$y'' - xy = 0$ dif. denk. x'in kuv. göre kuv. seri çöz. bulunuz.

$\underbrace{x\text{'in kuv. göre}}_{x=0 \text{ civarında}}$

$\hookrightarrow y = \sum_{n=0}^{\infty} a_n x^n$ çözümünü arıyoruz:

$$y' = \sum_{n=1}^{\infty} a_n . n . x^{n-1} = a_1 + \ldots$$

$$y'' = \sum_{n=2}^{\infty} a_n . n . (n-1) x^{n-2}$$

$$y'' - xy = \sum_{n=2}^{\infty} a_n n(n-1) x^{n-2} - x \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} a_n . n(n-1) x^{n-2} - \sum_{n=0+3}^{\infty} a_{n-3} x^{n+1-3} = 0$$

$$\sum_{n=2}^{\infty} a_n n(n-1) x^{n-2} - \sum_{n=3}^{\infty} a_{n-3} x^{n-2} = 0$$

İlk terimi serinin dışında yaz ($n=2$)

$$2a_2 + \sum_{n=3}^{\infty} a_n . n(n-1) x^{n-2} - \sum_{n=3}^{\infty} a_{n-3} x^{n-2} = 0$$

$$2a_2 + \sum_{n=3}^{\infty} \left[a_n n(n-1) - a_{n-3} \right] x^{n-2} = 0$$

$$\sum_{n=k}^{\infty} a_n (x-x_0)^n \mp \sum_{n=k}^{\infty} b_n (x-x_0)^n = \sum_{n=k}^{\infty} [a_n \mp b_n](x-x_0)^n$$

$$\sum_{n=k}^{\infty} a_n = a_k + a_{k+1} + \ldots = \sum_{n=k-2}^{\infty} a_{n+2} = a_k + a_{k+1} + \ldots = \sum_{n=k+3}^{\infty} a_{n-3} = a_k + a_{k+1} + \ldots$$

$$\sum_{n=k}^{\infty} a_n (x-x_0)^n = 0$$

$$a_n = 0$$

$\underline{\underline{a_2 = 0}}$ $\boxed{a_n . n(n-1) - a_{n-3} = 0}$ Rekürans Bağıntısı $(n \geq 3)$

$$a_n = \frac{a_{n-3}}{n(n-1)} \quad (n \geq 3) \Rightarrow$$

$n=3 \quad a_3 = \frac{a_0}{6}$, $n=4 \quad a_3 = \frac{a_1}{12}$

2 inci mert. Hom. L.D.D. genel çöz.

$y = c_1 y_1 + c_2 y_2$

keyfi sb.

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + \cancel{a_2 x^2} + a_3 x^3 + a_4 x^4 + \ldots$$

$$= a_0 + a_1 x + \frac{a_0}{6} x^3 + \frac{a_1}{12} x^4 + \ldots$$

$$= a_0 \left[1 + \frac{1}{6} x^3 + \ldots \right] + a_1 \left[x + \frac{1}{12} x^4 + \ldots \right]$$

G.Ç: $y = a_0 \cdot \overbrace{y_1} + a_1 \cdot \overbrace{y_2}$

$y'' + 2xy' + 2y = 0$ $\quad x_0=0$ civ. k.s çözümü?

$y = \sum_{n=0}^{\infty} a_n x^n$ çözümü aranır.

$$\sum_{n=2}^{\infty} a_n n(n-1)x^{n-2} + 2x \cdot \sum_{n=1}^{\infty} a_n n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2-2}^{\infty} a_{n+2}(n+2)(n+1)x^{n-2+2} + \sum_{n=1}^{\infty} 2a_n n x^n + \sum_{n=0}^{\infty} 2a_n x^n = 0$$

