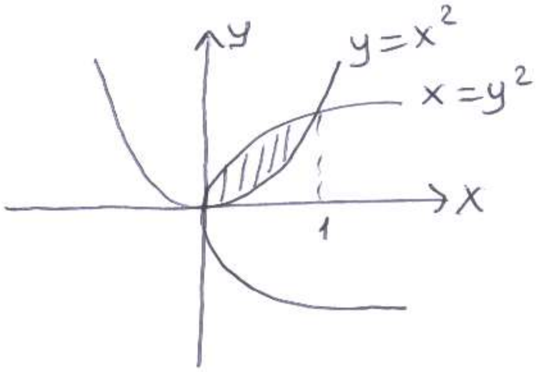
ÖR/ $y = x^2$ ve $x = y^2$ tarafından sınırlanan bölgenin alanını bulunuz.



$y = x^2,\ x = y^2$

$y = y^4$

$y - y^4 = 0$

$y(1-y^3) = 0$

$y=0 \quad y=1$

$$A = \int_0^1 (\sqrt{x} - x^2)\,dx = \left. \frac{x^{3/2}}{\frac{3}{2}} - \frac{x^3}{3} \right|_0^1$$

$$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}\ br^2$$

ÖR/ $f(x) = x^2 - 3x + x\ln x$ eğrisinin ekstremum noktasını bulunuz.

$f'(x) = 2x - 3 + \ln x + x \cdot \frac{1}{x}$

$f'(x) = 2x + \ln x - 2 = 0 \Rightarrow \ln x = 2 - 2x$

$x = e^{2-2x}$

$\underline{x = 1}$

$f''(x) = 2 + \frac{1}{x}$

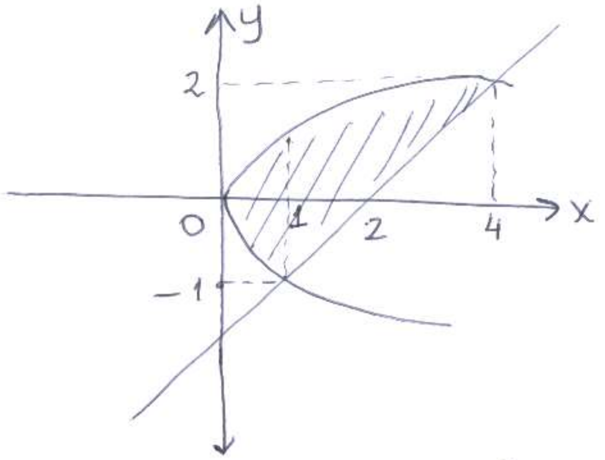
$f''(1) = 2 + \frac{1}{1} = 3 > 0$

$f''(1) = 3 > 0$

$x = 1 \Rightarrow f(1) = 1 - 3 + \underbrace{1 \cdot \ln 1}_{0} = -2$

$f(x)$ in $(1,-2)$ noktasında minimum değeri vardır.

ÖR/ $x=y^2$ ve $y=x-2$ tarafından sınırlanan bölgenin alanını bulunuz.



$y=x-2 \qquad x=y^2$

$y=y^2-2$

$y^2-y-2=0$

$2 \quad -1$

$$A=\int_{x=0}^{1}[\sqrt{x}-(-\sqrt{x})]\,dx+\int_{x=1}^{4}[\sqrt{x}-(x-2)]\,dx$$

$$=2\int_0^1\sqrt{x}\,dx+\int_1^4(\sqrt{x}-x+2)\,dx$$

$$=2\cdot\frac{x^{3/2}}{3/2}\Big|_0^1+\frac{x^{3/2}}{\frac{3}{2}}-\frac{x^2}{2}+2x\Big|_1^4$$

$$\frac{4}{3}x^{3/2}\Big|_0^1+\frac{2}{3}x^{3/2}-\frac{x^2}{2}+2x\Big|_1^4$$

$$=\frac{4}{3}+\frac{2}{3}(2^2)^{3/2}-\frac{4}{2}+8-\left(\frac{2}{3}-\frac{1}{2}+2\right)$$

$$=\frac{4}{3}+\frac{16}{3}\underbrace{-2+8}-\underset{(2)}{\frac{2}{3}}+\underset{(3)}{\frac{1}{2}}-2$$

