INS 3121
SOIL MECHANICS

Weight-Volume Relationships

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Weight-volume relationships

- Three-phase systems
  1. soil solids
  2. water
  3. Air

- $V = V_s + V_v = V_s + V_w + V_a$
- $W = W_s + W_w$

where Vs = volume of soil solids

Vv = volume of voids

Vw = volume of water in the voids

Va = volume of air in the voids

Ws = weight of soil solids

Ww = weight of water

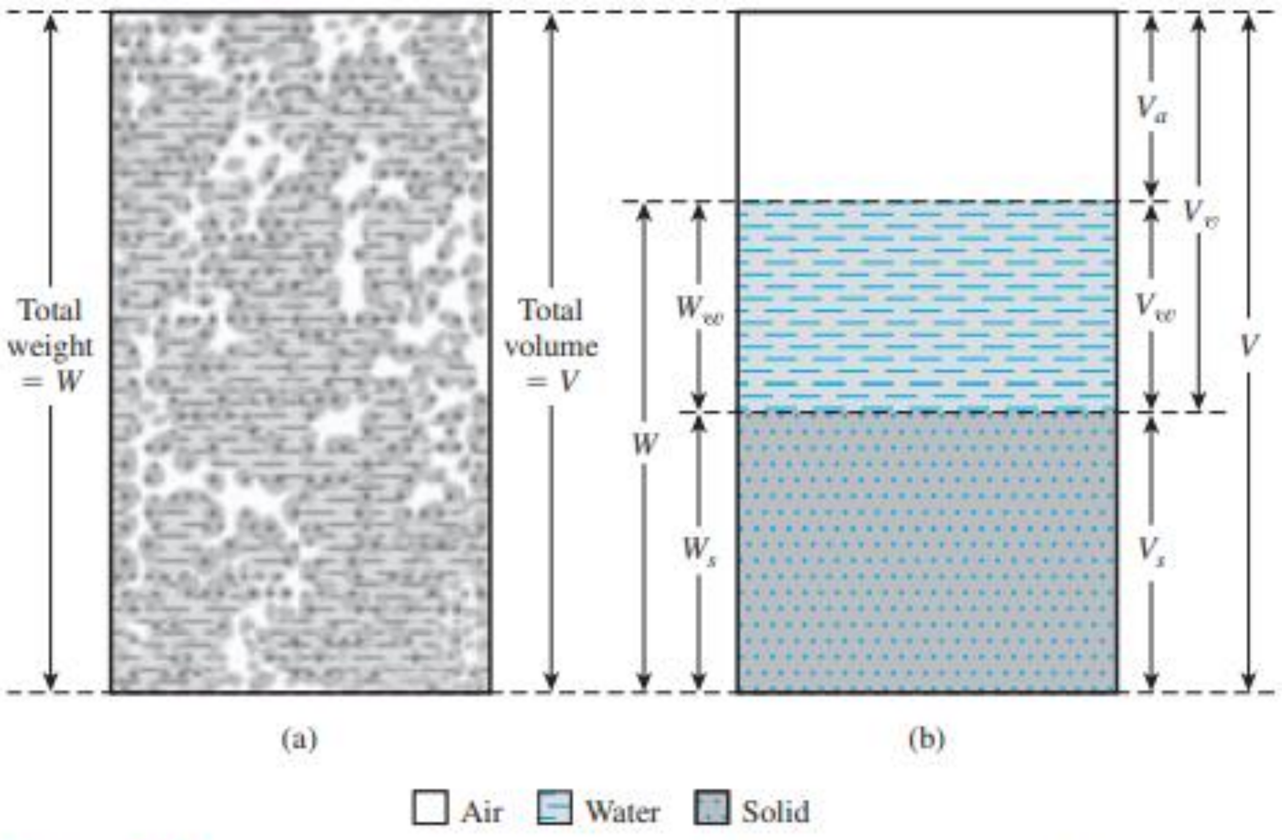
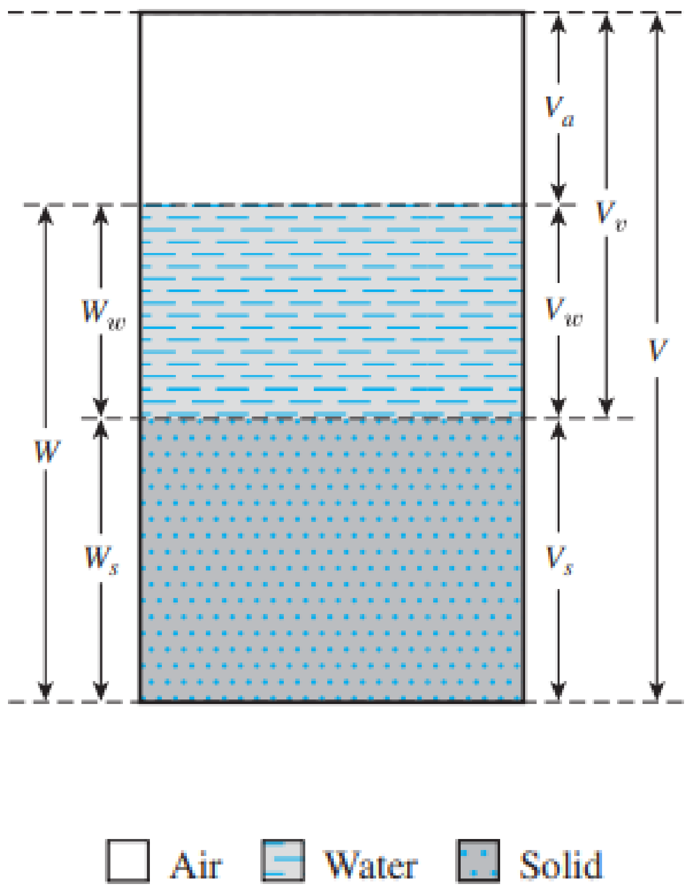