$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n + \sum_{n=1}^{\infty} 2a_n n x^n + \sum_{n=0}^{\infty} 2a_n x^n = 0$$

$$\sum_{n=3}^{\infty} a_n = a_3 + a_4 + \sum_{n=5}^{\infty} a_n$$

$n=0$ (ilk terimi serinin dışında yaz) $\quad n=0$

$$2a_2 + \sum_{n=1}^{\infty} a_{n+2}(n+2)(n+1)x^n + \sum_{n=1}^{\infty} 2a_n n x^n + 2a_0 + \sum_{n=1}^{\infty} 2a_n x^n = 0$$

$$\underbrace{2a_2 + 2a_0}_{0} + \sum_{n=1}^{\infty} \underbrace{\left[a_{n+2}(n+2)(n+1) + 2a_n n + 2a_n\right]}_{0} x^n = 0$$

$a_2 + a_0 = 0$

$a_2 = -a_0$

$$a_{n+2}(n+2)(n+1) + 2a_n(n+1) = 0$$

$$\boxed{a_{n+2} = -\frac{2a_n}{n+2}, \quad n \geq 1}$$

$n=1 \quad a_3 = -\frac{2a_1}{3}$

$n=2 \quad a_4 = -\frac{\cancel{2}a_2}{\cancel{4}_2} = \frac{a_0}{2}$

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots$$

$$= a_0 + a_1 x - a_0 x^2 - \frac{2}{3} a_1 x^3 + \frac{a_0}{2} x^4 + \dots$$

$$= a_0 \underbrace{\left[1 - x^2 + \frac{1}{2}x^4 \dots\right]}_{y_1} + a_1 \underbrace{\left[x - \frac{2}{3}x^3 \dots\right]}_{y_2}$$

$y'' - xy = 0$ $\quad (x-1)$'in kuv. göre bir kuvvet serisi çözümü bulunuz.

$y = \sum^{\infty} a_n (x-1)^n$ biç. k.s. çözüm aranır.

$y'' - xy = 0$ $(x-1)$'in kuv. göre bir kuvvet serisi çözümü bulunuz.

$y = \sum_{n=0}^{\infty} a_n (x-1)^n$ biç. k.s. çözüm aranır.

$$y' = \sum_{n=1}^{\infty} a_n . n (x-1)^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} a_n . n(n-1)(x-1)^{n-2}$$

$$y'' - xy = \sum_{n=2}^{\infty} a_n . n(n-1)(x-1)^{n-2} - x \sum_{n=0}^{\infty} a_n (x-1)^n = 0$$

$$\sum_{n=2}^{\infty} a_n . n . (n-1) . (x-1)^{n-2} - [(x-1)+1] \sum_{n=0}^{\infty} a_n (x-1)^n$$

$$\sum_{n=2-2}^{\infty} a_{n+2} (n+2)(n+1)(x-1)^{n-2+2} - \sum_{n=0+1}^{\infty} a_{n-1} (x-1)^{n+1-1} - \sum_{n=0}^{\infty} a_n (x-1)^n = 0$$

$$\sum_{n=0}^{\infty} a_{n+2} (n+2)(n+1)(x-1)^n - \sum_{n=1}^{\infty} a_{n-1} (x-1)^n - \sum_{n=0}^{\infty} a_n (x-1)^n = 0$$

$(n=0)$ $(n=0)$

$$\underbrace{2a_2 - a_0}_{0} + \sum_{n=1}^{\infty} \underbrace{\left[a_{n+2}(n+2)(n+1) - a_{n-1} - a_n \right]}_{0} (x-1)^n = 0$$

$a_2 = \frac{1}{2} a_0$ $a_{n+2} = \dfrac{a_{n-1} + a_n}{(n+2)(n+1)}$ $(n \geq 1)$ $n=1$ $a_3 = \dfrac{a_0 + a_1}{3.2}$

$$y = \sum_{n=0}^{\infty} a_n (x-1)^n = a_0 + a_1 (x-1) + \frac{1}{2} a_0 (x-1)^2 + \frac{a_0 + a_1}{6} (x-1)^3 + \dots$$

GÇ: $y = a_0 \left[1 + \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 \dots \right] + a_1 \left[(x-1) + \frac{1}{6}(x-1)^3 \dots \right]$

Ödev: $y'' - (x-2) y' + 2y = 0$ $x_0 = 2$ civ. k.s. çözümü bulunuz.

