Laplace Dönüşümü

Tanım: $f(t)$, $[0,\infty)$ tanımlı olsun

$$\int_0^\infty e^{-st} f(t)\,dt$$

improper integrali yakınsak olduğu taktirde bu integrale $f(t)$ fonksiyonunun Laplace dönüşümü, integralin yakınsadığı $F(s)$ fonksiyonuna, $f(t)$ nin Laplace dönüşümü denir ve

$$\int_0^\infty e^{-st} f(t)\,dt = L\{f(t)\} = F(s)$$

ile gösterilir.

çekirdek

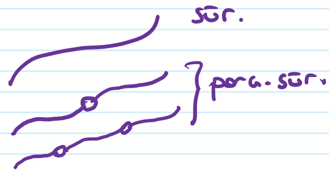
$$\int_0^\infty e^{-st} f(t)\,dt = \lim_{k\to\infty}\left[\int_0^k e^{-st} f(t)\,dt\right] = F(s)$$

1) $f(t)=1$ fonksiyonunun Laplace Dönüşümü

$$\int_0^\infty e^{-st}\, 1\, dt = \lim_{k\to\infty}\left[\int_0^k e^{-st}\,dt\right] = \lim_{k\to\infty}\left[\frac{e^{-st}}{-s}\Big|_{t=0}^{k}\right]$$

$$= -\frac{1}{s}\lim_{k\to\infty}\left[e^{-sk}-1\right] = -\frac{1}{s}\lim_{k\to\infty}\left[\underbrace{\frac{1}{e^{sk}}}_{\frac{1}{\infty}=0}-1\right] = \frac{1}{s} \qquad (s>0)$$

$$L\{1\} = \frac{1}{s} \qquad (s>0)$$



Teorem: $f(t)$, $[0,\infty)$ üzerinde parçalı sür. olsun.

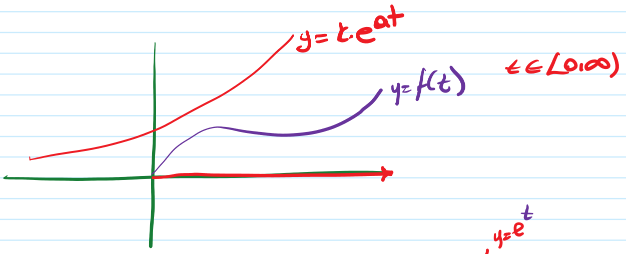
Eğer her $t\geq 0$ için

$$|f(t)| \leq K.e^{at}$$ olacak şekilde

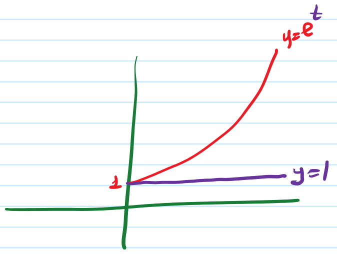
K ve a reel sabitleri varsa

$f(t)$ nin Laplace dönüşümü vardır:

