INS 3121
SOIL MECHANICS

Flow of Water in Soils

Assoc. Prof. Dr. Pelin ÖZENER
Civil Engineering Department
Geotechnics Division

6.1 Introduction

- Soils are permeable due to the existence of interconnected voids through which water can flow.

- It is necessary to identify the quantity of underground seepage under various hydraulic conditions.

- Pumping of water for underground construction, stability analysis of earth dams and earth-retaining structures under seepage forces.

6.2 Bernoulli's Equation

$$h = \frac{u}{\gamma_w} + \frac{v^2}{2g} + Z$$

Pressure Head — Velocity Head — Elevation Head

- elevation head, Z : the vertical distance of a given point above or below a datum plane
- pressure head: the water pressure, u, at that point divided by the unit weight of water, γ_w
- velocity head can be neglected ($v \approx 0$)

$$h = \frac{u}{\gamma_w} + Z$$

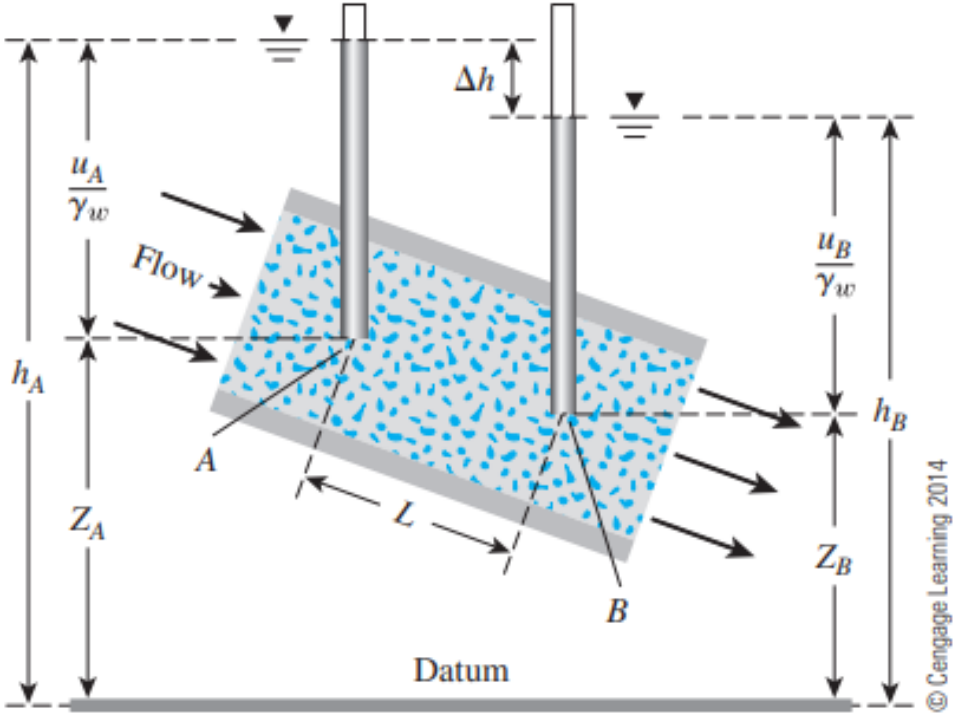


Figure 7.1 Pressure, elevation, and total heads for flow of water through soil

6.2 Bernoulli's Equation

- Piezometers
- *the loss of head*,

$$\Delta h = h_A - h_B$$

$$= \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)$$

- $i = \frac{\Delta h}{L}$: *hydraulic gradient*
- L = distance between point A and B

