Cevaplar

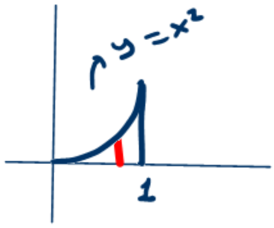
(1) $\int_0^1\int_{\sqrt{y}}^1 \frac{dx\,dy}{\sqrt{1+3x^3}}$ integrasyon sırasını değiştirirsek

D: $y=1$ $\quad y=0 \rightarrow$ x-eks.

$x=\sqrt{y}$ $\quad x=1$

$\downarrow$

$y=x^2$



$$I=\int_0^1\int_0^{x^2}\frac{dy\,dx}{\sqrt{1+3x^3}}$$

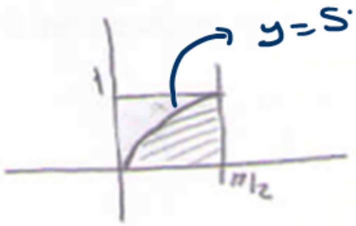
Cevap B

---

(2) $\int_0^1\int_{\arcsin y}^{\pi/2} e^{\cos x}\,dx\,dy$ int. sırasını değiştirirsek yeni integral?

D: $y=1$ $\quad y=0$

$x=\frac{\pi}{2}$ $\quad \underbrace{x=\arcsin y}_{y=\sin x}$



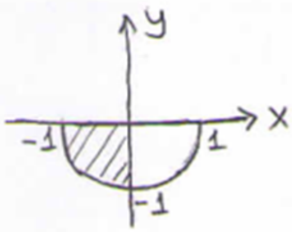
$$I=\iint_0 e^{\cos x}\,dy\,dx=\int_0^{\pi/2}\int_0^{\sin x} e^{\cos x}\,dy\,dx$$

Cevap C

---

(3) $\int_{-1}^0\int_{-\sqrt{1-x^2}}^0 \frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx$ kutupsal dönüşüm ile yazınca oluşan integral?

$R:\begin{cases} x=0, & x=-1 \\ y=0, & y=-\sqrt{1-x^2} \end{cases}$



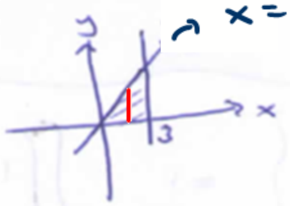
$$I=\int_{\pi}^{\frac{3\pi}{2}}\int_0^1 \frac{2}{1+\underbrace{\sqrt{r^2}}_{r}}\,r\,dr\,d\theta$$

Cevap B

④ $\int_0^1\int_{3y}^3 e^{x^2}\,dx\,dy$ int. sırasını değiştirince yeni integral?

x-ekseni

$D: y=0 \quad y=1 \quad x=3 \quad x=3y$



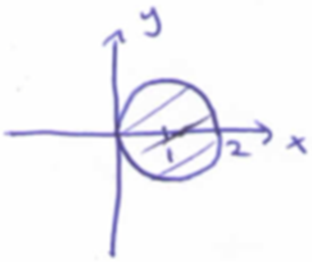
$\to x=3y \to y=\frac{x}{3}$

$I=\int_0^3\int_0^{x/3} e^{x^2}\,dy\,dx$

Cevap B

⑤ $D: x^2+y^2=2x$ bölgesinde $f(x,y)=x^2+y^2$ fonksiyonunun iki katlı integralini kutupsal koor. yazınız.

a) $\int_{-\pi/2}^{\pi/2}\int_0^1 r^3\,dr\,d\theta$ b) $\int_{-\pi/2}^{\pi/2}\int_0^2 r^3\,dr\,d\theta$ c) $\int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta} r^3\,dr\,d\theta$