$$=\underset{(2)}{\frac{20}{3}}+\underset{(6)}{\frac{4}{1}}-\frac{4}{6}+\frac{3}{6}=\frac{40+24-4+3}{6}$$

$$=\frac{64-1}{6}=\frac{63}{6}=\frac{21}{2}\ br^2$$

ÖR/ $\displaystyle\int \frac{2^{\ln x}}{x}\,dx = ?$

$\ln x = u$

$\frac{1}{x}dx = du$

$$\int 2^u \cdot du = \frac{2^u}{\ln 2} + c$$

$$= \frac{2^{\ln x}}{\ln 2} + c$$

ÖR/ $\displaystyle\int e^{e^x} \cdot e^x\,dx = ?$

$u = e^x \qquad du = e^x dx$

$$\int e^{e^x} \cdot e^x dx = \int e^u \cdot du$$

$$= e^u + c$$

$$= e^{e^x} + c$$

ÖR $\displaystyle\int x \sin x^2 dx = ?$

$x^2 = u$

$2x\,dx = du$

$x\,dx = \frac{du}{2}$

$$\int \sin u \cdot \frac{du}{2} = \frac{1}{2}\int \sin u\,du$$

$$= \frac{1}{2}(-\cos u) + c$$

$$= -\frac{1}{2}\cos x^2 + C$$

ÖR/ $\displaystyle\int e^x \sin x\,dx = ?$

$u = e^x \qquad \sin x\,dx = dv$

$du = e^x dx \qquad -\cos x = v$

$$\underbrace{\int e^x \sin x\,dx}_{I} = uv - \int v\,du = e^x(-\cos x) - \int -\cos x \cdot e^x dx$$

$$= -e^x \cos x + \int \cos x \cdot e^x dx$$

$e^x = u$

$e^x dx = du$

$\cos x\,dx = dv$

$\sin x = v$

$$= -e^x \cos x + \left[e^x \sin x - \int \sin x \cdot e^x dx\right]$$

$$I = -e^x \cos x + e^x \sin x - I$$

$$2I = -e^x \cos x + e^x \sin x +$$

$$I = \frac{-e^x \cos x + e^x \sin x}{2} + c$$

ÖR/ $y = -x^3 + 6x^2 - 9x + 3$ fonk grafiğini çiziniz.

$T.K = (-\infty, \infty)$

$y' = -3x^2 + 12x - 9 = 0$

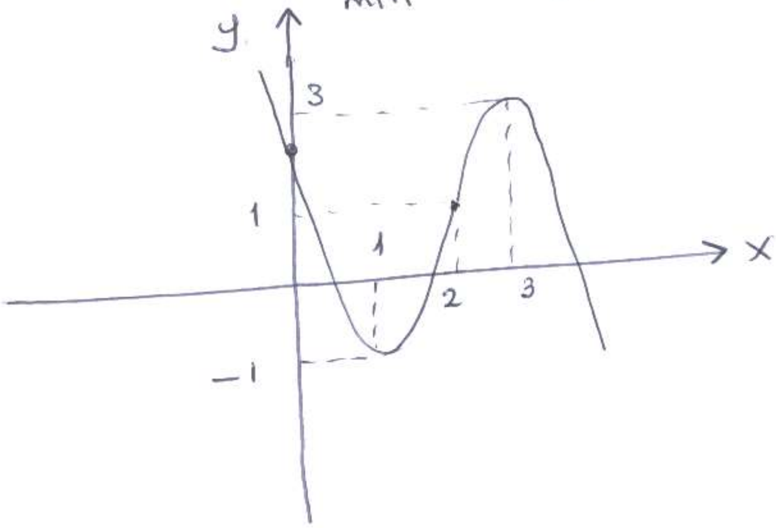
$-3(x^2 - 4x + 3) = 0$

$(x-1)(x-3) = 0$

$x_1 = 1 \quad x_2 = 3$

$y'' = -6x + 12 = 0 \Rightarrow x = 2$

x	$-\infty$	0	1	2	3	∞
y'	−	−	0	+	+ 0	− −
y''	+			+ 0 −		−
y	∞	3	−1 min	1	3 max	$-\infty$