$$L\{f(t)\} = F(s)$$

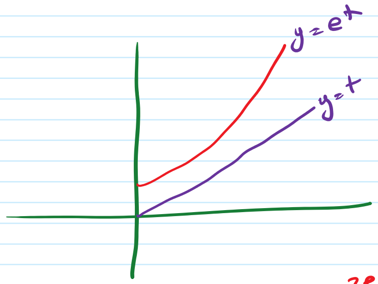


Örnek: $f(t) = 1$



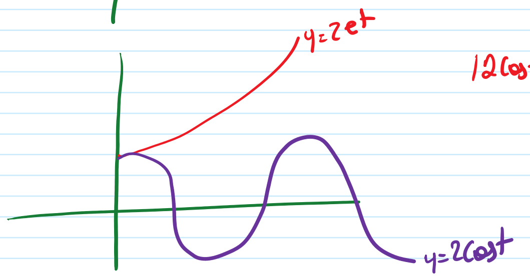
$|1| < e^t$

$f(t) = t$



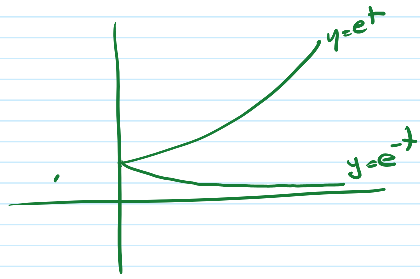
$|t| < e^t$

$f(t) = 2\cos t$



$|2\cos t| < 2e^t$

$f(t) = e^{-t}$



$|e^{-t}| < e^t$

$f(t) = e^{at}$ fonksiyonun Laplace Dönüşümü

$$\int_0^\infty e^{-st}\cdot e^{at}\,dt = \lim_{k\to\infty}\left[\int_0^k e^{(a-s)t}\,dt\right] = \lim_{k\to\infty}\left[\frac{e^{(a-s)t}}{(a-s)}\Big|_0^k\right]$$

$$= \frac{1}{a-s}\cdot\lim_{k\to\infty}\left[\underbrace{e^{(a-s)k}}_{\to 0} - 1\right] = \frac{-1}{a-s} = \frac{1}{s-a}, \quad s>a$$

$a-s<0$

$$\boxed{\mathcal{L}\{e^{at}\} = \frac{1}{s-a}, \quad s>a}$$

$f(t)=t$ fonksiyonunun Laplace Dönüşümü

$$\underbrace{\int_0^\infty e^{-st}\,t\,dt}\ =\ \underbrace{\lim_{k\to\infty}}\left[\int_0^k e^{-st}.t\,dt\right]=\lim_{k\to\infty}\left[-\frac{1}{s}te^{-st}\Big|_0^k-\int_0^k -\frac{1}{s}e^{-st}\,dt\right]$$

$t=u \qquad e^{-st}dt=dv$

$dt=du \qquad \frac{e^{-st}}{-s}=v$

$$=-\frac{1}{s}\lim_{k\to\infty}\left[+ke^{-sk}-\left(\frac{e^{-st}}{-s}\right)\Big|_0^k\right] \qquad (s>0)$$

$$=-\frac{1}{s}\lim_{k\to\infty}\left[\underbrace{\frac{k}{e^{sk}}}_{\to 0}+\frac{1}{s}\left(\underbrace{e^{-sk}}_{\to 0}-1\right)\right]=\frac{1}{s^2} \qquad s>0$$

L'H $\frac{\infty}{\infty}$

$$\lim_{k\to\infty}\frac{k}{e^{sk}}\overset{\frac{\infty}{\infty}}{=}\underset{L'H}{}\lim_{k\to\infty}\frac{1}{se^{sk}}=0$$

$$\mathcal{L}\{t\}=\frac{1}{s^2}\ ,\quad s>0$$

Laplace Dönüşümünün Lineerlik Özelliği

$\mathcal{L}\{f(t)\}=F(s) \quad , \ s>a$

$\mathcal{L}\{g(t)\}=G(s) \quad , \ s>b$

$$\mathcal{L}\{c_1f(t)+c_2g(t)\}=c_1\mathcal{L}\{f(t)\}+c_2\mathcal{L}\{g(t)\}$$
$$=c_1F(s)+c_2G(s)$$

(İnt. Lin. Öz: $\displaystyle\int_0^k\left(c_1f(t)+c_2g(t)\right)dt=c_1\int_0^k f(t)dt+c_2\int_0^k g(t)dt$)

$f(t)=\cos at$ fonksiyonun Laplace Dönüşümü :

Euler Formulü : $e^{iat}=\cos at+i\sin at \qquad (i=\sqrt{-1})$

$$+\ e^{-iat}=\cos at+-i\sin at$$

$$\cos at=\frac{e^{iat}+e^{-iat}}{2} \qquad \sin at=\frac{e^{iat}-e^{-iat}}{2i}$$