$y = \sum_{n=0}^{\infty} a_n (x-2)^n$. $\sum_{n=2}^{\infty} \dots - (x-2) \sum_{n=1}^{\infty} n . a_n . (x-2)^{n-1} + 2 \sum_{n=0}^{\infty} \dots = 0$

Ödev: $y'' - (x-2) y' + 2y = 0$ $x_0 = 0$ civ. k.s. çözümü

$y = \sum^{\infty} a_n . x^n$ $\sum^{\infty} \dots - (x-2) \sum n . a_n x^{n-1} + 2 \sum^{\infty} a_n x^n = 0$

$$y=\sum_{n=0}^{\infty} a_n x^n \qquad \sum_{n=2}^{\infty} \cdots - (x-2)\sum_{n=1}^{\infty} n a_n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$$

Ödev: $y'' - (x-2)y' + 2y = 0$ $\quad x_0 = 1$ civ. k.s. çözümü?

$$y=\sum_{n=0}^{\infty} a_n (x-1)^n \qquad \sum_{n=2}^{\infty} a_n n(n-1)(x-1)^{n-2} - \left[(x-1)-1\right]\sum_{n=1}^{\infty} a_n n (x-1)^{n-1} + 2\sum_{n=0}^{\infty} a_n (x-1)^n = 0$$

Ödev: $y'' + x(x-1)y' + (x+3)y = 0$ $\quad x_0 = 0$ civ. k.s çöz?

$$(x^2-x)\sum \quad + \quad (x+3)\sum_{n=0}^{\infty} a_n x^n$$

Ödev: $(x^2+1)y'' + xy' - y = 0$ $\quad x_0 = 0$ k.s. çöz bulunuz.

iki (lin.bsız) kuv. seri çözümlerini belirleyiniz.

$$y = a_0\underbrace{\left(1+\tfrac{1}{2}x^2+\cdots\right)}_{?} + a_1\underbrace{(\;x\;)}_{?}$$

Ödev: $y'' + (x-1)^2 y' - 4(x-1)y = 0$ $\quad x_0 = 1$ k.s çöz?

$\underbrace{xy'' = xy'^2 + 4y''}_{y \text{ yi içermeyen}}$, $\quad y' = ?$

$$\left.\begin{aligned} y' &= p \\ y'' &= p' \end{aligned}\right\}$$

$$xp' = xp^2 + 4p'$$

$$(x-4)p' = xp^2$$

$$\int \frac{dp}{p^2} = \int \frac{x-4+4}{x-4}\,dx = \int\left(1+\frac{4}{x-4}\right)dx$$

$$-\frac{1}{p} = x + 4\ln|x-4| + C$$

$$\underset{y'}{p} = -\frac{1}{x+4\ln|x-4|+c}$$

Hom. kısmına k.g. çözüm

\# $y'' - 2y' + y = xe^x \longrightarrow y_H = (c_1 + c_2 x)e^x$ veriliyor

Par. Değ. Yön (Lagr. Yönt) $c_2(x) = ?$

$$y = c_1 e^x + c_2 x e^x$$

GÇ $$y = c_1(x) y_1 + c_2(x) y_2$$

$y_1 = e^x \quad y_2 = x e^x$

$$\begin{cases} c_1' y_1 + c_2' y_2 = 0 \\ c_1' y_1' + c_2' y_2' = f(x) \end{cases}$$

$$\begin{array}{l} -1/\cancel{c_1' e^x} + c_2' \cancel{x e^x} = 0 \\ + \ \cancel{c_1' e^x} + c_2'(e^x + \cancel{x e^x}) = x e^x \\ \hline \end{array}$$