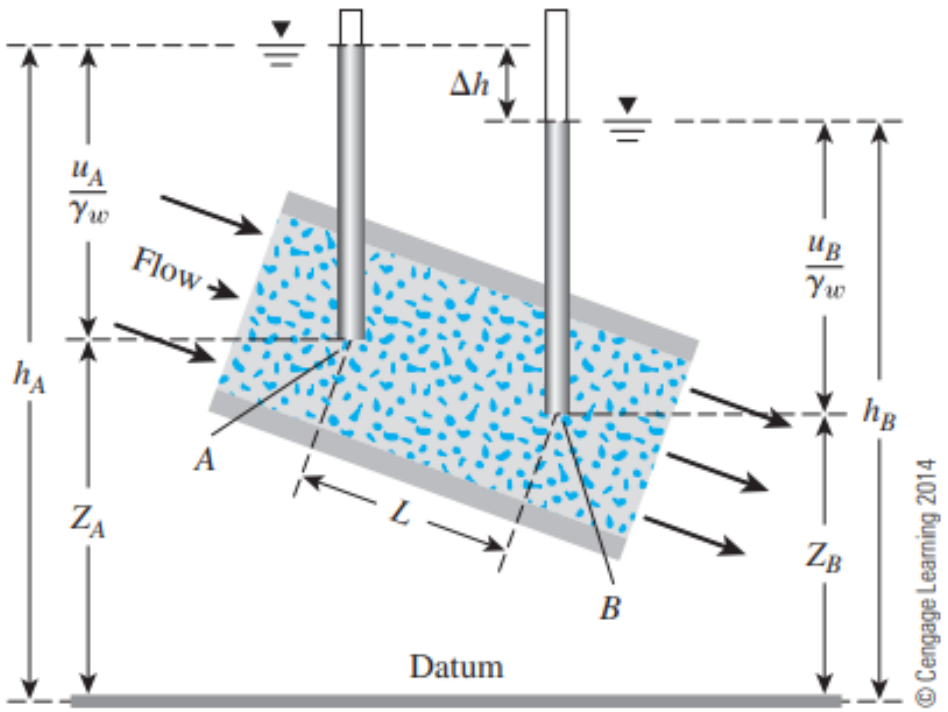


Figure 7.1 Pressure, elevation, and total heads for flow of water through soil

6.2 Bernoulli's Equation

➢ Three zones:

1. Laminar flow zone (Zone I)
2. Transition zone (Zone II)
3. Turbulent flow zone (Zone III)

• In most soils, the flow of water through the void spaces can be considered laminar and thus

$$v \propto i$$

※ In fractured rock, stones, gravels, and very coarse sands, turbulent flow conditions may exist, and then, the Eq. may not be valid.

