Hypergeometric Distribution

* Discrete Probability Distribution
* applications same as binomial dist.
* sampling is different (SAMPLING WITHOUT REPLACEMENT)
* independence among trials NOT required

Probability Function

$$h(x; N, n, k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} \qquad x = 0, 1, 2, \dots, n$$

X random variable (the number of success in n)

n random sample

N number of items

k success in N

$N-k$ failure

EXAMPLE (Probability Using Combinatorial Analysis)

A box contains 8 red, 3 white and 9 blue balls. If 3 balls are drawn at random without replacement determine the probability that (a) all 3 are red (b) 2 are red and 1 is white a)

Method 1

R_1 red ball on 1st draw

R_2 " " " 2nd "

R_3 " " " 3rd "

$R_1 \cap R_2 \cap R_3$ all three balls drawn are red

$$P(R_1 \cap R_2 \cap R_3) = P(R_1)\,P(R_2 \mid R_1)\,P(R_3 \mid R_1 \cap R_2)$$

$$= \left(\frac{8}{20}\right)\left(\frac{7}{19}\right)\left(\frac{6}{18}\right) = \frac{14}{285}$$

Method 2

$$\text{Required probability} = \frac{\text{number of selections of 3 out of 8 red balls}}{\text{number of selections of 3 out of 20 balls}}$$

$$= \frac{{}_8C_3}{{}_{20}C_3} = \frac{14}{285}$$

b) P (2 are red and 1 is white)

$$= \frac{(\text{selections of 2 out of 8 red balls})\,(\text{selections of 1 out of 3 white balls})}{\text{number of selections of 3 out of 20 balls}}$$

$$= \frac{({}_8C_2)({}_3C_1)}{({}_{20}C_3)} = \frac{7}{95}$$

EXAMPLE

A committee of size 5 is to be selected at random from 3 chemists and 5 physicicists. Find the probability distribution for the number of chemists on the committee.

SOLUTION

X number of chemists on the committee

$$P(X=0) = h(0;8,5,3) = \frac{\binom{3}{0}\binom{5}{5}}{\binom{8}{5}} = \frac{1}{56}$$

$$P(X=1) = h(1;8,5,3) = \frac{\binom{3}{1}\binom{5}{4}}{\binom{8}{5}} = \frac{15}{56}$$

$$P(X=2) = h(2;8,5,3) = \frac{\binom{3}{2}\binom{5}{3}}{\binom{8}{5}} = \frac{30}{56}$$

$$P(X=3) = h(3;8,5,3) = \frac{\binom{3}{3}\binom{5}{2}}{\binom{8}{5}} = \frac{10}{56}$$

In tabular form

x	0	1	2	3
$h(x;8,5,3)$	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

$$h(x;8,5,3)=\frac{\binom{3}{x}\binom{5}{5-x}}{\binom{8}{5}} \qquad x=0,1,2,3$$

EXAMPLE

Lots of 40 components each are called acceptable if they contain no more than 3 defectives. The procedure for sampling the lot is to select 5 components at random and to reject the lot if a defective is found. What is the probability that exactly 1 defective will be found in the sample if there are 3 defectives in the entire lot.

$n=5$, $N=40$, $k=3$, and $x=1$

$$P=h(1;40,5,3)=\frac{\binom{3}{1}\binom{37}{4}}{\binom{40}{5}}=0.3011$$

Mean and Variance

$$\mu=\frac{nk}{N} \quad \text{and} \quad \sigma^2=\frac{N-n}{N-1}\cdot n\cdot\frac{k}{N}\left(1-\frac{k}{N}\right)$$

Relationship to Binomial Distribution when n is small compared to N (i.e. $N=5000$, $n=10$)