Vector Algebra

Usefull Info

$$\begin{aligned}
\vec a_R\cdot\vec a_x &= \sin\theta\ \cos\phi & \vec a_R\cdot\vec a_y &= \sin\theta\ \sin\phi & \vec a_R\cdot\vec a_z &= \cos\theta\\
\vec a_\theta\cdot\vec a_x &= \cos\theta\ \cos\phi & \vec a_\theta\cdot\vec a_y &= \cos\theta\ \sin\phi & \vec a_\theta\cdot\vec a_z &= -\sin\theta\\
\vec a_\phi\cdot\vec a_x &= -\sin\phi & \vec a_\phi\cdot\vec a_y &= \cos\phi & \vec a_\phi\cdot\vec a_z &= 0
\end{aligned}$$

$$\begin{aligned}
\vec a_r\cdot\vec a_x &= \cos\phi & \vec a_r\cdot\vec a_y &= \sin\phi & \vec a_r\cdot\vec a_z &= 0\\
\vec a_\phi\cdot\vec a_x &= -\sin\phi & \vec a_\phi\cdot\vec a_y &= \cos\phi & \vec a_\phi\cdot\vec a_z &= 0\\
\vec a_z\cdot\vec a_x &= 0 & \vec a_z\cdot\vec a_y &= 0 & \vec a_z\cdot\vec a_z &= 1
\end{aligned}$$

$$\begin{aligned}
x &= R\ \sin\theta\ \cos\phi & R &= \sqrt{x^2+y^2+z^2}\\
y &= R\ \sin\theta\ \sin\phi & \theta &= \cos^{-1}\big(z/R\big)\\
z &= R\ \cos\theta & \phi &= \tan^{-1}\big(y/x\big)
\end{aligned}$$

$$\begin{aligned}
x &= r\ \cos\phi & r &= \sqrt{x^2+y^2}\\
y &= r\ \sin\phi & \phi &= \tan^{-1}\big(y/x\big)\\
z &= z & z &= z
\end{aligned}$$

$$\begin{aligned}
\vec{d\ell} &= \vec a_x\ dx+\vec a_y\ dy+\vec a_z\ dz & dV &= dx\ dy\ dz\\
\vec{d\ell} &= \vec a_r\ dr+\vec a_\phi\ r\ d\phi+\vec a_z\ dz & dV &= r\ dr\ d\phi\ dz\\
\vec{d\ell} &= \vec a_R\ dR+\vec a_\theta\ R\ d\theta+\vec a_\phi\ R\ \sin\theta\ d\phi & dV &= R^2\ \sin\theta\ dR\ d\theta\ d\phi
\end{aligned}$$

$$\begin{aligned}
\vec{dS} &= \vec a_x\ dy\ dz & &\vec a_y\ dx\ dz & &\vec a_z\ dx\ dy\\
\vec{dS} &= \vec a_r\ r\ d\phi\ dz & &\vec a_\phi\ dr\ dz & &\vec a_z\ r\ dr\ d\phi\\
\vec{dS} &= \vec a_R\ R^2\ \sin\theta\ d\theta\ d\phi & &\vec a_\theta\ R\ \sin\theta\ dR\ d\phi & &\vec a_\phi\ R\ dR\ d\theta
\end{aligned}$$

$$\int_V\big[\nabla\cdot\vec A(\vec r)\big]\ dV=\oint_S\vec A(\vec r)\cdot d\vec S \qquad\qquad \int_S\big[\nabla\times\vec A(\vec r)\big]\cdot d\vec S=\oint_C\vec A(\vec r)\cdot d\vec\ell$$

$$\vec A = \nabla\Phi+\nabla\times\vec F \qquad \nabla\times\nabla\Phi \equiv 0 \qquad \nabla\cdot\nabla\times\vec A \equiv 0$$

$$\nabla\big(\Phi\Psi\big) = \Phi\ \nabla\Psi+\Psi\ \nabla\Phi \qquad \nabla\times\nabla\times\vec A = \nabla\big(\nabla\cdot\vec A\big)-\nabla^2\vec A$$

$$\nabla\times\big(\Phi\vec A\big) = \Phi\ \big(\nabla\times\vec A\big)+\nabla\Phi\times\vec A \qquad \nabla\cdot\big(\Phi\vec A\big) = \Phi\ \big(\nabla\cdot\vec A\big)+\nabla\Phi\cdot\vec A$$

$$\nabla\left(\frac{1}{|\vec R-\vec R'|}\right)=-\nabla'\left(\frac{1}{|\vec R-\vec R'|}\right)=-\frac{\vec R-\vec R'}{|\vec R-\vec R'|^3}$$