ÖR/ $\lim\limits_{x \to 1} \dfrac{1-\sqrt{x}}{\ln x} = ?$ $\dfrac{0}{0}$ belirsizlik.

$$\lim_{x \to 1} \frac{\frac{-1}{2\sqrt{x}}}{\frac{1}{x}} = \lim_{x \to 1} \frac{-x}{2\sqrt{x}.} = -\frac{1}{2}$$

ÖR/ $\lim\limits_{x \to 0} \dfrac{x + \arcsin x}{\sin 2x} = ?$ $\dfrac{0}{0}$

$$\lim_{x \to 0} \frac{1 + \frac{1}{\sqrt{1-x^2}}}{2\cos 2x} = \frac{2}{2} = 1$$

ÖR/ $f(x) = 2x-1$ $g(x) = \dfrac{x}{2} - \dfrac{1}{x}$ olduğuna göre

$\lim\limits_{x \to 2} \dfrac{f(g(x))}{x-2}$ limitinin değeri kaçtır?

$$.(g(2)) = \frac{2}{2} - \frac{1}{2} = \frac{1}{2}$$

$$g'(x) = \frac{1}{2} + \frac{1}{x^2}$$

$$g'(2) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$\lim_{x \to 2} \frac{f\left(\frac{1}{2}\right)}{2-2} = \frac{0}{0}$$

$$\lim_{x \to 2} \frac{f'(g(x))\, g'(x)}{1} = \frac{f'(\overbrace{g(2)}^{1/2})\, g'(2)}{1} =$$

$$= \underbrace{f'(1/2)}_{2} . \frac{3}{4} = \frac{3}{2}$$

ÖR/

$$\lim_{x \to 0^+} \frac{1-\cos\sqrt{x}}{x} = ? \quad \frac{0}{0}$$

$$\lim_{x \to 0^+} \frac{\frac{1}{2\sqrt{x}} \cdot \sin\sqrt{x}}{1} = \quad 0/0$$

$$\lim_{x \to 0^+} \frac{\sin\sqrt{x}}{2\sqrt{x}} = \lim_{x \to 0^+} \frac{\cancel{\frac{1}{2\sqrt{x}}} \cos\sqrt{x}}{2 \cdot \cancel{\frac{1}{2\sqrt{x}}}} = \frac{1}{2}$$

veya $$\lim_{x \to 0^+} \frac{1}{2} \frac{\sin\sqrt{x}}{\sqrt{x}} = \frac{1}{2} \underbrace{\lim_{x \to 0^+} \frac{\sin\sqrt{x}}{\sqrt{x}}}_{1} = 1/2$$

ÖR/ Gerçel sayılar kümesi üzerinde tanımlı ve türevlenebilir bir f fonksiyonu için

$$f(x+y) = f(x) + f(y) + xy$$

$\lim_{h \to 0} \frac{f(h)}{h} = 3$ olduğuna göre $f'(1) = ?$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$\lim_{h \to 0} \frac{\cancel{f(x)} + f(h) + xh - \cancel{f(x)}}{h} = \underbrace{\lim_{h \to 0} \frac{f(h)}{h}}_{3} + \frac{xh}{h} = f'(x)$$

$$3 + x = f'(x)$$

$$f'(1) = 1 + 3 = 4$$

ÖR/ $\lim\limits_{x \to 0} \dfrac{\sin 3x}{2-\sqrt{4-x}} = ?\ \dfrac{0}{0}$ belirsizliği

$$\lim_{x \to 0} \frac{3 \cdot \cos 3x}{\dfrac{-(-1)}{2\sqrt{4-x}}} = \frac{3}{\dfrac{1}{4}} = 12$$

