\# $y^{IV} - 2y''' + y'' - 2y' = 0$ G.S=?

$r^4 - 2r^3 + r^2 - 2r = 0$

$r(r^3 - 2r^2 + r - 2) = 0$

$r(r^2(r-2) + (r-2)) = 0$

$r.(r-2)(r^2+1) = 0$

$r_1 = 0 \quad r_2 = 2 \quad r_{3,4} = \pm i$

$a = 0 \quad b = 1$

$y = c_1 + c_2 e^{2x} + c_3 \cos x + c_4 \sin x$

\# Solve $D^2(D^2+9)^2 y = 0$. that is introduced operator notation

$D \leftrightarrow r$

$r^2(r^2+9)^2 = 0$

$r_1 = r_2 = 0 \quad r_{3,4} = \pm 3i = r_{5,6} \quad (a=0, b=3)$

two fold root (real repeated roots)

two fold root the conjugate complex root

$y = c_1 + c_2 x + (c_3 + c_4 x)\cos 3x + (c_5 + c_6 x)\sin 3x$

\# $y^{IV} - 2y'' + y = 0$

$r^4 - 2r^2 + 1 = (r^2-1)^2 = 0$

$r_1 = r_2 = 1 \quad r_3 = r_4 = -1$

$y_h = (c_1 + c_2 x)e^x + (c_3 + c_4 x)e^{-x}$

\# $y^{IV} + 2y'' + y = 0$

$(r^2+1)^2 = 0$

$r_{1,2} = \pm i = r_{3,4}$

$y = (c_1 + c_2 x)\cos x + (c_3 + c_4 x)\sin x$

If $W(f,g) = 3e^{4t}$ and $f(t) = e^{2t}$, then find $g(t) = ?$

$$W(f,g) = \begin{vmatrix} f(t) & g(t) \\ f'(t) & g'(t) \end{vmatrix} = \begin{vmatrix} e^{2t} & g(t) \\ 2e^{2t} & g'(t) \end{vmatrix} = 3e^{4t}$$

$$g' \cdot e^{2t} - 2e^{2t} g = 3e^{4t}$$

$$\boxed{g' - 2g = 3e^{2t}}$$ 1st order Lin. D.E.

The Method of Variation of Parameters

$$g' - 2g = 0 \qquad \frac{dg}{g} = 2dt \Rightarrow \ln g = 2t + \ln c \Rightarrow g = c.e^{2t}$$

$$\left.\begin{aligned} g &= c(t).e^{2t} \\ g' &= c'.e^{2t} + 2c.e^{2t} \end{aligned}\right\}$$

$$c'e^{2t} + \cancel{2ce^{2t}} - \cancel{2c.e^{2t}} = 3e^{2t} \Rightarrow c' = 3 \Rightarrow c = 3t + k$$

$$g(t) = 3te^{2t} + k.e^{2t}$$

Determine whether (or not) the funct $y = c_1 e^{-3x} + c_2 e^{2x}$ is a general solution of DE $y'' + y' - 6y = 0$, find a particular solution for $y(0) = 1$ $y'(0) = 1$

(I) $y_1 = e^{-3x}$ $\quad y_2 = e^{2x}$

$$\left.\begin{array}{l} y_1'' + y_1' - 6y_1 = 9e^{-3x} - 3e^{-3x} - 6e^{-3} = 0 \\ y_2'' + y_2' - 6y_2 = 4e^{2x} + 2e^{2x} - 6e^{2x} = 0 \end{array}\right\} \; y_1 \text{ and } y_2 \text{ are solutions of DE}$$

(II) y_1 and y_2 are lin. ind:

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-3x} & e^{2x} \\ -3e^{-3x} & 2e^{2x} \end{vmatrix} = 5e^{-x} \neq 0$$

(III) $y = c_1 e^{-3x} + c_2 e^{2x}$ $\qquad y' = -3c_1 e^{-3x} + 2c_2 e^{2x}$

$y(0) = c_1 + c_2 = 1$ $\qquad y'(0) = -3c_1 + 2c_2 = 1$

$$\left.\begin{array}{l} c_1 + c_2 = 1 \\ -3c_1 + 2c_2 = 1 \end{array}\right\} \quad \left.\begin{array}{l} \cancel{3c_1} + 3c_2 = 3 \\ -\cancel{3c_1} + 2c_2 = 1 \end{array}\right\} \quad c_2 = 4/5 \quad c_1 = 1/5$$