$x^2+y^2-2x=0 \Rightarrow (x-1)^2+y^2=1$

$$\left.\begin{aligned} x&=r\cos\theta\\ y&=r\sin\theta\\ dx\,dy&=r\,dr\,d\theta\\ x^2+y^2&=r^2\end{aligned}\right\}$$

$I=\int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta} r^2\cdot r\,dr\,d\theta$

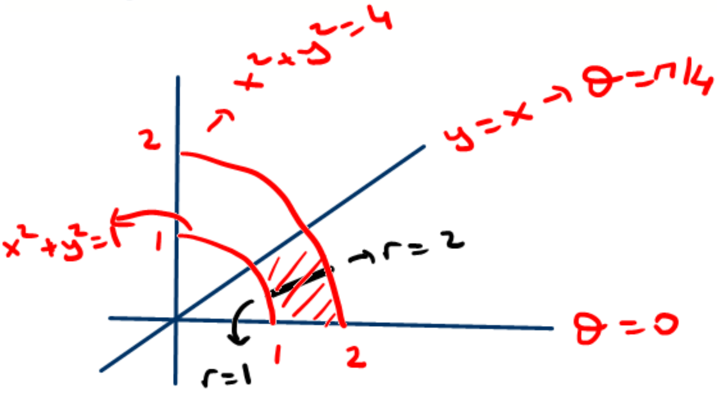
Cevap C

$x^2+y^2-2x=0$

$r^2-2r\cos\theta=0 \quad r=0 \quad r=2\cos\theta$

⑥ R bölgesi : $\{(x,y)\in\mathbb{R}^2 \mid 1\le x^2+y^2\le 4,\ 0\le y\le x\}$ olmak üzere $\iint_R \operatorname{Arctan}\left(\frac{y}{x}\right) dA$ integralini kutupsal koor. yazınız.

a) $\int_0^{\pi/2}\int_1^4 \theta \, dr\, d\theta$ b) $\int_0^{\pi/4}\int_1^2 \theta\, r\, dr\, d\theta$ c) $\int_0^{\pi/4}\int_1^2 r \tan\theta\, dr\, d\theta$



$$I=\int_0^{\pi/4}\int_1^2 \theta . r . dr\, d\theta$$

Cevap B

$$\left.\begin{array}{l} x=r\cos\theta \\ y=r\sin\theta \end{array}\right\} \frac{y}{x}=\tan\theta$$

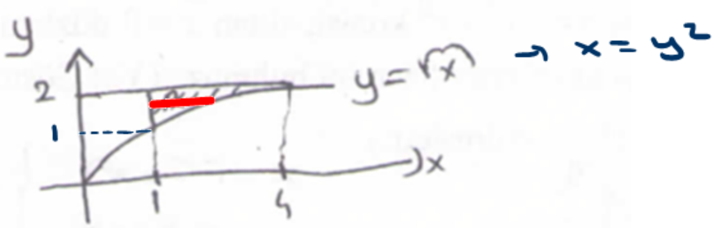
$dx\,dy = r\,dr\,d\theta$

$\operatorname{Arctan}\frac{y}{x} = \operatorname{Arctan}(\tan\theta)=\theta$

---

⑦ $\int_1^4\int_{\sqrt{x}}^2 \frac{e^y}{y+1}\, dy\, dx$ int. sırasını değiştirince yeni int.?

R: $\begin{cases} y=\sqrt{x}, & y=2 \\ x=1, & x=4 \end{cases}$

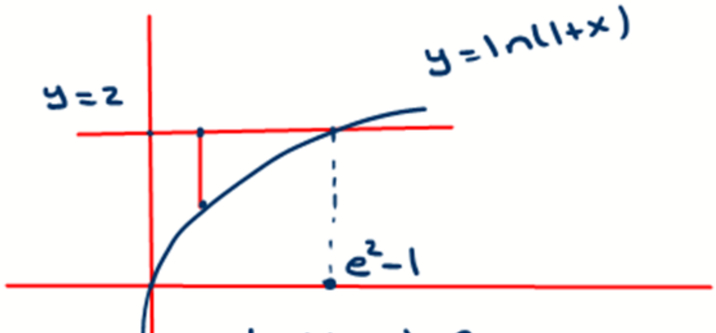


$$I=\int_1^2\int_1^{y^2} \frac{e^y}{y+1}\, dx\, dy$$

Cevap B

⑧ $y=\ln(1+x)$, $x=0$, $y=2$ ile sınırlı bölgenin alanını veren integral hangisi(leri) olabilir?