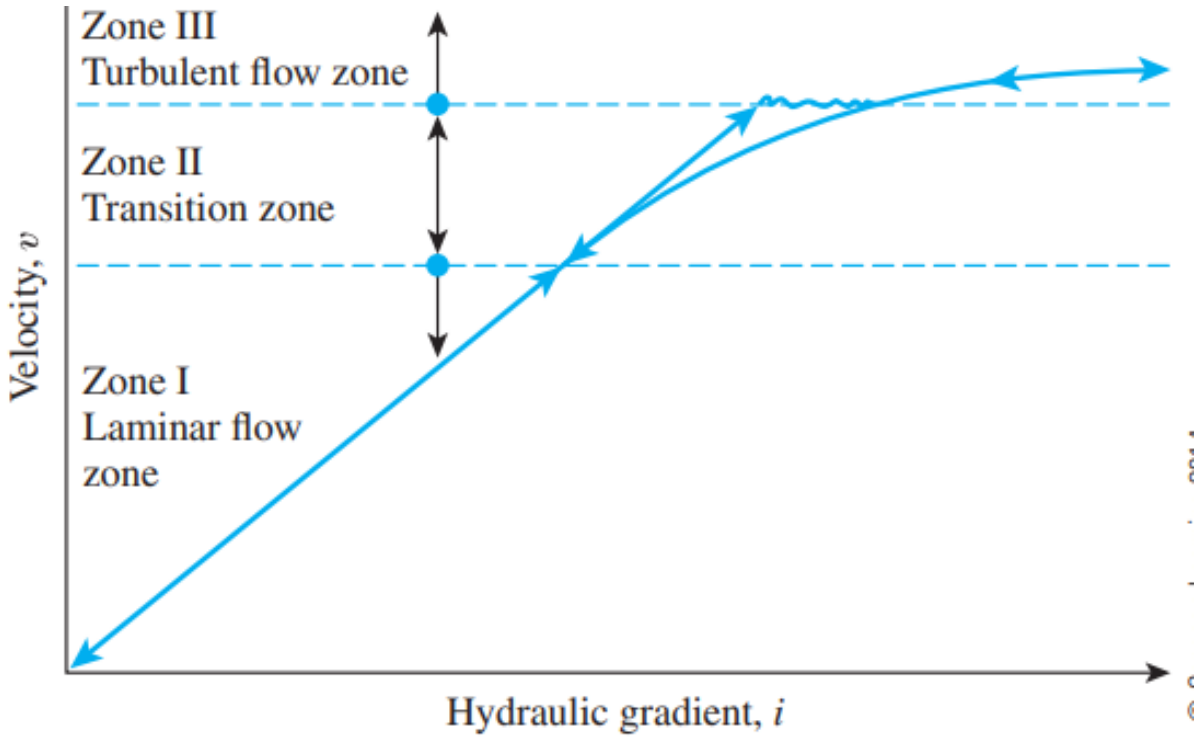


Figure 7.2 Nature of variation of v with hydraulic gradient, i

6.3 Darcy's Law

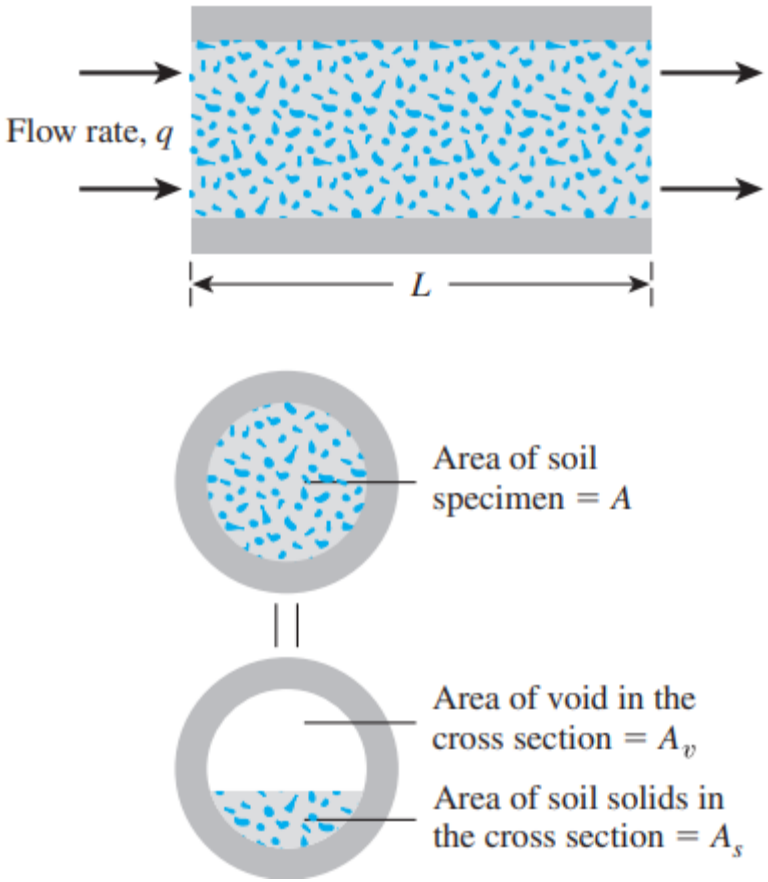
- Darcy (1856) Law : Eq. for discharge velocity of water through saturated soils

$$v = ki$$

Where v = discharge velocity

k = hydraulic conductivity (coefficient of permeability)