Figure 3.1 (a) Soil element in natural state; (b) three phases of the soil element

Volume relationships

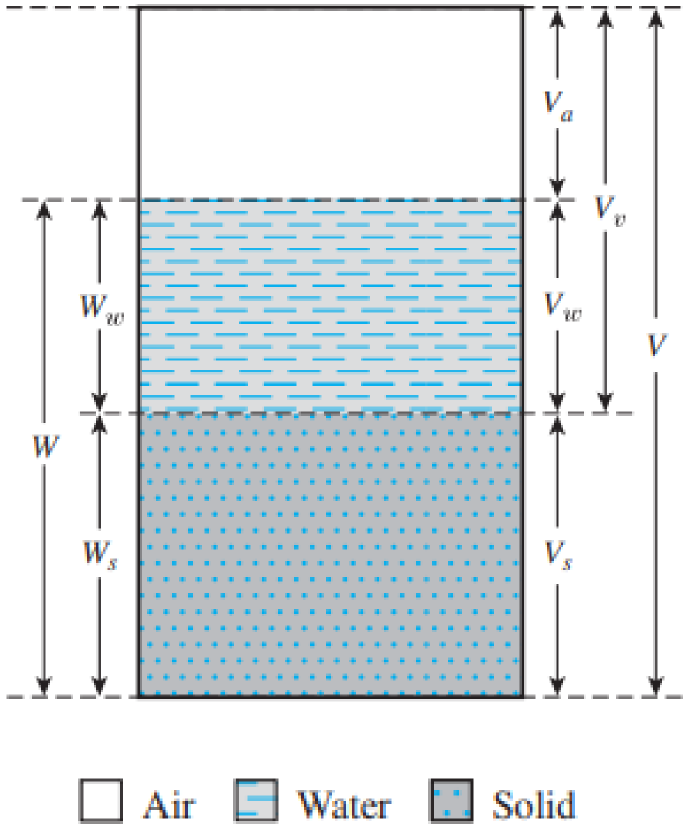
- Void ratio: $e = Vv/Vs$
- Porosity: $n = Vv/V$
- Degree of saturation (%): $S = Vw/Vv$
- $e = \frac{V_v}{V_s} = \frac{V_v}{(V - V_v)} = \frac{\frac{V_v}{V}}{(1 - \frac{V_v}{V})} = \frac{n}{1-n}$
- $n = \frac{e}{e+1}$