I. $\displaystyle\int_0^2\int_0^{e^y} dx\,dy$ II. $\displaystyle\int_0^{e^2-1}\int_{\ln(1+x)}^{2} dy\,dx$ III. $\displaystyle\int_0^2\int_0^{e^y-1} dx\,dy$

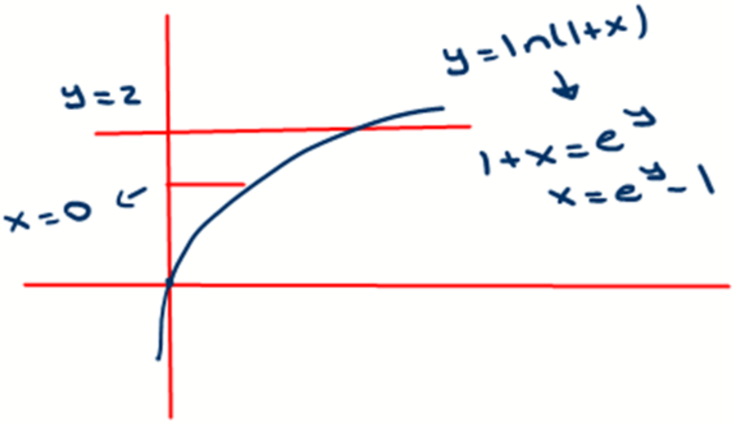


$\ln(1+x)=2$

$1+x=e^2 \rightarrow x=e^2-1$

★ x'e göre

$$\int_0^{e^2-1}\int_{\ln(1+x)}^{2} dy\,dx \rightarrow \text{II}$$



★ y'ye göre

$$\int_0^{2}\int_0^{e^y-1} dx\,dy \rightarrow \text{III}$$

Cevap C

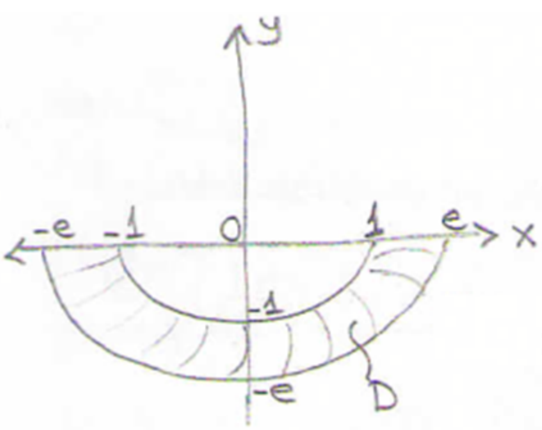
---

⑨ $D=\{(x,y)\in\mathbb{R}^2 \mid 1\le x^2+y^2\le e^2 \;,\; y\le 0\}$ bölgesinin alanını veren kutupsal integral?