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{1}{r} \frac{\partial V}{\partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla V = \vec{a}_R \frac{\partial V}{\partial R} + \vec{a}_\theta \frac{1}{R} \frac{\partial V}{\partial \theta} + \vec{a}_\phi \frac{1}{R\sin\theta} \frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r}\big(rA_r\big) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2} \frac{\partial}{\partial R}\big(R^2 A_R\big) + \frac{1}{R\sin\theta} \frac{\partial}{\partial \theta}\big(\sin\theta A_\theta\big) + \frac{1}{R\sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \vec{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \vec{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \vec{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla \times \vec{A} = \vec{a}_r \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \vec{a}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \vec{a}_z \frac{1}{r}\left(\frac{\partial}{\partial r}\big[rA_\phi\big] - \frac{\partial A_r}{\partial \phi}\right)$$

$$\nabla \times \vec{A} = \vec{a}_R \frac{1}{R\sin\theta} \left(\frac{\partial}{\partial \theta}\big[\sin\theta A_\phi\big] - \frac{\partial A_\theta}{\partial \phi}\right) + \vec{a}_\theta \frac{1}{R} \left(\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}\big[RA_\phi\big]\right)$$
$$+ \vec{a}_\phi \frac{1}{R}\left(\frac{\partial}{\partial R}\big[A_\theta R\big] - \frac{\partial A_R}{\partial \theta}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{R^2} \frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta} \frac{\partial^2 V}{\partial \phi^2}$$

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{1}{4} \sin 2x \qquad \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\frac{x}{a}$$

$$\int \cos^2 x \, dx = \frac{x}{2} + \frac{1}{4} \sin 2x \qquad \int \log_e x \, dx = x \log_e x - x$$

$$\int \sin x \, \cos x \, dx = \frac{1}{2} \sin^2 x \qquad \int x \, e^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1)$$

$$\int \sin^2 x\, dx = \frac{x}{2} - \frac{1}{4} \sin 2x$$

$$\int \cos^2 x\, dx = \frac{x}{2} + \frac{1}{4} \sin 2x$$

$$\int \sin x \cos x\ dx = \frac{1}{2} \sin^2 x$$

$$\int \frac{dx}{[x^2 \pm a^2]^{\frac{3}{2}}} = \pm \frac{x}{a^2\sqrt{x^2 \pm a^2}}$$

$$\int \frac{x dx}{[x^2 \pm a^2]^{\frac{3}{2}}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log_e \left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{dx}{x\sqrt{x^2 + a^2}} = -\frac{1}{a} \log_e \left(\frac{a + \sqrt{x^2 + a^2}}{x}\right)$$

$$\int \frac{x dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a}$$

$$\int \frac{x dx}{c^4 + x^4} = \frac{1}{2c^2} \tan^{-1} \frac{x^2}{c^2}$$

$$\int \frac{dx}{x^2\sqrt{x^2 \pm a^2}} = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2x^2}$$

$$\int \frac{x^2}{x^2 + c^2} dx = x - c \tan^{-1} \frac{x}{c}$$

$$\int x^2 e^{ax} dx = e^{ax}\left[\frac{x^2}{2} - \frac{2}{a^2}x + \frac{2}{a^3}\right]$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \log_e x\, dx = x \log_e x - x$$

$$\int x\, e^{ax}\, dx = \frac{e^{ax}}{a^2} (ax - 1)$$

$$\int_0^{2\pi} \sin^2 x dx = \int_0^{2\pi} \cos^2 x\, dx = \pi$$

$$\int_0^\infty x^n e^{-ax}\, dx = \frac{n!}{a^{n+1}} \quad (a > 0)$$

$$\int_0^\infty x e^{-ax^2}\, dx = \frac{1}{2a} \quad (a > 0)$$

$$\int_0^\infty x^3 e^{-ax^2}\, dx = \frac{1}{2a^2} \quad (a > 0)$$

$$\int_0^\infty x^{2n+1} e^{--ax^2}\, dx = \frac{n!}{2a^{n+1}} \quad (a > 0)$$

$$\int_0^\infty x^2 e^{-ax^2}\, dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}} \quad (a > 0)$$

