2.1 A transformer is made up of a 1200-turn primary coil and an open-circuited 75-turn secondary coil wound around a losed core of cross-sectional area 42 cm 2. The core material can be considered to saturate when the rms applied flux density eaches 1.45 T. What maximum 60-Hz rms primary voltage is possible without reaching this saturation level? What is the orresponding secondary voltage? How are these values modified if the applied frequency is lowered to 50 Hz?

At 60 Hz, $\omega = 120\pi$.

$$\text{primary:} \quad (V_{\rm rms})_{\max} = N_1 \omega A_c (B_{\rm rms})_{\max} = 2755 \quad \text{V, rms}$$

$$\text{secondary:} \quad (V_{\rm rms})_{\max} = N_2 \omega A_c (B_{\rm rms})_{\max} = 172 \quad \text{V, rms}$$

At 50 Hz, $\omega = 100\pi$. Primary voltage is 2295 V, rms and secondary voltage is 143 V, rms.

2.2 A magnetic circuit with a cross-sectional area of 15 cm 2 is to be operated at 60 Hz from a 120-V rms supply. Calculate the number of turns required to achieve a peak magnetic flux density of 1.8 T in the core.

$$N = \frac{\sqrt{2} V_{\rm rms}}{\omega A_c B_{\rm peak}} = 167 \quad \text{turns}$$

2.3 A transformer is to be used to transform the impedance of a 8-fl resistor to an impedance of 75 ft. Calculate the required turns ratio, assuming the transformer to be ideal.

$$N = \sqrt{\frac{75}{8}} = 3 \quad \text{turns}$$

2.4 A 100-fl resistor is connected to the secondary of an idea transformer with a turns ratio of 1:4 (primary to secondary). A 10-V rms, 1-kHz voltage source is connected to the primary.

Resistance seen at primary is $R_1 = (N_1/N_2)^2 R_2 = 6.25\Omega$. Thus

$$I_1 = \frac{V_1}{R_1} = 1.6 \quad \text{A}$$

2.5 A source which can be represented by a voltage source of 8 V rms in series with an internal resistance of 2 kOhm is connected to a 50-Ohm load resistance through an ideal transformer. Calculate the value of turns ratio for which maximum power is supplied to the load and the corresponding load power?

The maximum power will be supplied to the load resistor when its impedance, as reflected to the primary of the ideal transformer, equals that of the source (2 kΩ). Thus the transformer turns ratio N to give maximum power must be

$$N = \sqrt{\frac{R_s}{R_{\rm load}}} = 6.32$$

Under these conditions, the source voltage will see a total resistance of $R_{\rm tot} = 4$ kΩ and the current will thus equal $I = V_s/R_{\rm tot} = 2$ mA. Thus, the power delivered to the load will equal

$$P_{\rm load} = I^2 (N^2 R_{\rm load}) = 8 \quad \text{mW}$$

1.1 A magnetic circuit with a single air gap is shown in Fig. 1.24. The core dimensions are:

Cross-sectional area Ac = 1.8 × 10 -3 m 2
Mean core length lc = 0.6 m
Gap length g = 2.3 x 10 -3 m
N = 83 turns

Assume that the core is of infinite permeability ($\mu \to \infty$) and neglect the effects of fringing fields at the air gap and leakage flux. (a) Calculate the reluctance of the core R_c and that of the gap R_g. For a current of i = 1.5 A, calculate (b) the total flux $\emptyset$, (c) the flux linkages λ of the coil, and (d) the coil inductance L.

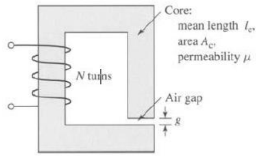

Figure 1.24 Magnetic circuit for Problem 1.1.

Part (a):

$$\mathcal{R}_c = \frac{l_c}{\mu A_c} = \frac{l_c}{\mu_r \mu_0 A_c} = 0 \quad \mathrm{A/Wb}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_c} = 1.017 \times 10^6 \quad \mathrm{A/Wb}$$

part (b):

$$\Phi = \frac{NI}{\mathcal{R}_c + \mathcal{R}_g} = 1.224 \times 10^{-4} \quad \mathrm{Wb}$$

part (c):

$$\lambda = N\Phi = 1.016 \times 10^{-2} \quad \mathrm{Wb}$$

part (d):

$$L = \frac{\lambda}{I} = 6.775 \quad \mathrm{mH}$$

1.5 The magnetic circuit of Problem 1.1 has a nonlinear core material whose permeability as a function of Bm is given by

$$\mu = \mu_0 \left(1 + \frac{3499}{\sqrt{1 + 0.047(B_m)^{7.8}}}\right)$$

where Bm is the material flux density. Find the current required to achieve a flux density of 2.2 T in the core.

$$\mu_r = 1 + \frac{3499}{\sqrt{1 + 0.047(2.2)^{7.8}}} = 730$$

$$I = B\left(\frac{g + \mu_0 l_c/\mu}{\mu_0 N}\right) = 65.8 \quad \mathrm{A}$$

1.6 The magnetic circuit of Fig. 1.25 consists of a core and a moveable plunger of width lp, each of permeability/z. The core has cross-sectional area Ac and mean length Ic. The overlap area of the two air gaps Ag is a function of the plunger position x and can be assumed to vary as

$$A_g = A_c\left(1 - \frac{x}{X_0}\right)$$

You may neglect any fringing fields at the air gap and use approximations consistent with magnetic-circuit analysis.
a. Assuming that $\mu \rightarrow \infty$, derive an expression for the magnetic flux
density in the air gap Bg as a function of the winding current I and as the plunger position is varied (0 < x < 0.8X0). What is the corresponding
flux density in the core?
b. Repeat part (a) for a finite permeability μ.