$x=r\cos\theta$

$y=r\sin\theta$

$dx\,dy=r\,dr\,d\theta$



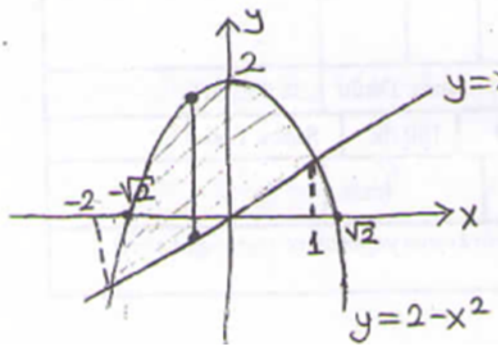
$$A=\iint_D dx\,dy$$

$$=\int_{\pi}^{2\pi}\int_1^{e} r\,dr\,d\theta \qquad r\,d\theta$$

Cevap C

(10) D bölgesi $y=2-x^2$ ve $y=x$ ile sınırlı bölge olsun. $z=2x^2y$ ile üstten, $z=0$ daki D bölgesi ile alttan sınırlanan cismin hacmini veren integral?

a) $\int_0^1 \int_x^{2-x^2} 2x^2y \, dy\, dx$ b) $\int_{-2}^1 \int_x^{2-x^2} 2x^2 \, dy\, dx$ c) $\int_{-2}^2 \int_0^{2-x^2} 2x^2y \, dy\, dx$



$2-x^2=x$

$x^2+x-2=0$

$x=-2\ ,\ x=1$

$$V=\iint_D 2x^2y \, dx\, dy$$

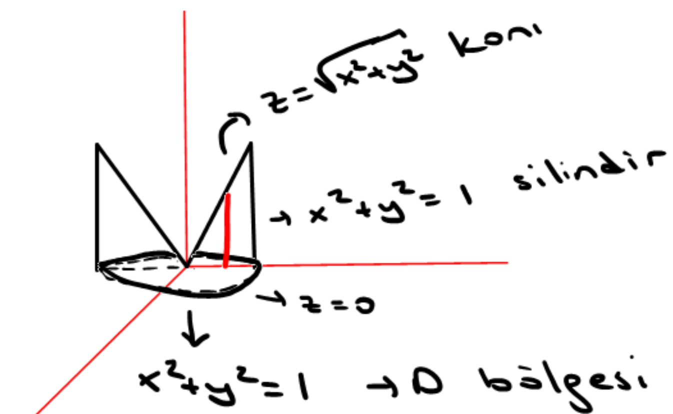
$$=\int_{-2}^1 \int_x^{2-x^2} 2x^2y \, dy\, dx$$

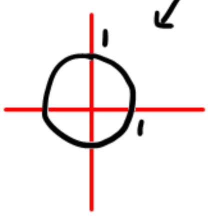
Cevap B

---

(11) üstten $z=\sqrt{x^2+y^2}$ yüzeyi, alttan $z=0$ düzlemi, yandan $x^2+y^2=1$ yüzeyi ile sınırlı cismin hacmini veren integral?

a) $\int_0^{2\pi} \int_0^1 r\, dr\, d\theta$ b) $\int_0^{2\pi} \int_0^1 r^2\, dr\, d\theta$ c) $\int_0^{\pi} \int_0^1 r^2\, dr\, d\theta$