$$\int_0^\infty x^4 e^{-ax^2}\, dx = \frac{3}{8a^2}\sqrt{\frac{\pi}{a}} \quad (a > 0)$$

$$\int_0^\infty x^{2n} e^{-ax^2}\, dx = \frac{1.3.5\ldots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}} \quad (a > 0)$$

$$\int_0^\infty \frac{e^{-nx}}{\sqrt{x}}\, dx = \sqrt{\frac{\pi}{n}}$$

$$\int_0^\infty \cos(x^2) dx = \int_0^\infty \sin(x^2) dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$$

$$(1+x)^m = 1 + m\, x + \frac{m(m-1)}{2!}\, x^2 + \ldots + \frac{m(m-1)(m-2)\ldots(m-k+1)}{k!}\, x^k$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots \qquad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

Coulomb's law:	$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$
Electric field:	$\vec{\mathbf{E}} = \dfrac{Q\vec{\mathbf{a}}_R}{4\pi\epsilon R^2}$ or $\vec{\mathbf{E}} = \dfrac{1}{4\pi\epsilon}\displaystyle\int_v \dfrac{\rho\vec{\mathbf{a}}_R}{R^2}\,dv$
Gauss's law:	$\nabla\cdot\vec{\mathbf{D}} = \rho$ or $\displaystyle\oint_s \vec{\mathbf{D}}\cdot\overrightarrow{d\mathbf{s}} = Q$
Conservative $\vec{\mathbf{E}}$ field:	$\nabla\times\vec{\mathbf{E}} = 0$ or $\displaystyle\oint_c \vec{\mathbf{E}}\cdot\overrightarrow{d\ell} = 0$
Potential function:	$\vec{\mathbf{E}} = -\nabla V$ or $V_{ba} = -\displaystyle\int_a^b \vec{\mathbf{E}}\cdot\overrightarrow{d\ell}$
Poisson's equation:	$\nabla^2 V = -\dfrac{\rho}{\epsilon}$
Laplace's equation:	$\nabla^2 V = 0$
Energy density:	$w_e = \frac{1}{2}\vec{\mathbf{D}}\cdot\vec{\mathbf{E}}$
Constitutive relationship:	$\vec{\mathbf{D}} = \epsilon\vec{\mathbf{E}}$
Ohm's law:	$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$

Force equation:	$\vec{\mathbf{F}} = q\vec{\mathbf{u}}\times\vec{\mathbf{B}}$	or	$\overrightarrow{d\mathbf{F}} = I\overrightarrow{d\ell}\times\vec{\mathbf{B}}$
Biot–Savart law:	$\overrightarrow{d\mathbf{B}} = \dfrac{\mu}{4\pi}\dfrac{I\overrightarrow{d\ell}\times\vec{\mathbf{a}}_r}{r^2}$		
Ampère's law:	$\nabla\times\vec{\mathbf{H}} = \vec{\mathbf{J}}$	or	$\displaystyle\oint_c \vec{\mathbf{H}}\cdot\overrightarrow{d\ell} = I$
Gauss's law:	$\nabla\cdot\vec{\mathbf{B}} = 0$	or	$\displaystyle\oint_s \vec{\mathbf{B}}\cdot\overrightarrow{ds} = 0$
Magnetic vector potential:	$\vec{\mathbf{B}} = \nabla\times\vec{\mathbf{A}}$	or	$\vec{\mathbf{A}} = \dfrac{\mu}{4\pi}\displaystyle\int_c \dfrac{I\overrightarrow{d\ell}}{r}$
Magnetic flux:	$\Phi = \displaystyle\int_s \vec{\mathbf{B}}\cdot\overrightarrow{ds}$	or	$\Phi = \displaystyle\oint_c \vec{\mathbf{A}}\cdot\overrightarrow{d\ell}$
Magnetic energy:	$w_m = \frac{1}{2}\vec{\mathbf{B}}\cdot\vec{\mathbf{H}}$		
Poisson's equation:	$\nabla^2\vec{\mathbf{A}} = -\mu\vec{\mathbf{J}}$		
Constitutive relationship:	$\vec{\mathbf{B}} = \mu\vec{\mathbf{H}}$		