Weigth relationships

- Moisture content: $\omega = \frac{W_w}{W_s}$
- Unit weigth or moist unit weigth: $\gamma = \frac{W}{V}\ \left(\frac{kN}{m^3}, \frac{t}{m^3}\right)$
- $\gamma = \frac{W}{V} = \frac{W_s+W_w}{V} = \frac{W_s[1+\left(\frac{W_w}{W_s}\right)]}{V} = \frac{W_s(1+\omega)}{V}$
- Dry unit weight: $\gamma_d = \frac{W_s}{V} \rightarrow \frac{\gamma}{1+\omega}$
- Mass densities of soil: $\rho = \frac{m}{V} (kg/m^3)$

$$\gamma_d = p_d g = 9.81 p_d \ (N/m^3)$$



Weigth relationships

- Conversion factors for unit weight from English to SI units
- 1 $^{Ib}/_{ft^3}$ =0.1572 kN/m^3
- 1 $^{Ib}/_{ft^3}$ $= 16.0256\ kgf/m^3$
- Unit weigth of water,

$$\gamma_w = \begin{cases} 62.4\ ^{Ib}/_{ft^3} \\ 9.81\ ^{kN}/_{m^3} \\ 1000\ kgf/m^3 \end{cases}$$

3.2 Relationships among Unit Weight, Void Ratio, Moisture Content, and Specific Gravity

- $\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s\gamma_w + \omega G_s\gamma_w}{1+e}$