$$c_2' \cancel{e^x} = x \cancel{e^x} \qquad c_2' = x$$

$$c_2(x) = \frac{x^2}{2} + k_2$$

\# $y'' + \frac{y'}{y} = 0$ $\boxed{y' = ?}$

x'i içermeyen dif denk

$$\boxed{y' = p} \quad y'' = p\frac{dp}{dy}$$

$$p\frac{dp}{dy} + \frac{p}{y} = 0$$

$$\underset{\neq 0}{p}\underbrace{\left(\frac{dp}{dy} + \frac{1}{y}\right)}_{0} = 0$$

$$\frac{dp}{dy} + \frac{1}{y} = 0$$

$$\int dp + \int \frac{dy}{y} = \int 0$$

$$p + \ln y = \ln c$$

$$p = \ln\frac{c}{y}$$

$$\downarrow$$

$$y' = \ln\frac{c}{y}$$

\# $y''' - 2y'' + y' - 2y = x$ Lagr. Yönt (Par. Değ. Yönt) Sist. As. Hesaplayın?

$$r^3-2r^2+r-2=0$$
$$r^2(r-2)+(r-2)=0$$
$$(r-2)(r^2+1)=0$$
$$r_1=2 \quad r_{2,3}=\mp i$$

$$y_1=e^{2x} \quad y_2=\cos x \quad y_3=\sin x$$
$$y=c_1(x)e^{2x}+c_2(x)\cos x+c_3(x)\sin x$$

$$\left.\begin{aligned} &c_1' e^{2x}+c_2'\cos x+c_3'\sin x=0\\ &c_1' 2e^{2x}+c_2'(-\sin x)+c_3'\cos x=0\\ &c_1' 4e^{2x}+c_2'(-\cos x)+c_3'(-\sin x)=x \end{aligned}\right\}$$

Ödev: $y''+ay'^2=0$ G.Ç: $y=\frac{1}{2}\ln(2ax+c_2) \Rightarrow a=?$ ②

Ödev: $y''-3y'+2y = 2x^2+e^x+2xe^x+4e^{3x}$

$r^2-3r+2=0$

$(r-2)(r-1)=0$

$y_H=c_1y_1+c_2y_2$: $S=\{y_1,y_2\}=\{e^x, e^{2x}\}$

$y_p=y_1+y_2+y_3$

$y_1=Ax^2+Bx+C \quad r\neq 0$

$y_2=(1+2x)e^x \Rightarrow$ 1.yol $y_2=x(Dx+E)e^x \quad r=1$ ✓

$y_3=Fe^{3x} \quad r\neq 3$

Not: $f(x)=e^{\alpha x}\cdot P_m(x)$

2.yol: $f(x)=e^{\alpha x}(A\cos\lambda x+B\sin\lambda x)$

Değ. Dön. Yönt

$y=z\,e^{\alpha x}$ değ. dön. uyg.

z nin yeni dif. denk. bulunur →

z_p özel çözümü belirlenir. →

$y_2(x)=(1+2x)e^x \rightarrow y_{p_2}=?$

$y=z\cdot e^x$ değ. dön.

$\hookrightarrow y'=z'e^x+ze^x$

$y''=z''e^x+\underline{z'e^x+z'e^x}+ze^x$

$y''-3y'+2y=(1+2x)e^x$ uygun dif. denk. $y_p\left(=ze^x\right)$ değ. dön. uyg. elde edilecek dif. denk. ös. homojendir