$x=r\cos\theta$

$y=r\sin\theta$

$x^2+y^2=r^2$

$dx\,dy=r\,dr\,d\theta$

$$V=\iint_D \underbrace{(\text{yükseklik})}_{\sqrt{x^2+y^2}-0} . dx\, dy$$

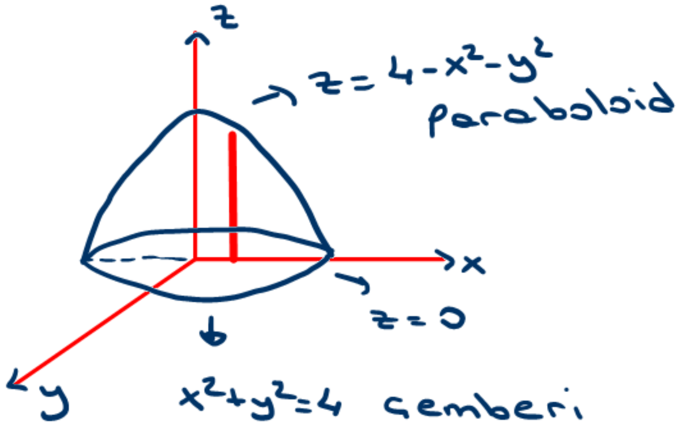
$$=\iint_D \sqrt{x^2+y^2}\, dx\, dy$$

$$=\int_0^{2\pi} \int_0^1 r.r\, dr\, d\theta$$

Cevap B

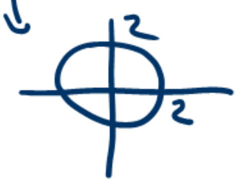
(12) $z = 4 - x^2 - y^2$ ve $z = 0$ yüzeylerinin sınırladığı cismin hacmini veren int.?

a) $\int_0^{2\pi}\int_0^4 r\,dr\,d\theta$ b) $\int_0^{2\pi}\int_0^2 (4r - r^3)\,dr\,d\theta$ c) $\int_0^{2\pi}\int_0^1 (4r - r^3)\,dr\,d\theta$



$x^2 + y^2 = 4$ çemberi

D bölgesi



$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$dx\,dy = r\,dr\,d\theta$

$$V = \iint_D (\text{yük.})\,dx\,dy$$

$$= \iint_D (4 - x^2 - y^2)\,dx\,dy$$

$$= \int_0^{2\pi}\int_0^2 (4 - r^2)\,r\,dr\,d\theta$$

Cevap B

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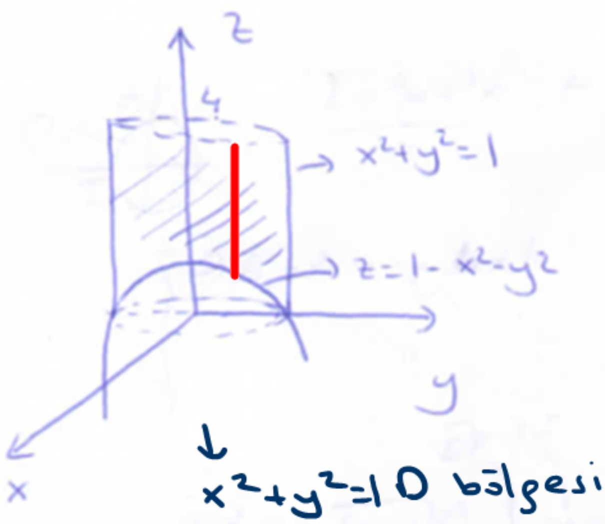
(13) $x^2 + y^2 = 1$, $z = 4$, $z = 1 - x^2 - y^2$ yüzeyleri ile sınırlı cismin hacmini veren integral?



$x^2 + y^2 = 1$ D bölgesi

$x^2 + y^2 = r^2$

$dx\,dy = r\,dr\,d\theta$

$x = r\cos\theta$

$y = r\sin\theta$

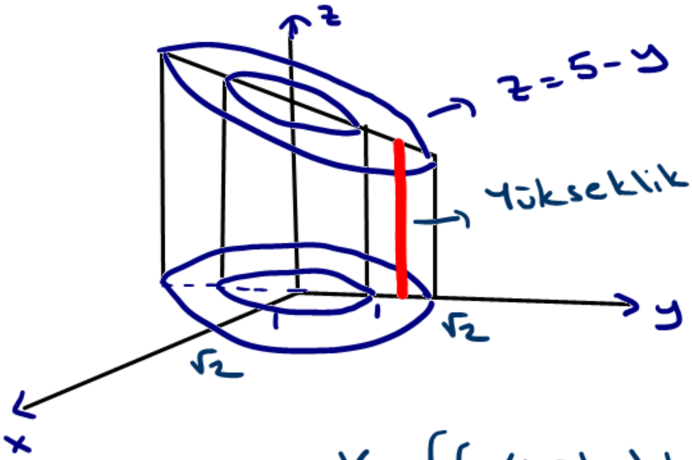
$$V = \iint_D \underbrace{(\text{yük.})}_{4 - (1 - x^2 - y^2)}\,dx\,dy$$