Euler Formulü : $e^{iat} = \cos at + i \sin at$ $\quad (i=\sqrt{-1})$

$$\pm \; e^{-iat} = \cos at + -i \sin at$$

$$\cos at = \frac{e^{iat} + e^{-iat}}{2} \qquad \sin at = \frac{e^{iat} - e^{-iat}}{2i}$$

$$\mathcal{L}\{\cos at\} = \mathcal{L}\left\{\frac{e^{iat} + e^{-iat}}{2}\right\} = \frac{1}{2}\left[\mathcal{L}\{e^{iat}\} + \mathcal{L}\{e^{-iat}\}\right]$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \quad (s>a)$$

$$= \frac{1}{2}\left[\frac{1}{s-ia}_{(s+ia)} + \frac{1}{s+ia}_{(s-ia)}\right]$$

$$= \frac{1}{\cancel{2}} \frac{\cancel{2}s}{s^2-(ia)^2}$$

$$\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}, \quad (s>a)$$

$f(t)=\sin at$ fonksiyonunun Laplace Dönüşümü

$$\mathcal{L}\{\sin at\} = \mathcal{L}\left\{\frac{e^{iat} - e^{-iat}}{2i}\right\} = \frac{1}{2i}\left[\mathcal{L}\{e^{iat}\} - \mathcal{L}\{e^{-iat}\}\right]$$

$$= \frac{1}{2i}\left(\frac{1}{s-ia}_{(s+ia)} - \frac{1}{s+ia}_{(s-ia)}\right) = \frac{1}{2i}\left(\frac{2ia}{s^2-(ia)^2}\right)$$

$$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}, \quad s>a$$

Örnekler: $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$ $\quad \mathcal{L}\{1\} = \frac{1}{s}$ $\quad \mathcal{L}\{t\} = \frac{1}{s^2}$ $\quad \mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$

$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$

1) $\mathcal{L}\{2e^{3t}\} = \frac{2}{s-3}$

2) $\mathcal{L}\{e^{4t} + e^{5t}\} = \frac{1}{s-4} + \frac{1}{s-5}$

1) $\mathcal{L}\{2e^{3t}\} = \dfrac{2}{s-3}$

2) $\mathcal{L}\{e^{4t} + e^{5t}\} = \dfrac{1}{s-4} + \dfrac{1}{s-5}$

3) $\mathcal{L}\{5t\} = 5 \cdot \dfrac{1}{s}$

4) $\mathcal{L}\{e^{7t} + 4t\} = \dfrac{1}{s-7} + \dfrac{4}{s}$ $\qquad \cos 2t = 2\cos^2 t - 1$

5) $\mathcal{L}\{\cos 2t + \sin 3t\} = \dfrac{s}{s^2+4} + \dfrac{3}{s^2+9}$

6) $\mathcal{L}\{\cos^2 t\} = \mathcal{L}\left\{\dfrac{\cos 2t + 1}{2}\right\} = \dfrac{1}{2}\dfrac{s}{s^2+4} + \dfrac{1}{2}\dfrac{1}{s}$

7) $\mathcal{L}\{\sin t \cdot \cos t\} = \mathcal{L}\left\{\dfrac{\sin 2t}{2}\right\} = \dfrac{1}{2}\dfrac{2}{s^2+4}$

Laplace Dönüşümünün Öteleme Özelliği (e^{at} ile Çarpma Öz)

$\mathcal{L}\{f(t)\} = F(s)$ $\quad s > b$ olsun

$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$

İspat: $\mathcal{L}\{e^{at} f(t)\} = \displaystyle\int_0^\infty e^{-st} e^{at} f(t)\,dt = \int_0^\infty e^{-(s-a)t} f(t)\,dt = F(s-a)$

$\mathcal{L}\{f(t)\} = \displaystyle\int_0^\infty e^{-st} f(t)\,dt = F(s)$ olduğundan $s \to s-a$ yazılır.

Örnek: $\mathcal{L}\{\sin t\} = \dfrac{1}{s^2+1} = F(s)$

$\mathcal{L}\{e^{2t}\sin t\} = F(s-2) = \dfrac{1}{(s-2)^2+1}$

Örnek: $\mathcal{L}\{e^{-3t}\sin\sqrt{2}t\} = ?$ $\quad F(s-a) = \dfrac{\sqrt{2}}{(s+3)^2+2}$ $\quad (s > -3)$

öteleme Öz.