$$= \frac{(1+\omega)G_s\gamma_w}{1+e} = \frac{G_s + Se}{1+e}\gamma_w$$

- $\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w}{1+e}$

- $e = \frac{G_s\gamma_w}{\gamma_d} - 1$

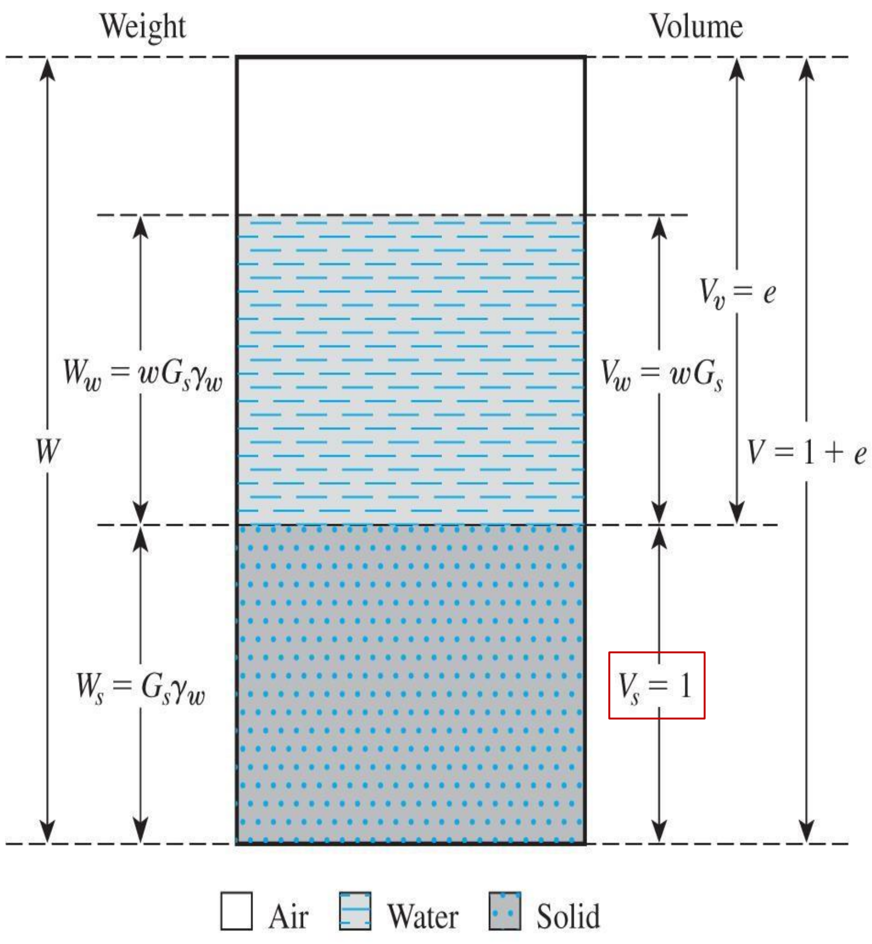


Figure 3.2 Three separate phases of a soil element with volume of soil solids equal to one

3.2 Relationships among Unit Weight, Void Ratio, Moisture Content, and Specific Gravity

- $V_w = \frac{W_w}{\gamma_w} = \frac{\omega G_s \gamma_w}{\gamma_w} = \omega \cdot G_s$
- $S = \frac{V_w}{V_v} = \frac{\omega \cdot G_s}{e}$
- $S \cdot e = \omega \cdot G_s$
- $G_s = \frac{W_s}{V_s \gamma_w}$
- $\gamma_{sat} = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + e\gamma_w}{1+e} = \frac{(G_s + e)\gamma_w}{1+e}$
- $e = \omega \cdot G_s$

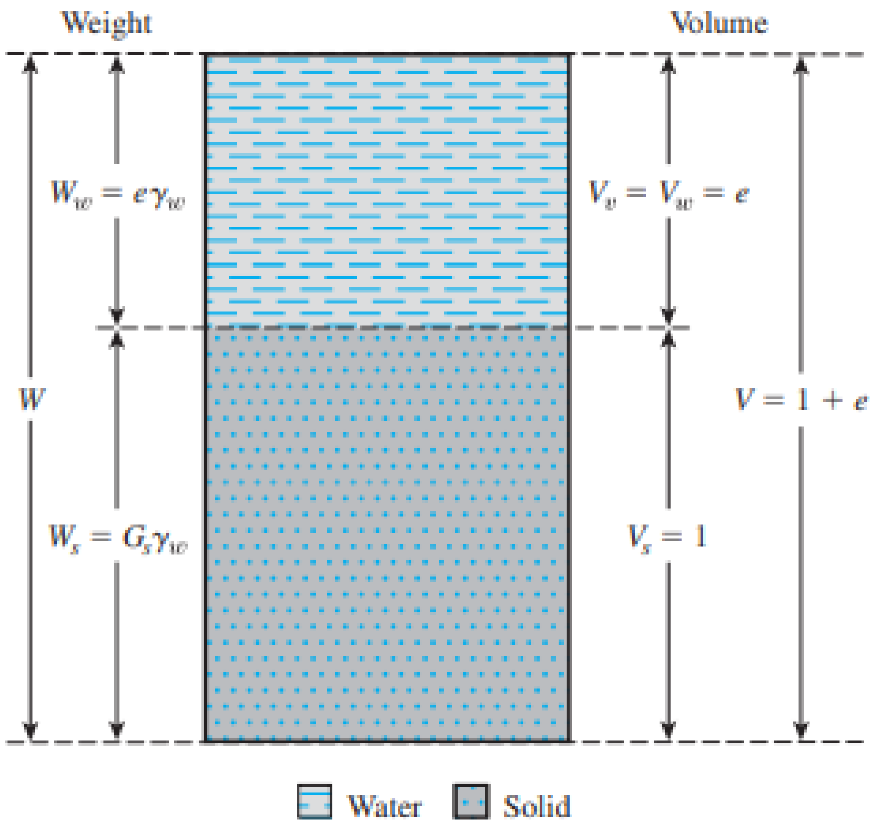


Figure 3.3
Saturated soil element with volume of soil solids equal to one

3.3 Relationships among Unit Weight, Porosity, and Moisture Content

- $\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w (1-n)}{1} = G_s \gamma_w (1-n)$