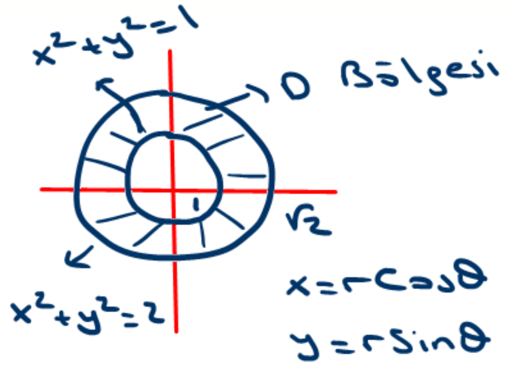
$$= \iint_D (3 + x^2 + y^2)\,dx\,dy$$

$$= \int_0^{2\pi}\int_0^1 (3 + r^2)\,r\,dr\,d\theta$$

Cevap C

(14) $5=z+y$ düzleminin altında, $z=0$ düzleminin üstünde, $x^2+y^2=2$ ve $x^2+y^2=1$ silindirleri arasında kalan cismin hacmini veren int.?





$x=r\cos\theta$

$y=r\sin\theta$

$dx\,dy=r\,dr\,d\theta$

$$V=\iint_D (\text{yük.})\,dx\,dy$$

$$=\iint_D (5-y)\,dx\,dy=\int_0^{2\pi}\int_1^{\sqrt{2}}(5-r\sin\theta)\,r\,dr\,d\theta$$

Cevap A

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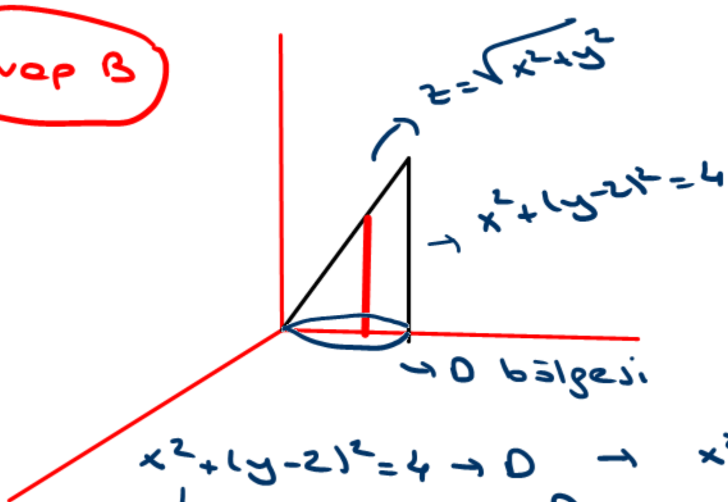
(15) $x^2+y^2-4y=0$, $z=0$, $z=\sqrt{x^2+y^2}$ yüzeyler, arasındaki cismin hacmini veren integral?

a) $\int_0^{2\pi}\int_0^{4} r^2\,dr\,d\theta$ b) $\int_0^{\pi}\int_0^{4\sin\theta} r^2\,dr\,d\theta$ c) $\int_0^{\pi}\int_0^{2} r^2\,dr\,d\theta$