$\mathcal{L}\{\sin\sqrt{2}t\} = \dfrac{\sqrt{2}}{s^2+2} = F(s)$

$$\mathcal{L}\{\sin\sqrt{2}\,t\} = \frac{\sqrt{2}}{s^2+2} = F(s)$$

Örnek: $\mathcal{L}\{e^{t} \cos 3t\} = \underset{\text{öteleme}}{F(s-1)} = \frac{s-1}{(s-1)^2+9} \quad (s>1)$

$$\mathcal{L}\{\cos 3t\} = \frac{s}{s^2+9} = F(s)$$

Örnek: $\mathcal{L}\{e^{4t} \cdot t\} = \underset{\text{öteleme}}{F(s-4)} = \frac{1}{(s-4)^2} \quad (s>4)$

$$\mathcal{L}\{t\} = \frac{1}{s^2} = F(s) \quad (s>0)$$

t^n ile Çarpma Özelliği

$\mathcal{L}\{f(t)\} = F(s)$ olsun

$$\mathcal{L}\{t \cdot f(t)\} = -F'(s)$$

$$\mathcal{L}\{t^2 f(t)\} = F''(s)$$

$$\mathcal{L}\{t^3 f(t)\} = -F'''(s)$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$$

$\mathcal{L}\{t f(t)\} = -F'(s)$ olduğunu gösterelim:

$\underbrace{\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\,dt}_{s\text{'ye göre türevini alalım}}$ olduğunu biliyoruz:

$$F'(s) = \int_0^\infty -t e^{-st} f(t)\,dt = \int_0^\infty e^{-st} (-t f(t))\,dt = \mathcal{L}\{-t f(t)\}$$

$$F'(s) = -\mathcal{L}\{t f(t)\}$$

$$\mathcal{L}\{t \cdot f(t)\} = -F'(s)$$

$\mathcal{L}\{t.\cos t\} = ?$

t ile çarpılma

$\mathcal{L}\{t\, f(t)\} = -F'(s)$

$\mathcal{L}\{\cos t\} = \dfrac{s}{s^2+1}$ $\quad F'(s) = \dfrac{s^2+1-s\,2s}{(s^2+1)^2} = \dfrac{-s^2+1}{s^2+1}$

$\mathcal{L}\{t.\cos t\} = \dfrac{s^2-1}{s^2+1}$

$\mathcal{L}\{t^2 \sin 2t\} = ?$ $F''(s)$

$\mathcal{L}\{\sin 2t\} = \dfrac{2}{s^2+4} = F(s)$ $\quad F'(s) = \dfrac{-2.\,2s}{(s^2+4)^2}$

$F''(s) = \dfrac{-4\,(s^2+4)^{\cancel{2}} + 4s\; 2.\,2s\,\cancel{(s^2+4)}}{(s^2+4)^{\cancel{4}\,3}} = \dfrac{-4s^2-16+16s^2}{(s^2+4)^3}$

$\mathcal{L}\{t^2 \sin 2t\} = \dfrac{12s^2-16}{(s^2+4)^3}$

$\mathcal{L}\{t.e^t\}$ $\nearrow$ 1. yol öteleme $\searrow$ 2. yol t ile çarpılma

1. yol: $\mathcal{L}\{t.e^t\} = \mathcal{L}\{e^t.\,t\} = F(s-1) = \dfrac{1}{(s-1)^2}$

$\mathcal{L}\{t\} = \dfrac{1}{s^2}$

2. yol: $\{t.e^t\} = -F'(s) = +\dfrac{1}{(s-1)^2}$

$\mathcal{L}\{e^t\} = \dfrac{1}{s-1} = F(s)$

$f(t)=t^n$ fonksiyonlarının Laplace Dönüşümü

$f(t) = t^2$ alalım.

