm{v}}\cdot d\vec{a}$ **Rotational (Stokes) Theorem:** $\int_{\mathcal{S}}(\nabla\times\vec{\mathrm{v}})\cdot d\vec{a} = \oint_{\mathcal{P}}\vec{\mathrm{v}}\cdot d\vec{l}$

Orthogonality Relation: $\int_0^a \sin kx\sin nx\,dx = \begin{cases} 0 \to If\ k \neq n \\ \frac{a}{2} \to If\ k = n \end{cases}$; $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$

$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\int_{\mathcal{V}}\frac{\rho}{r^2}\hat{r}d\tau$; $\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}Q_{enc.}$; $\nabla\cdot\vec{\mathrm{E}} = \frac{1}{\varepsilon_0}\rho$; $V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$

$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$; $\int_0^\pi P_l(\cos\theta)\,P_m(\cos\theta)\ sin\theta d\theta = \begin{cases} 0 \to If\ l \neq m \\ \frac{2}{2m+1} \to If\ l = m \end{cases}$

$P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \dfrac{1}{2}(3x^2 - 1)$, $P_3(x) = \dfrac{1}{2}(5x^3 - 3x)\cdots$ $\quad$ *here,* $x = \cos\theta$.

$$V_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

Energy: $\mathrm{W} = \frac{1}{2}\sum_{i=1}^{n} q_i \mathrm{V}(\mathbf{r_i})$; $\mathrm{W} = \frac{\varepsilon_0}{2}\int_{All\ space} \mathrm{E}^2 \mathrm{d}\tau$; $\mathrm{W} = \frac{1}{2}\int \rho V\, \mathrm{d}\tau$

Electrostatic Boundary Conditions:

1) $\vec{E}_1 - \vec{E}_2 = \frac{\sigma}{\varepsilon_0}\hat{n}$, 2) $V_1 = V_2$, 3) $\frac{\partial V_1}{\partial n} - \frac{\partial V_2}{\partial n} = -\frac{\sigma}{\varepsilon_0}$

Linear Dielectric:

$\rho_b = -\vec{\nabla}\cdot\vec{\mathrm{P}}$ $\sigma_b = \vec{\mathrm{P}}\cdot\hat{n}$ $\vec{\mathrm{P}} = \varepsilon_0\chi_e\vec{E}$

$\vec{D} = \varepsilon_0\vec{E} + \vec{\mathrm{P}}$ $\vec{D} = \varepsilon\vec{E}$ $\oint\vec{D}\cdot \mathrm{d}\vec{a} = \mathrm{Q}_{\mathrm{enc.}}^{\mathrm{free}}$

$\mathrm{D}_{\mathrm{above}}^{\perp} - \mathrm{D}_{\mathrm{below}}^{\perp} = \sigma_f$; $\mathrm{D}_{\mathrm{above}}^{\parallel} - \mathrm{D}_{\mathrm{below}}^{\parallel} = \boldsymbol{P}_{\boldsymbol{above}}^{\parallel} - \boldsymbol{P}_{\boldsymbol{below}}^{\parallel}$; $\varepsilon = \varepsilon_0(1+\chi_e)$; $\varepsilon_r = 1+\chi_e = \frac{\varepsilon}{\varepsilon_0}$

The Biot-Savart Law: $\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\hat{r}}{r^2}$

Ampere's Law: $\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{enc.}$

Vector Potential:

$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}}{r}d\tau$, $\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{K}}{r}da$, $\vec{A} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}}{r}$; $\vec{A}_{above} = \vec{A}_{below}$; $\frac{\partial\vec{A}_{above}}{\partial n} - \frac{\partial\vec{A}_{below}}{\partial n} = -\mu_0\vec{K}$

$\vec{K} = \sigma\vec{v}$, $\vec{J} = \rho\vec{v}$

$$\mathbf{A}_{\mathrm{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2},$$