$$y_p = \frac{1}{5} e^{-3x} + \frac{4}{5} e^{2x}$$

Solve $y^{(6)} - y''' = 0$

This DE is a 6^{th} order Hom. Lin. with Constant Coefficient DE

The characteristic equation: $r^6 - r^3 = 0$

$r^3(r^3-1) = r^3(r-1)(r^2+r+1)$ $\quad a=-1/2 \quad b=\sqrt{3}/2$

$r_1 = r_2 = r_3 = 0$ $\quad r_4 = 1$ $\quad r_{5,6} = \frac{-1}{2} \mp \frac{\sqrt{3}}{2} i$

three fold root 0 $\quad$ real root $\quad$ conjugate complex roots

The part of general solution corresponding to

three fold root 0 is $(c_1 + c_2 x + c_3 x^2) . e^{0x}$

the simple root 1 is $c_4 e^x$

the conj. compl. roots is $e^{-\frac{1}{2}x} \left(c_5 \cos \frac{\sqrt{3}}{2} x + c_6 \sin \frac{\sqrt{3}}{2} x \right)$

General solution =>

$$y = c_1 + c_2 x + c_3 x^2 + c_4 e^x + e^{-\frac{1}{2}x} . \left(c_5 \cos \frac{\sqrt{3}}{2} x + c_6 \sin \frac{\sqrt{3}}{2} x \right)$$

Find a hom. 2nd order constant coef. lin. dif. eq whose general solution is given by

$$y(t) = c_1 e^{-3t} + c_2 t . e^{-3t}$$

(1)

$$r_1 = r_2 = -3$$

$$(r+3)^2 = 0$$

$$r^2 + 6r + 9 = 0.$$

$$y'' + 6y' + 9y = 0.$$

(5)

Find a homogeneous constant-coefficient linear diff eq whose general solution is given by, ($c_1, c_2, c_3, c_4, c_5, c_6$ are constants)

(2) $y = (c_1 + c_2 x + c_3 x^2) e^{2x} + c_4 e^{-2x} + (c_5 \cos 3x + c_6 \sin 3x)$

Solution:

The roots of characteristic func. are

$r_1 = r_2 = r_3 = 2$

$r_4 = -2$

$r_{5,6} = \pm 3i$

Characteristik func is $(r-2)^3 \cdot (r+2)(r^2+9) = 0$

D.E. can be written as operator. equation

$(D-2)^3 \cdot (D+2)(D^2+9) y = 0.$

(3) $y = e^x ((c_1 + c_2 x) \cos 2x + (c_3 + c_4 x) \sin 2x)$

$r_{1,2} = \underset{a=1}{1} \mp \underset{b=2}{2i} = r_{3,4}$ conj. complex double roots

$(r^2 - (r_1 + r_2) r + r_1 . r_2)^2 = 0.$

$(r^2 - 2r + 5)^2 = 0$

$(D^2 - 2D + 5)^2 y = 0$

Find the dif eq whose characteristic func is $\lambda(\lambda-1)^2(\lambda-2)=0$ and find the general solution of it

$$\lambda(\lambda^2-2\lambda+1)(\lambda-2)=0$$

$$(\lambda^3-2\lambda^2+\lambda)(\lambda-2)=0$$

$$\lambda^4-\underline{2\lambda^3}+\lambda^2-\underline{2\lambda^3}+4\lambda^2-2\lambda=0$$

$$\lambda^4-4\lambda^3+5\lambda^2-2\lambda=0$$

$$y^{IV}-4y'''+5y''-2y'=0$$

$$\lambda_1=0 \qquad \lambda_2=\lambda_3=1 \qquad \lambda_4=2$$

$$y=c_1+(c_2+c_3x)e^x+c_4e^{2x}$$

Given that the some solutions of a dif. eq. are

t^2e^t, e^{-t}, t, $\cos t$.

write characteristic equation and gen sol. of the DE.

if t^2e^t is a solution of d.e, then

e^t, te^t are also solutions of D.E

Because $r=1$ is three fold root of CE.

if t is a solution of DE. $r=0$ is double root

so 1 is another solution

if $\cos t$ is a sol then $\sin t$ must be a sol.

$$y_{GC}=c_1+c_2te^t+c_3t^2e^t+c_4e^{-t}+(c_5+c_6t)+c_7\cos t+c_8\sin t$$

$$CE=(r-1)^2\cdot(r+1)\cdot r^2\cdot(r^2+1)=0$$