- v: the discharge velocity of water based on the gross cross-sectional area of the soil
- seepage velocity: actual velocity of water through the void space
- $q = vA = A_v v_s$
- $A = A_v + A_s$



6.3 Darcy's Law

$$q = v(A_v + A_s) = A_v v_s$$

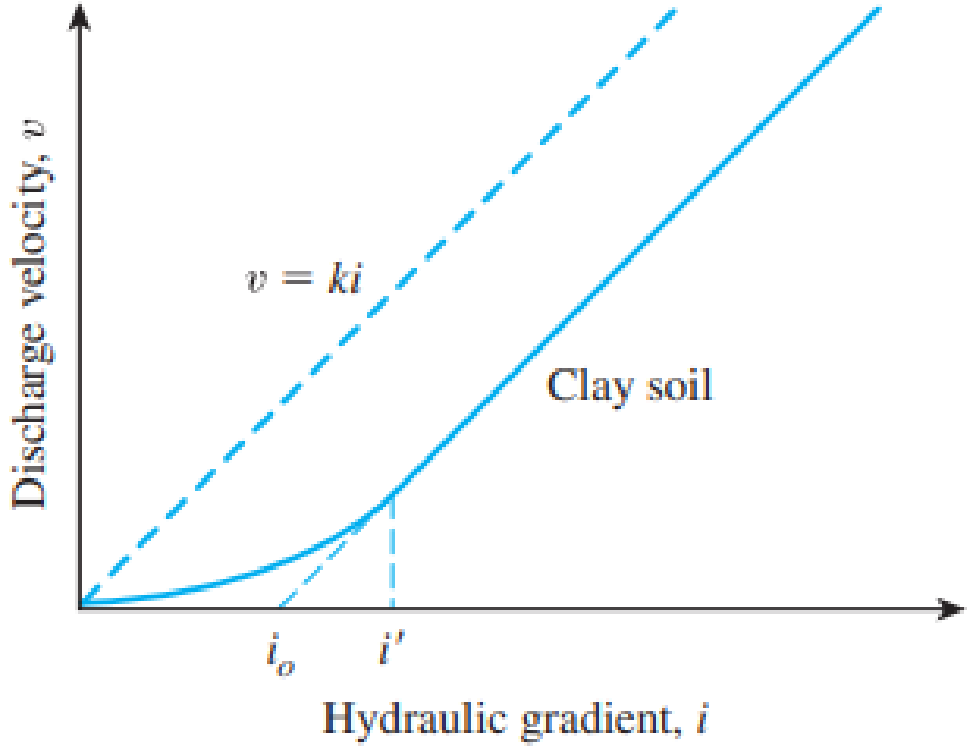
$$v_s = \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)L}{A_v L} = \frac{v(V_v + V_s)}{V_v}$$