$\mathcal{L}\{t^2.\,1\} = F''(s) = \dfrac{2}{s^3}$

$\mathcal{L}\{1\} = \dfrac{1}{s} = F(s)$ $\quad F'(s) = -\dfrac{1}{s^2}$ $\quad F''(s) = \dfrac{2}{s^3}$

$f(t)=t^3$ alalım

$$\mathcal{L}\{t^3\cdot 1\} = -F'''(s) = \frac{3!}{s^4}$$

$\mathcal{L}\{1\}=\frac{1}{s}=F(s)$ $\quad \underline{F'(s)=\frac{-1}{s^2}}$ $\quad \underline{F''(s)=\frac{2}{s^3}}$ $\quad \underline{F'''(s)=\frac{-2\cdot 3}{s^4}}$

$f(t)=t^4$ alalım $\qquad F^{IV}=\frac{2\cdot3\cdot4}{s^5}$

$$\mathcal{L}\{t^4\cdot 1\} = F^{IV}(s) = \frac{4!}{s^5}$$

$f(t)=t^n$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \qquad (n\in\mathbb{N})$$

f(t) fonksiyonunun türevlerinin Laplace Dönüşümü

$\mathcal{L}\{f(t)\}=F(s)$ olsun

$$\mathcal{L}\{f'(t)\} = sF(s)-f(0)$$

İspat:

$$\mathcal{L}\{f'(t)\} = \int_0^\infty e^{-st}f'(t)\,dt = \lim_{K\to\infty}\left[\int_0^K e^{-st}f'(t)\,dt\right]$$

$u=e^{-st}$ $\quad dv=f'(t)dt$

$du=-se^{-st}$ $\quad v=f(t)$

$$=\lim_{K\to\infty}\left[e^{-st}f(t)\Big|_0^K+\int_0^K se^{-st}f(t)\,dt\right]$$

$$=\lim_{K\to\infty}\left(\frac{f(t)}{e^{st}}\Big|_0^K\right)+s\underbrace{\int_0^\infty e^{-st}f(t)\,dt}$$

$(s>0)$

$$=\lim_{K\to\infty}\left(\underbrace{\frac{f(K)}{e^{sK}}}_{\to 0}-f(0)\right)+s\;F(s) \quad = sF(s)-f(0)$$

$L \to 0$

$$L\{f'(t)\} = sF(s) - f(0)$$

$$L\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$L\{f'''(t)\} = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$$

$$L\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-1}f'(0) - \dots - f^{(n-1)}(0)$$

Ters Laplace Dönüşümü

$L\{f(t)\} = F(s)$ $f(t)$ nin Laplace dönüşümü $F(s)$ ise

$F(s)$ nin ters Laplace Dönüşümü de $f(t)$ olur:

$L^{-1}\{F(s)\} = f(t)$ ile gösterilir.

Örnekler:

$L\{t^n\} = \dfrac{n!}{s^{n+1}}$

$L\{e^{at}\} = \dfrac{1}{s-a}$

$L\{\sin at\} = \dfrac{a}{s^2+a^2}$

$L\{1\} = \dfrac{1}{s}$

$L\{3t^4 + 2e^{-3t} - 4\sin 2t + 5\} =$

$3L\{t^4\} + 2L\{e^{-3t}\} - 4L\{\sin 2t\} + 5L\{1\} =$

$3 \cdot \dfrac{4!}{s^5} + 2\dfrac{1}{s+3} - 4\dfrac{2}{s^2+4} + 5 \cdot \dfrac{1}{s}$

$L^{-1}\left\{\dfrac{1}{s+3}\right\} = e^{-3t}$

$L^{-1}\left\{\dfrac{1}{s}\right\} = 1$

$L^{-1}\left\{\dfrac{1}{s^2+4}\right\} = \dfrac{1}{2}L^{-1}\left\{\dfrac{2}{s^2+4}\right\} = \dfrac{1}{2}\sin 2t$

$L^{-1}\left\{\dfrac{1}{s^5}\right\} = \dfrac{1}{4!}L^{-1}\left\{\dfrac{4!}{s^5}\right\} = \dfrac{1}{4!}t^4$