$y''-3y'+2y=0$

$$(z''+2z'+z)\cancel{e^x}-3.(z'+z)\cancel{e^x}+2ze^{\cancel{x}}=(1+2x)\cancel{e^x}$$

$$z''+2z'+\cancel{z}-3z'-\cancel{3z}+\cancel{2z}=1+2x$$

$$\boxed{z''-z'=1+2x} \quad z_p=?$$

$r^2-r=r(r-1)=0$

$z'' - z' = 1+2x$ $\qquad z_p = ?$

$r^2 - r = r(r-1) = 0$

$g(x) = 1+2x \qquad z_p = (ax+b)x \qquad r=0$

$z = ax^2 + bx$

$z' = 2ax + b$

$z'' = 2a$

$2a - 2ax - b = 1 + 2x$

$a = -1 \qquad 2a - b = 1$

$-2 - b = 1$

$b = -1$

$z_p = -x^2 - x \Rightarrow y = z \cdot e^x$

$$\boxed{y_{p_2} = (-x^2 - x)e^x}$$

Ödev: $3y''' + 4y'' - 5y' - 2y = 2x\cos\frac{x}{3} - 3e^{-2x} + e^x \sin x$

$y_{Gen} = y_H + y_{p_1} + y_{p_2} + y_{p_3}$

$y''x + 3y' = \frac{\ln x}{x^2} - \frac{1}{x}y$ $\qquad$ (Sb. Kats D.D) den?

$x \,/\, y''x + \frac{1}{x}y + 3y' = \frac{\ln x}{x^2}$ $\qquad$ Değ. Kats. L D.D.

C. Euler ?

$x^2y'' + 3xy' + y = \frac{\ln x}{x}$ (Euler

$\ln x = t$

$x = e^t$

$y' = e^{-t}Dy$

$y'' = e^{-2t}D(D-1)y$

$\cancel{e^{2t}} \cdot \cancel{e^{-2t}} D(D-1)y + 3\cancel{e^{t}} \cdot \cancel{e^{-t}} Dy + y = \frac{t}{e^t}$

$D = \frac{d}{dt}$

$D^2y - Dy + 3Dy + y = e^{-t} \cdot t$

$Dy = \frac{dy}{dt}$

$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = t e^{-t}$

$x^2 \,/\, y'' - \frac{3}{x}y' + \frac{4}{x^2}y = x$ $\qquad$ Lin. Değ. Kats D.D. $\rightarrow$ Sb Kats Dön. G.Ç bul.

$x^2y'' - 3xy' + 4y = x^3$ $\qquad$ C. Euler ?

$x = e^t$

$3ye^{2xy}dx - 4bxe^{2xy}dy = x\,dx$ tam olması $b=?$

$$Pdx + Qdy = 0 \qquad P_y \stackrel{?}{=} Q_x$$

$$\underbrace{(3ye^{2xy} - x)}_{P}dx - \underbrace{4bxe^{2xy}}_{Q}dy = 0$$

$$\left.\begin{array}{l} P_y = 3e^{2xy} + 6yxe^{2xy} \\ Q_x = -4be^{2xy} - 8bxye^{2xy} \end{array}\right] \qquad -4b = 3 \qquad b = -\frac{3}{4}$$

$y'^2 - yy'' = 0$ $\quad \underbrace{y(0) = y'(0) = 1}_{x=0 \quad y=1 \quad y'=1}$ çözüm?

x yok

$$\left.\begin{array}{l} y' = p \\ y'' = p\dfrac{dp}{dy} \end{array}\right\} \qquad p^2 - y.p\frac{dp}{dy} = 0 \qquad p\left(p - y\frac{dp}{dy}\right) = 0$$

① $p = 0 \Rightarrow y' = 0 \Rightarrow y = c$

② $p - y\dfrac{dp}{dy} = 0 \qquad \displaystyle\int \frac{dy}{y} = \int \frac{dp}{p} \Rightarrow \ln c + \ln y = \ln p \quad \rightarrow p = cy$

$y = 1,\ y' = 1 \Rightarrow \quad y' = cy \quad c = 1 \quad \Rightarrow y' = y$

$$\frac{dy}{dx} = y \quad \rightarrow \int \frac{dy}{y} = \int dx$$

$x = 0,\ y = 1$ $\quad \ln y = x + k$

$$\left.\begin{array}{l} \ln y = x + k \\ \ln 1 = 0 + k \Rightarrow k = 0 \end{array}\right\} \ln y = x \qquad y = e^x$$

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