- $\gamma = \frac{W_s + W_w}{V} = G_s \gamma_w (1-n)(1+\omega)$

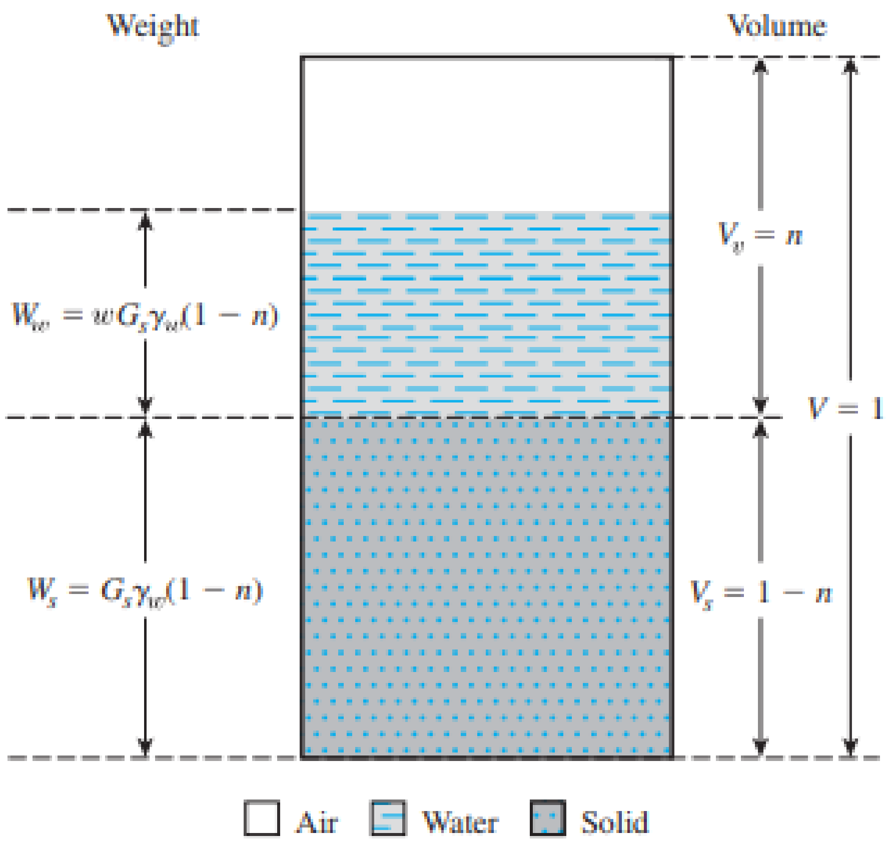


Figure 3.5
Soil element with total volume equal to one

3.3 Relationships among Unit Weight, Porosity, and Moisture Content

- $\gamma_{sat} = \frac{W_s + W_w}{V} = \frac{(1-n)G_s\gamma_w + n\gamma_w}{1}$
  $= [(1-n)G_s + n]\gamma_w$

- $\omega = \frac{W_w}{W_s} = \frac{n\gamma_w}{(1-n)\gamma_w G_s} = \frac{n}{(1-n)G_s}$