\# $\mathcal{L}\{e^{t}\sin 2t\} = \frac{2}{(s-1)^2+4}$

ötelewe

\# $\mathcal{L}\{e^{3t}\sin 4t\} = \frac{4}{(s-3)^2+16}$

\# $\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2+1}\right\} = ?\; e^{2t}\sin t$

ötelewe

$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin t$

\# $\mathcal{L}^{-1}\left\{\frac{1}{(s-5)^2+9}\right\} = ?\; \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s-5)^2+9}\right\} = \frac{1}{3}e^{5t}\sin 3t$

e^{5t} ötelewe

$\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1\cdot 3}{s^2+9}\right\} = \frac{1}{3}\sin 3t$

\# $\mathcal{L}\{t \cdot e^{2t}\} = \frac{1}{(s-2)^2}$

e^{2t} ötelewe

$\mathcal{L}\{t\} = \frac{1}{s^2}$

\# $\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2}\right\} = e^{3t}\cdot t$

ötelewe e^{3t}

$\frac{1}{s^2}$

\# $\mathcal{L}^{-1}\left\{\frac{1}{(s-7)^4}\right\} = ?\; \frac{1}{3!}\mathcal{L}^{-1}\left\{\frac{3!}{(s-7)^4}\right\} = \frac{1}{3!}\cdot e^{7t}\cdot t^3$

e^{7t} ötelewe

$\frac{1}{3!}\mathcal{L}^{-1}\left\{\frac{1\cdot 3!}{s^4}\right\} = \frac{1}{3!}\cdot t^3$

\# $\mathcal{L}\{\cos 2t\} = \dfrac{s}{s^2+4}$

\# $\mathcal{L}\{e^{3t}\cos 2t\} = \dfrac{(s-3)}{(s-3)^2+4}$

\# $\mathcal{L}^{-1}\left\{\dfrac{s-2}{(s-2)^2+1}\right\} = e^{2t}\cos t$

$\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+1}\right\} = \cos t$

\# $\mathcal{L}^{-1}\left\{\dfrac{s}{(s-2)^2+1}\right\} = ?$ $\quad \mathcal{L}^{-1}\left\{\dfrac{s-2+2}{(s-2)^2+1}\right\} = \underbrace{\mathcal{L}^{-1}\left\{\dfrac{s-2}{(s-2)^2+1}\right\}} + 2\,\mathcal{L}^{-1}\left\{\dfrac{1}{(s-2)^2+1}\right\}$

$= e^{2t}\cos t + 2e^{2t}\sin t$

\# $\mathcal{L}^{-1}\left\{\dfrac{s}{(s-1)^2+4}\right\} = ?$ $\quad \mathcal{L}^{-1}\left\{\dfrac{s-1+1}{(s-1)^2+4}\right\} = \underbrace{\mathcal{L}^{-1}\left\{\dfrac{s-1}{(s-1)^2+4}\right\}} + \dfrac{1}{2}\mathcal{L}^{-1}\left\{\dfrac{1.2}{(s-1)^2+4}\right\}$

$= e^{t}\cdot\cos 2t + \dfrac{1}{2}e^{t}\sin 2t$

\# $f(t) = t.e^{-t}$ $\quad \mathcal{L}\{f''(t)\} = ?$

$\mathcal{L}\{f(t)\} = F(s)$ $\quad \mathcal{L}\{f''(t)\} = s^2F(s) - s\,f(0) - f'(0)$

$\mathcal{L}\{t.e^{-t}\} = \dfrac{1}{(s+1)^2} = F(s)$

e^{-t} ötelem

$\mathcal{L}\{t\} = \dfrac{1}{s^2}$

$f(0) = 0$

$f'(t) = e^{-t} - te^{-t}$ $\quad f'(0) = 1$

$\mathcal{L}\{f''(t)\} = s^2\dfrac{1}{(s+1)^2} - s.0 - 1$

$\mathcal{L}\{f''(t)\} = \dfrac{s^2}{(s+1)^2} - 1$

Başlangıç Değer Probleminin Laplace Dönüşümü ile Çözümü

$$\left.\begin{aligned} a_0 y'' + a_1 y' + a_2 y = f(t) \\ y(0) = k_1 \quad y'(0) = k_2 \end{aligned}\right\}$$ Başl. Değ. Pr çözümü $y(t) = ?$