$$v_s = v\left(\frac{1 + \left(\frac{V_v}{V_s}\right)}{\frac{V_v}{V_s}}\right) = v\left(\frac{1 + e}{e}\right) = \frac{v}{n}$$

6.3 Darcy's Law

- Darcy's law implies that the discharge velocity v bears a linear relationship to the hydraulic gradient i and will pass through the origin as shown in the figure.

- Hydraulic conductivity (k) has the same unit as velocity, [cm/sec], [m/sec].



6.3 Darcy's Law

- The hydraulic conductivity of soils is dependent on several factors:

  1. Fluid viscosity
  2. pore-size distribution
  3. Grain-size distribution
  4. Void ratio
  5. Roughness of mineral particles
  6. Degree of soil saturation

6.3 Darcy's Law

- In clayey soils, structure plays an important role in the hydraulic conductivity.

- The value of the hydraulic conductivity (k) varies widely for different soils.

- $k = \frac{\gamma_w}{\eta} K$

where η = viscosity of water

K = absolute permeability, unit of L^2

6.3 Darcy's Law

- The value of the hydraulic conductivity (k) varies widely for different soils.

Table 7.1 Typical Values of Hydraulic Conductivity of Saturated Soils

Soil type	k cm/sec	k ft/min
Clean gravel	100−1.0	200−2.0
Coarse sand	1.0−0.01	2.0−0.02
Fine sand	0.01−0.001	0.02−0.002
Silty clay	0.001−0.00001	0.002−0.00002
Clay	<0.000001	<0.000002

6.4 Laboratory Determination of Hydraulic Conductivity

- Two standard laboratory tests
  - Constant head test
  - Falling head test
- Constant head test:

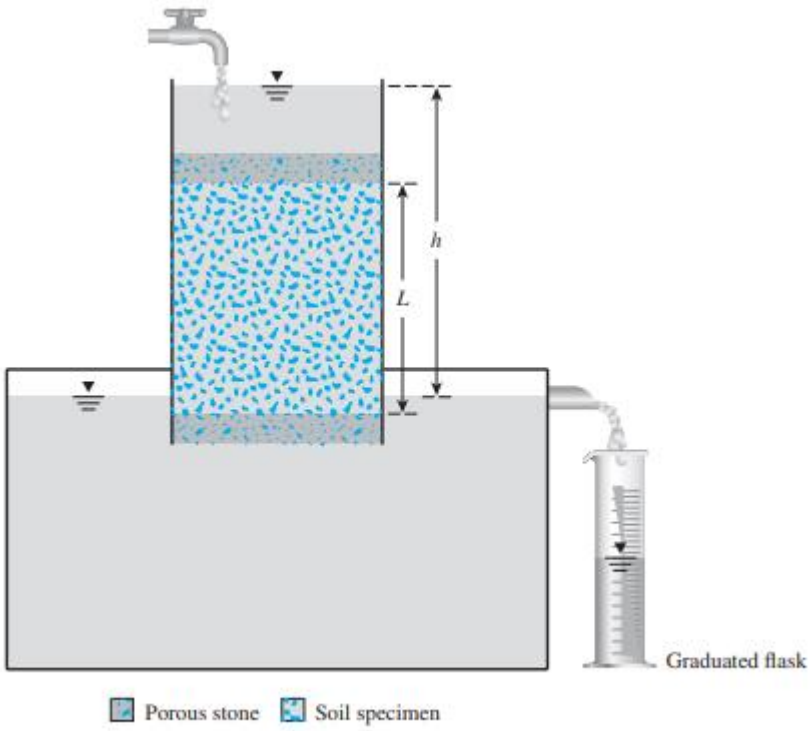


Figure 7.5 Constant-head permeability test

6.4 Laboratory Determination of Hydraulic Conductivity

- Constant head test

$$Q = Avt = A(ki)t$$

$$i = \frac{h}{L}$$

Substitution

$$Q = A\left(k\frac{h}{L}\right)t \qquad k = \frac{QL}{Aht}$$

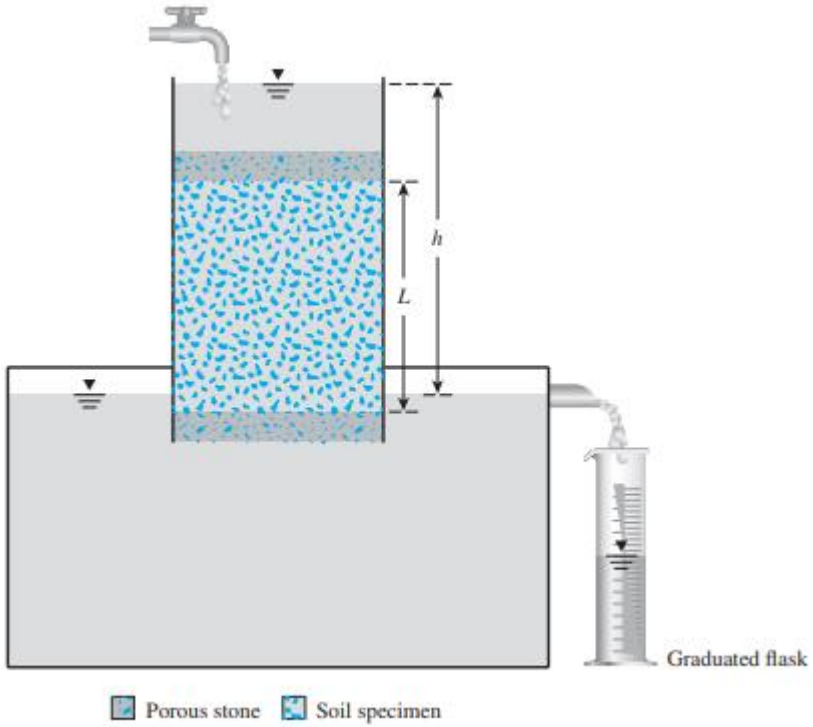


Figure 7.5 Constant-head permeability test

6.4 Laboratory Determination of Hydraulic Conductivity

- Constant head test

$$k = \frac{QL}{Aht}$$

- constant head test: more suitable for coarse-grained soils that have higher coefficients of permeability -> k = 10^{-2} ~ 10^{-3} cm/sec

6.4 Laboratory Determination of Hydraulic Conductivity

- Falling Head Test

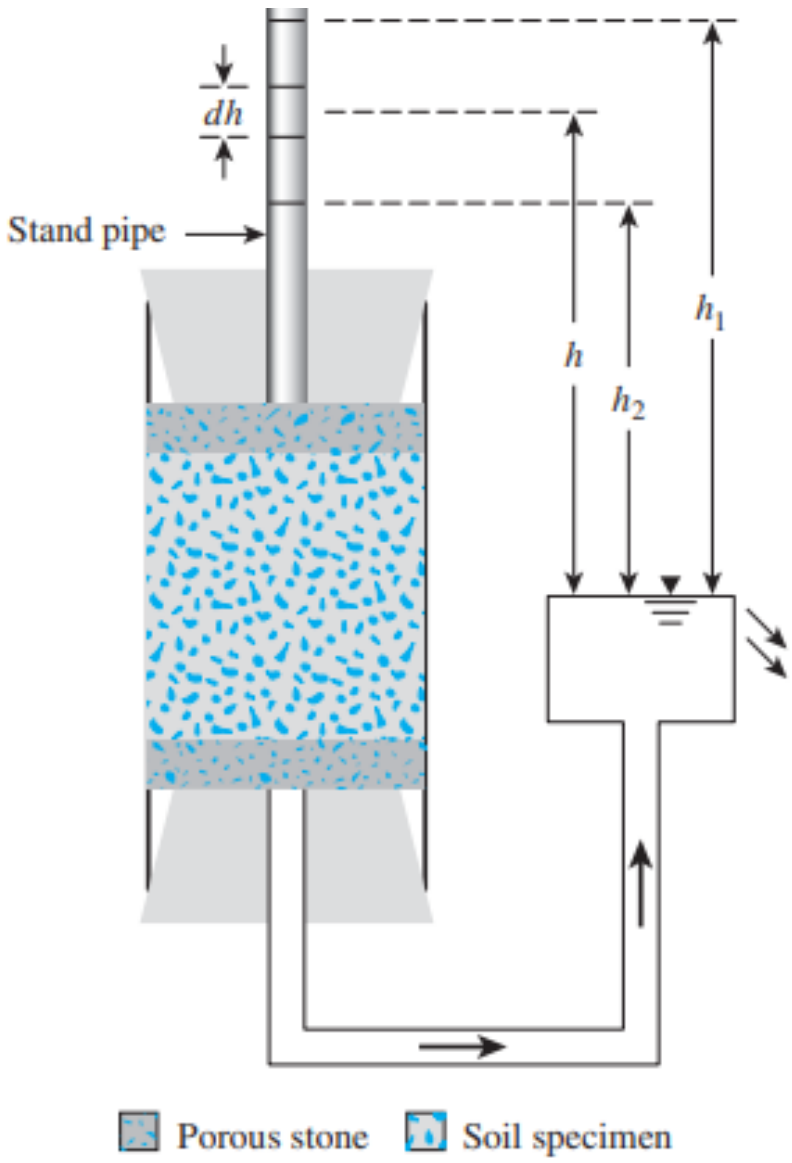
The rate of flow of the water through the specimen at any time t