ÖR/ $\lim\limits_{x \to 1^+} (x-1)\ln(x^2-1) = ?$ $\quad 0(-\infty)$

$\ln 0 = x$
$e^x = 0$
$x = -\infty$

$$\lim_{x \to 1^+} \frac{\ln(x^2-1)}{\dfrac{1}{x-1}} \overset{\infty/\infty}{=} \lim_{x \to 1^+} \frac{\dfrac{2x}{x^2-1}}{\dfrac{-1}{(x-1)^2}}$$

$$= \lim_{x \to 1^+} \frac{-2x}{\cancel{(x-1)}(x+1)} \cdot (\cancel{x-1})^2$$

$$= \lim_{x \to 1^+} \frac{-2x(x-1)}{x+1} = \frac{0}{2} = 0$$

ÖR/

$\lim\limits_{x \to \infty} \dfrac{e^{-3x}+e^{2x}}{\ln x + 3e^{2x}} = ?\ \dfrac{\infty}{\infty}$ belirsizliği var.

$$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$\ln\infty = x \Rightarrow e^x = \infty$$
$$x = \infty$$

$$\ln\infty = \infty$$

$$\lim_{x \to \infty} \frac{-3\overset{\to 0}{e^{-3x}} + 2e^{2x}}{\underset{\downarrow 0}{\dfrac{1}{x}} + 3 \cdot 2e^{2x}} =$$

$$\lim_{x \to \infty} \frac{2\cancel{e^{2x}}}{6\cancel{e^{2x}}} = \frac{1}{3}$$

ÖR/ $f'(x+2) = 2x-4$ ise $f(x) = ?$

$x+2=u \qquad x=u-2$

$f'(u) = 2.(u-2)-4$

$\int f'(u)du = \int (2u-8)du$

$f(u) = \frac{2u^2}{2} - 8u + C$

$f(u) = u^2 - 8u + c$

$f(x) = x^2 - 8x + c$

ÖR/ $\lim\limits_{x \to 1^+} x^{\frac{1}{x-1}} = ?$ 1^∞ belirsizliği

$y = x^{\frac{1}{x-1}} \qquad \ln y = \ln x^{1/x-1} = \frac{1}{x-1} \ln x$

$\lim\limits_{x \to 1^+} \ln y = \lim\limits_{x \to 1^+} \frac{1}{x-1} \ln x$

$= \lim\limits_{x \to 1^+} \frac{\frac{1}{x}}{1} = 1$

$\lim\limits_{x \to 1^+} \ln y = 1$

$\lim\limits_{x \to 1^+} y = e^1 = e$ olur.

ÖR/

$$\lim_{x \to 0^+} x^{\sin x} = ? \qquad 0^0 \text{ belirsizliği}$$

$$y = x^{\sin x} \qquad \ln y = \ln x^{\sin x}$$

$$\ln y = \sin x \ln x$$

$$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \sin x \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{\sin x}}$$

$$= \lim_{x \to 0^+} \frac{1/x}{\frac{-\cos x}{\sin^2 x}}$$

$$= \lim_{x \to 0^+} \frac{\sin x}{x} \cdot \frac{\sin x}{\cos x}$$

$$= \underbrace{\lim_{x \to 0^+} \frac{\sin x}{x}}_{1} \; \lim_{x \to 0^+} \frac{\sin x}{\cos x}$$

$$1 \cdot 0 = 0$$

$$\lim_{x \to 0^+} \ln y = 0 \Rightarrow \lim_{x \to 0^+} y = e^0 = 1 \text{ olur.}$$

ÖR/

$$\lim_{x \to \infty} x^{1/\ln x} = ? \qquad \infty^0 \text{ belirsizliği}$$

$$y = x^{1/\ln x} \Rightarrow \ln y = \ln x^{1/\ln x} = \frac{1}{\ln x} \cdot \ln x = 1$$

$$\ln y = 1$$

$$\lim_{x \to \infty} \ln y = 1 \Rightarrow y = e^1 = e$$

ÖR/ $x=\sqrt{t+1}$, $y=\sin t$ ise $\dfrac{dy}{dx}=?$

$$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\cos t}{\frac{1}{2\sqrt{t+1}}}=2\sqrt{t+1}\cdot\cos t$$
$$=2x\cdot\sqrt{1-y^2}$$

$y=\sin t$

$y^2=\sin^2 t$

$1-y^2=\cos^2 t$

$\sqrt{1-y^2}=\cos t$

ÖR/ $y=x^{\sqrt{x}}$ $\quad y'=?$

$\ln y=\sqrt{x}\,\ln x$

$$\frac{y'}{y}=\frac{1}{2\sqrt{x}}\ln x+\sqrt{x}\cdot\frac{1}{x}$$

$$y'=x^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}\ln x+\frac{\sqrt{x}}{x}\right]$$