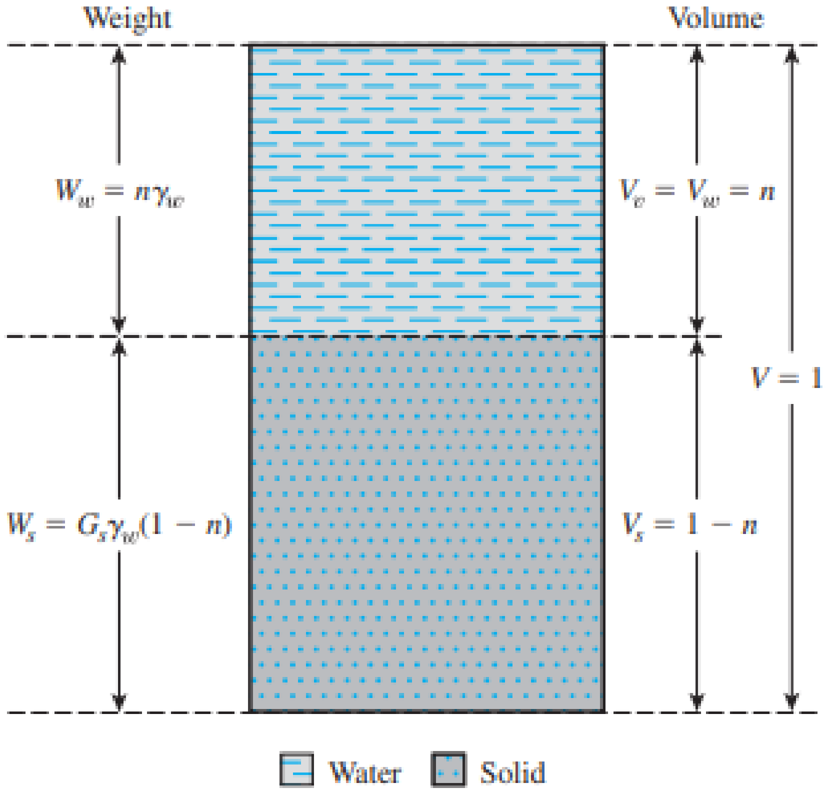


Figure 3.6 Saturated soil element with total volume equal to one

3.4 Relative Density

- Relative density: $D_r = \frac{e_{max}-e}{e_{max}-e_{min}}$, $e = e_{max}; D_r = 0$

- Compacting a granular soil to a relative density greater than about 85% is difficult.

- $D_r = \frac{\frac{1}{\gamma_{dmin}}-\frac{1}{\gamma_d}}{\frac{1}{\gamma_{dmin}}-\frac{1}{\gamma_{dmax}}} = (\frac{\gamma_d-\gamma_{dmin}}{\gamma_{dmax}-\gamma_{dmin}})(\frac{\gamma_{dmax}}{\gamma_d})$ (3.35)

3.4 Relative Density

Table 3.3 Qalitative Description of Granular Soil Deposits

Relative density (%)	Description of soil deposit
0–15	Very loose
15–50	Loose
50–70	Medium
70–85	Dense
85–100	Very dense

3.4 Relative Density

- ASTM test Designation D-4253 (2007) provides procedure for determining the minimum and maximum dry unit dry unit weights of granular soils so that they can be used in Eq. (3.35) to measure the relative density of compaction in the field. For sands this procedure involves using a mold with a volume of 2830 cm^3.

minimum dry unit weight

- Sand is poured loosely into the mold from a funnel with a 12.7mm diameter spout
- The average height of the fall of sand into the mold is maintained at about 25.4mm.

3.4 Relative Density

minimum dry unit weight

- The value of γ_{dmin} then can be calculated by using the following equation;

$$\gamma_{dmin} = \frac{W_s}{V_m} = \frac{G_s \gamma_w}{1+e_{max}}$$

where Ws = weight of sand required to fill the mold

Vm = volume of the mold

3.4 Relative Density

maximum dry unit weight

- The maximum dry unit weight is determined by vibrating sand in the mold for 8 min.
- a surcharge of 14 kN/m^2 is added to the top of the sand in the mold.
- the mold is placed on a table that vibrates at a frequency of 3600 cycles/min and that has an amplitude of vibration of 0.635 mm.
- the value of $\gamma d(max)$ can be determined at the end of the vibrating period with knowledge of the weight and volume of the sand

Summary&Essential Points

- This chapter discussed weight-volume relationships.
- Weight-volume relationships include relationships among parameters such as void ratio, porosity, degree of saturation, moisture content, and weight. The parameters are fundamental to the study of geotechnical engineering.
- Relative density is commonly used to indicate the in situ denseness or looseness of granular soil.