$$q = k\frac{h}{L}A = -a\frac{dh}{dt}$$

where q = rate of flow

a = cross-sectional area of the standpipe

A = cross-sectional area of the soil specimen



6.4 Laboratory Determination of Hydraulic Conductivity

- Falling Head Test

$$q = k\frac{h}{L}A = -a\frac{dh}{dt}$$

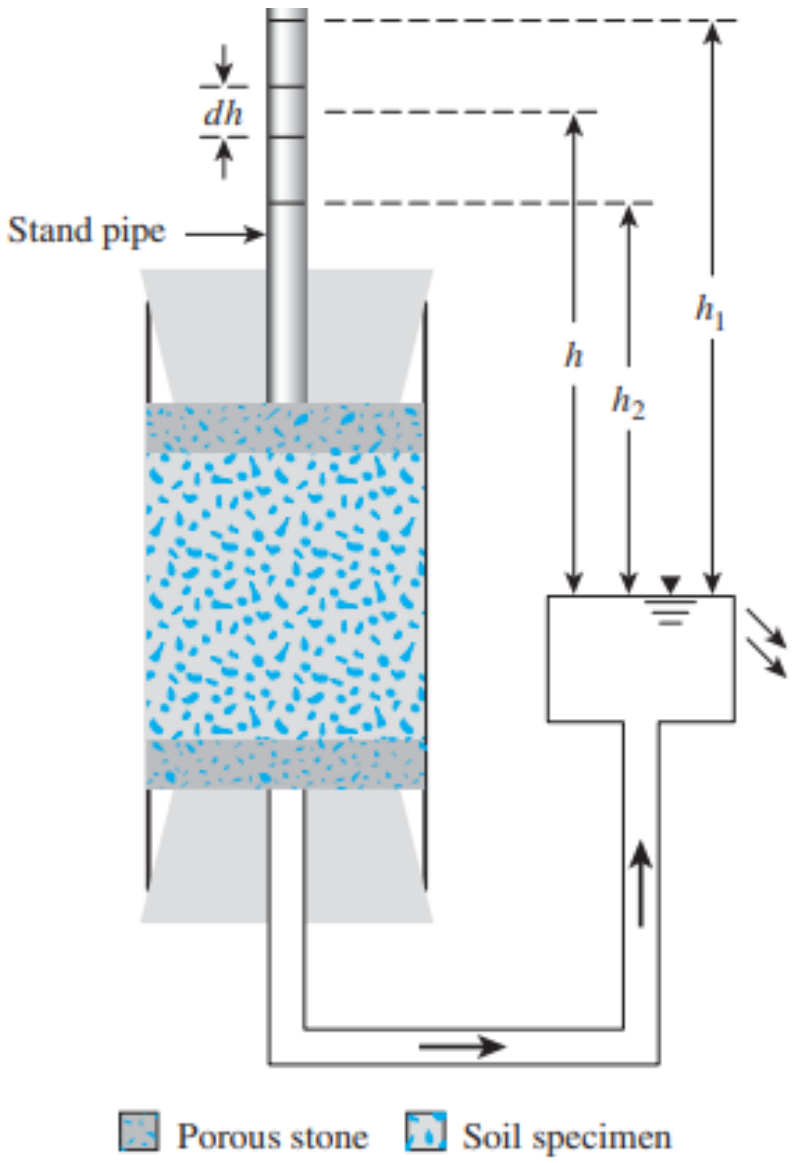
Rearrangement of the Equation,

$$dt = \frac{aL}{Ak}\left(-\frac{dh}{h}\right)$$

time: 0 → t, head: $h1$ → $h2$

$$t = \frac{aL}{Ak}\log_e\frac{h_1}{h_2}$$

$$k = 2.303\frac{al}{At}log_{10}\frac{h_1}{h_2}$$



6.4 Laboratory Determination of Hydraulic Conductivity

- Falling Head Test

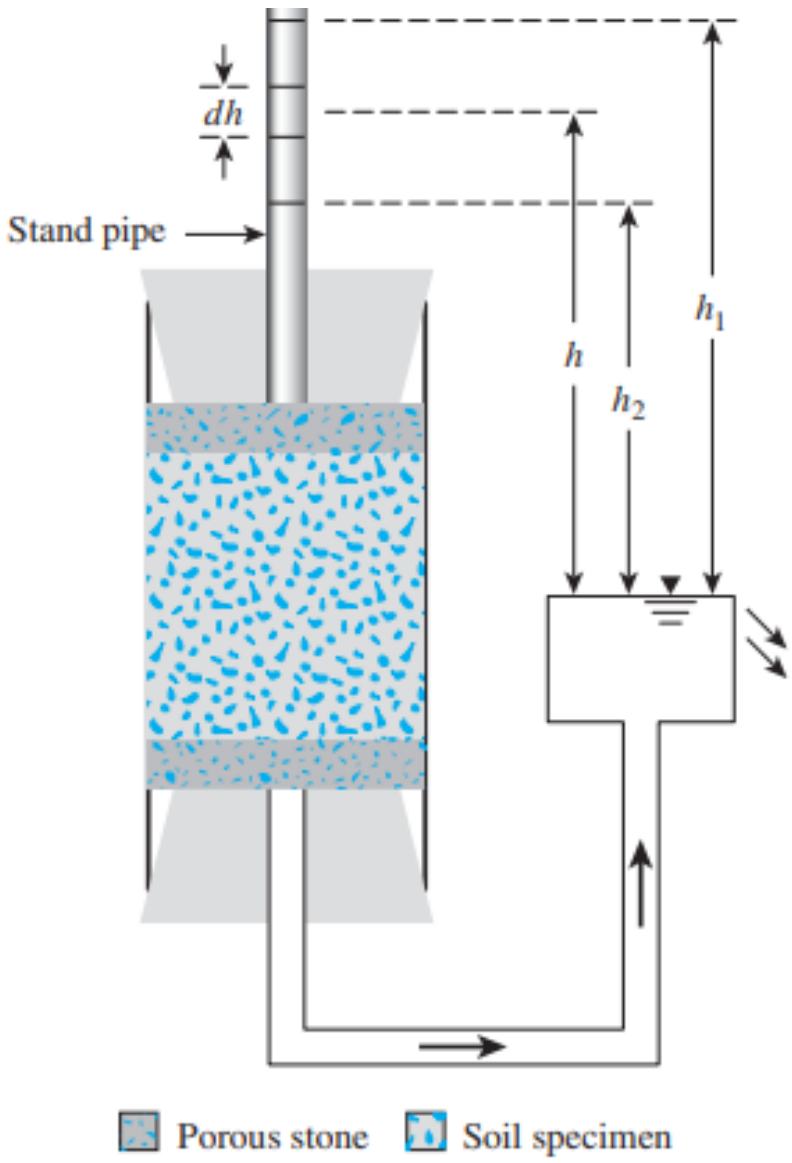
$$k = 2.303\frac{al}{At}log_{10}\frac{h_1}{h_2}$$

Falling head test: more appropriate for fine –grained soils with a low coefficient of permeability -> k = 10^{-3} ~ 10^{-6} cm/sec

- For fairly uniform sand, Hazen(1930) proposed
- $k\ (cm/sec) = c \cdot D10^2$

where c = a constant that varies from 1.0 to 1.5

D_{10} = the effective size, in mm



6.4 Laboratory Determination of Hydraulic Conductivity

- Another form of equation that gives fairly good results in estimation the hydraulic conductivity of sandy soils is based on the Kozeny-Carman equation (Kozeny, 1927; Carman, 1938, 1956).