$\mathcal{L}\{y(t)\} = Y(s)$ gösterelim. Dif denk her iki tarafının Lapl. dön alalım:

$$\mathcal{L}\{a_0 y'' + a_1 y' + a_2 y\} = \mathcal{L}\{f(t)\}$$

Lineerlik Öz. $\quad a_0 \mathcal{L}\{y''\} + a_1 \mathcal{L}\{y'\} + a_2 \mathcal{L}\{y\} = \mathcal{L}\{f(t)\}$

Türevlerin Lap. Dön Form: $\quad a_0\left(s^2 Y(s) - s y(0) - y'(0)\right) + a_1\left(s Y(s) - y(0)\right) + a_2 Y(s) = F(s)$

$y(0) = k_1 \quad y'(0) = k_2$

$$a_0\left(s^2 Y(s) - s k_1 - k_2\right) + a_1\left(s Y(s) - k_1\right) + a_2 Y(s) = F(s)$$

$Y(s)$ nin cebirsel bir denklemi elde edilir.

Lapl. dön sayesinden y nin dif. denklemini, Y nin cebirsel bir denk. dönüştürdü.

Bu cebirsel denklemden $Y(s)$ yi çözeriz

$$Y(s) = \frac{F(s) + a_0 s k_1 + a_0 k_2 + a_1 k_1}{a_0 s^2 + a_1 s + a_2}$$

Ters laplace dön uygulanarak $y(t) = \mathcal{L}^{-1}\{Y(s)\}$ çözüme ulaşılır.

Örnek: $\quad y'' + 4y = 0 \quad y(0) = 5 \quad y'(0) = 3 \quad$ BDPr çöz?

$\mathcal{L}\{y(t)\} = Y(s)$: $\quad \mathcal{L}\{y'' + 4y\} = \mathcal{L}\{0\} \quad$ (I)

$$s^2 Y(s) - s y(0) - y'(0) + 4 Y(s) = 0 \quad \text{(II)}$$

$$s^2 Y(s) - 5s - 3 + 4 Y(s) = 0$$

$$Y(s) = \frac{5s + 3}{s^2 + 4} \quad \text{(III)}$$

$$Y(s) = \frac{\;\;}{s^2+4}$$

(IV) $y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{5s+3}{s^2+4}\right\} = 5\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} + \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1.2}{s^2+4}\right\}$

$$y(t) = 5\cos 2t + \frac{3}{2}\sin 2t$$

$y'' - y = -t \quad y(0)=0 \quad y'(0)=1$ BDPr ?

$\mathcal{L}\{y(t)\} = Y(s) = ?$

$(\mathcal{L}\{y\} = Y)$ $\mathcal{L}\{y'' - y\} = \mathcal{L}\{-t\}$

$$s^2Y(s) - s\cancel{y(0)}^{\,0} - \underbrace{y'(0)}_{1} - Y(s) = -\frac{1}{s^2}$$

$$s^2Y(s) - 1 - Y(s) = -\frac{1}{s^2}$$

$$(s^2-1)\,Y(s) = 1 - \frac{1}{s^2}$$

$$Y(s) = \frac{\cancel{s^2-1}}{s^2\cancel{(s^2-1)}} = \frac{1}{s^2}$$

$$\underbrace{\mathcal{L}^{-1}\{Y(s)\}}= \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$$

$$y(t) = t$$

$y'' + 3y' + 2y = 0 \quad y(0)=2 \quad y'(0)=3$

$(y(t) = e^{-2t} + e^{-t})$

$y'' - 6y' + 9y = t^2e^{3t} \quad y(0)=2 \quad y'(0)=6$

$(y(t) = \frac{1}{12}e^{3t}.t^4 + 2e^{3t})$