part (a):

$$H_g = \frac{NI}{2g}; \qquad B_c = \left(\frac{A_g}{A_c}\right) B_g = B_g \left(1 - \frac{x}{X_0}\right)$$

part (b): Equations

$$2gH_g + H_c l_c = NI; \qquad B_g A_g = B_c A_c$$

and

$$B_g = \mu_0 H_g; \qquad B_c = \mu H_c$$

can be combined to give

$$B_g = \left(\frac{NI}{2g + \left(\frac{\mu_0}{\mu}\right)\left(\frac{A_g}{A_c}\right)(l_c + l_p)}\right) = \left(\frac{NI}{2g + \left(\frac{\mu_0}{\mu}\right)\left(1 - \frac{x}{X_0}\right)(l_c + l_p)}\right)$$

1.7 The magnetic circuit of Fig. 1.25 and Problem 1.6 has the following dimensions
$A_c=8.2cm^2$ lc=23cm
lp = 2.8 cm g = 0.8 mm
X0 = 2.5 cm N = 430 turns
a. Assuming a constant permeability of $\mu_r = 2800$, calculate the current required to achieve a flux density of 1.3 T in the air gap when the plunger is fully retracted (x = 0).
b. Repeat the calculation of part (a) for the case in which the core and plunger are composed of a nonlinear material whose permeability is given by

$$\mu = \mu_0 \left(1 + \frac{1199}{\sqrt{1 + 0.05 B_m^8}}\right)$$

where Bm is the magnetic flux density in the material.

part (a):

$$I = B\left(\frac{g + \left(\frac{\mu_0}{\mu}\right)(l_c + l_p)}{\mu_0 N}\right) = 2.15 \quad \text{A}$$

part (b):

$$\mu = \mu_0 \left(1 + \frac{1199}{\sqrt{1 + 0.05 B^8}}\right) = 1012\, \mu_0$$

$$I = B\left(\frac{g + \left(\frac{\mu_0}{\mu}\right)(l_c + l_p)}{\mu_0 N}\right) = 3.02 \quad \text{A}$$

1.8 An inductor of the form of Fig. 1.24 has dimensions: Cross-sectional area $A_c = 3.6\ cm^2$ Mean core length lc = 15 cm N = 75 turns Assuming a core permeability of $\mu_r = 2100$ and neglecting the effects of leakage flux and fringing fields, calculate the air-gap length required to achieve an inductance of 6.0 mH.

$$g = \left(\frac{\mu_0 N^2 A_c}{L}\right) - \left(\frac{\mu_0}{\mu}\right) l_c = 0.353 \quad \text{mm}$$

1.9 The magnetic circuit of Fig. 1.26 consists of rings of magnetic material in a stack of height h. The rings have inner radius Ri and outer radius Ro. Assume that the iron is of infinite permeability ($\mu \to \infty$) and neglect the effects of magnetic leakage and fringing. For:

Ri = 3.4 cm
Ro = 4.0 cm
H = 2cm
g = 0.2 cm
calculate:
a. the mean core length lc and the core cross-sectional area Ac.
b. the reluctance of the core $\mathcal{R}_c$ and that of the gap $\mathcal{R}_g$.
For N = 65 turns, calculate:
c. the inductance L.
d. current i required to operate at an air-gap flux density of Bg = 1.35T.
e. the corresponding flux linkages ~. of the coil.

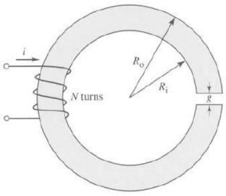

Figure 1.26 Magnetic circuit for Problem 1.9.

part (a):

$$l_c = 2\pi(R_o - R_i) - g = 3.57 \quad \text{cm}; \qquad A_c = (R_o - R_i)h = 1.2 \quad \text{cm}^2$$

part (b):

$$\mathcal{R}_g = \frac{g}{\mu_0 A_c} = 1.33 \times 10^7 \quad \text{A/Wb}; \qquad \mathcal{R}_c = 0 \quad \text{A/Wb};$$

part (c):

$$L = \frac{N^2}{\mathcal{R}_g + \mathcal{R}_g} = 0.319 \quad \text{mH}$$

part (d):

$$I = \frac{B_g(\mathcal{R}_c + \mathcal{R}_g)A_c}{N} = 33.1 \quad \text{A}$$

part (e):

$$\lambda = N B_g A_c = 10.5 \quad \text{mWb}$$

1.13 The inductor of Fig. 1.27 has the following dimensions:
Ac = 1.0 cm 2
lc = 15 cm
g = 0.8 mm
N = 480 turns
Neglecting leakage and fringing and assuming $\mu_r = 1000$, calculate the inductance.

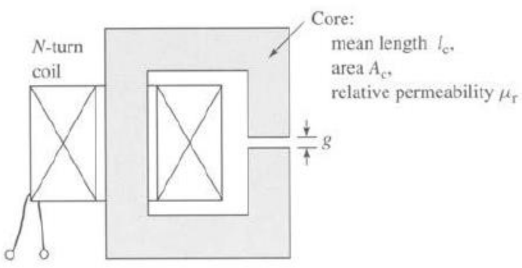

Figure 1.27 Inductor for Problem 1.12.

$$L = \frac{\mu_0 N^2 A_\mathrm{c}}{g + l_\mathrm{c}/\mu_\mathrm{r}} = 30.5 \quad \mathrm{mH}$$