$$k = \frac{1}{C_s S_s^2 T^2} \frac{\gamma_w}{\eta} \frac{e^3}{1 + e}$$

where Cs = shape factor, which is a function of the shape of flow channels

Ss = specific surface area per unit volume of particles

T = tortuosity of flow channels

γ_w = unit weight of water

η = viscosity of permeant

e = void ratio

6.5 Relationships for Hydraulic conductivity – Cohesive Soil

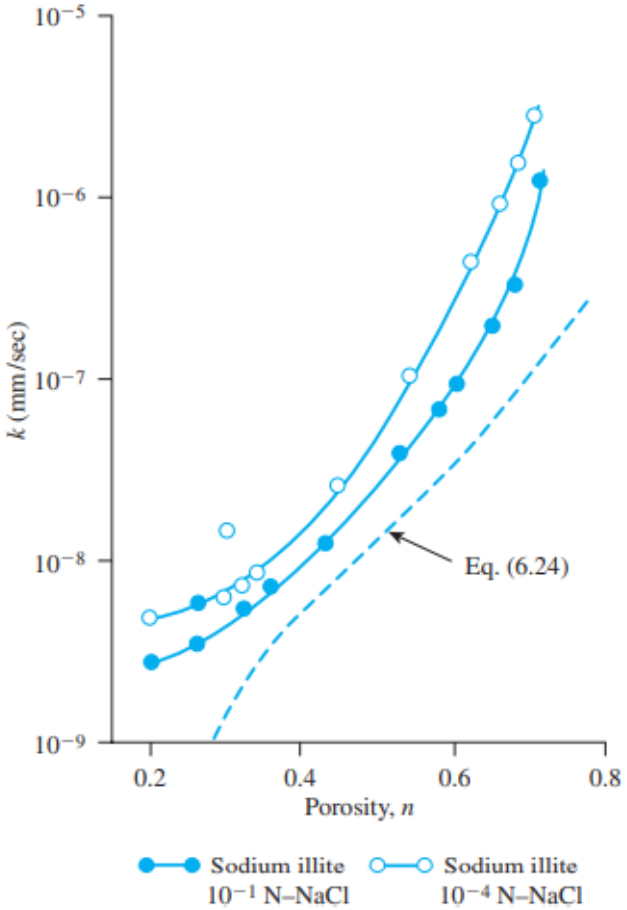


Figure 7.11 Coefficient of permeability for sodium illite (*Based on Olsen, 1961*)

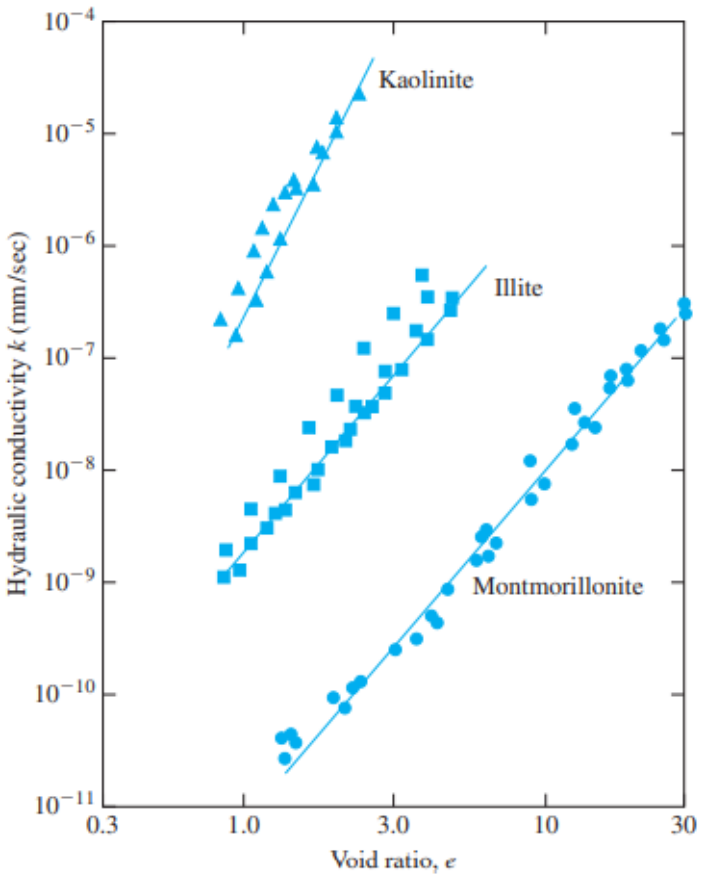


Figure 7.12 Variation of hydraulic conductivity of sodium clay minerals (*Based on Mesri and Olson, 1971*)

6.5 Relationships for Hydraulic conductivity – Cohesive Soil

- Taylor (1948) proposed a linear relationship between the logarithm of k and the void ratio as,

$$\log k = \log k_0 - \frac{e_0 - e}{C_k}$$

where k_0 = in situ hydraulic conductivity at a void ratio e_0

k = hydraulic conductivity at a void ratio e

C_k = hydraulic conductivity change index