ÖR/ $2\sqrt{y}=x-y^2$ $\quad y'=?$

$$2\cdot\frac{y'}{2\sqrt{y}}=1-2yy'$$

$$y'=\sqrt{y}\,(1-2yy')$$

$$y'=\sqrt{y}-2y\sqrt{y}\,y'$$

$$y'\left[1+2y\sqrt{y}\right]=\sqrt{y}\Rightarrow y'=\frac{\sqrt{y}}{1+2y\sqrt{y}}$$

$2\sqrt{y}-x+y^2=0$

$$\frac{dy}{dx}=y'=-\frac{F_x}{F_y}$$

$$y'=-\frac{-1}{\cancel{2}\cdot\frac{1}{\cancel{2}\sqrt{y}}+2y}$$

ÖR/ $f(x)=x^3-3x^2-9x+1$ fonksiyonunun $[1,5]$ aralığındaki en küçük değerini bulunuz.

$f'(x)=3x^2-6x-9$

$3(x^2-2x-3)=0$

$(x-3)(x+1)=0$

$x=3$

$x=-1$

$f(1)=1^3-3.1^2-9.1+1$

$f(1)=1-3-9+1$

$=-10$

$f(5)=5^3-3.5^2-9.5+1$

$=125-75-45+1$

$=50-45+1$

$=6$

$f(1)=-10$

$f(5)=6$

$f(3)=-26$

$f(-1)=6$

$f(x)$ in $[1,5]$ aralığında en küçük değeri -26 dır.

ÖR/ $a \in R$ olmak üzere $f: R \to R$, $f(x)=x^3-9x^2+ax+5$ fonksiyonunun $x=2$ de bir yerel maksimumu olduğuna göre $f(1)$ değerini bulunuz.

$f(x)=x^3-9x^2+ax+5$

$f'(x)=3x^2-18x+a=0$

$f'(2)=3.2^2-18.2+a=0$

$12-36+a=0$

$a=24$

$f(x)=x^3-9x^2+24x+5$

$f(1)=1^3-9.1^2+24+5=21$

f fonksiyonunun $x = x_0$ noktasındaki türevi

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} \quad \text{veya}$$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \quad \text{limit değerlerinin}$$

var olması gerekir.

ÖR/ $f(x) = x^2 - x + 1$ olduğuna göre $\lim_{x \to 2} \frac{f(x) - f(2)}{x-2}$ değerini bulunuz.

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x-2} = f'(2)$$

$$f'(2) = \lim_{x \to 2} \frac{x^2 - x + 1 - 3}{x-2}$$

$$f'(2) = \lim_{x \to 2} \frac{x^2 - x - 2}{x-2}$$

$$f'(2) = \lim_{x \to 2} \frac{(x-2)(x+1)}{x-2} = 3 \text{ tür.}$$

ÖR/ $f: \mathbb{R} \to \mathbb{R}$ $f(x) = \begin{cases} x^2 - x + 4, & x < 3 \\ ax^2 + x, & x \geq 3 \end{cases}$

fonksiyonu $x = 3$ te türevli olduğuna göre a değerini bulunuz.

f fonksiyonu $x = 3$ te türevli ise

$f'(3^-) = f'(3^+)$ dır.

$$\lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$$

$$\lim_{x \to 3^-} \frac{x^2 - x + 4 - 10}{x - 3} = \lim_{x \to 3^+} \frac{ax^2 + x - 9a + 3}{x - 3}$$

$$\lim_{x \to 3^-} \frac{x^2 - x - 6}{x - 3} = \lim_{x \to 3^+} \frac{a(x^2 - 9) + (x - 3)}{x - 3}$$

$$\lim_{x \to 3^-} \frac{\cancel{(x-3)}(x+2)}{\cancel{x-3}} = \lim_{x \to 3^+} \frac{\cancel{(x-3)}\left[a(x+3) + 1\right]}{\cancel{x-3}}$$

$$5 = 6a + 1$$

$$6a = 4 \qquad a = \frac{2}{3}$$