Cevap B



(*) $x^2+y^2-4y+4-4=0$

$x^2+(y-2)^2=4$

ötelenmiş silindir

(*) $z=\sqrt{x^2+y^2}$ → Koni

$x^2+(y-2)^2=4 \to D \to x^2+y^2=4y \to r^2=4r\sin\theta$

$r=4\sin\theta$

$x=r\cos\theta$

$y=r\sin\theta$

$dx\,dy=r\,dr\,d\theta$

$$V=\iint_D \sqrt{x^2+y^2}\,dx\,dy$$

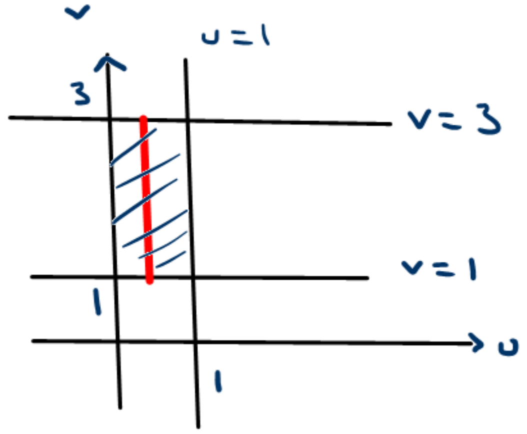
$$=\int_0^{\pi}\int_0^{4\sin\theta} r.r\,dr\,d\theta$$

(16) $R: \left\{ \begin{matrix} x+y=1 \\ x+y=3 \\ x-y=0 \\ x-y=1 \end{matrix} \right\}$ bölgesinde $I = \iint_R \frac{x^2-y^2}{\sqrt{1+(x-y)^2}}\, dA$ integralini $u = x-y$ $v = x+y$ dönüşümü ile yeniden yazınız.

$u = x-y \qquad v = x+y$

(1)

R		R'
$x+y=1$	$\longrightarrow$	$v=1$
$x+y=3$	$\longrightarrow$	$v=3$
$x-y=0$	$\longrightarrow$	$u=0$
$x-y=1$	$\longrightarrow$	$u=1$



(2) $J = \dfrac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}} = \dfrac{1}{2} \Rightarrow dx\,dy = \dfrac{1}{2} du\,dv$

(3) $$I = \iint_R \frac{\overbrace{x^2-y^2}^{u.v}}{\sqrt{1+\underbrace{(x-y)^2}_{u^2}}} \overbrace{dx\,dy}^{\frac{1}{2}du\,dv} = \int_0^1 \int_1^3 \frac{uv}{\sqrt{1+u^2}} \cdot \frac{1}{2}\, dv\, du$$

Cevap A

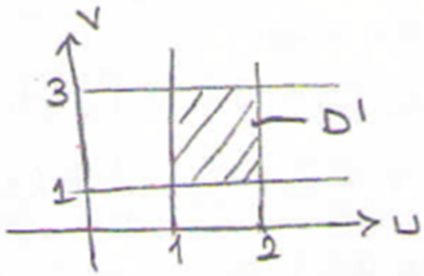
(17) $D: \left\{ \begin{array}{l} y=x^2 \\ y=2x^2 \\ x=y^2 \\ x=3y^2 \end{array} \right\}$ bölgesinin alanını veren integrali, $u=\dfrac{y}{x^2}$ $v=\dfrac{x}{y^2}$ dönüşümü ile yazarsak integral?

a) $\displaystyle\int_1^2\int_1^3 \frac{1}{3u^2v^2}\,dv\,du$ b) $\displaystyle\int_1^2\int_1^3 dv\,du$ c) $\displaystyle\int_1^3\int_1^3 \frac{1}{3u^2v^2}\,du\,dv$

$u=\dfrac{y}{x^2}$

$v=\dfrac{x}{y^2}$

D		D'
$\dfrac{y}{x^2}=1$	$\rightarrow$	$u=1$
$\dfrac{y}{x^2}=2$	$\rightarrow$	$u=2$
$\dfrac{x}{y^2}=1$	$\rightarrow$	$v=1$
$\dfrac{x}{y^2}=3$	$\rightarrow$	$v=3$



$$\frac{D(x,y)}{D(u,v)} = \frac{1}{\dfrac{D(u,v)}{D(x,y)}} = \frac{1}{\begin{vmatrix} -2y/x^3 & 1/x^2 \\ 1/y^2 & -2x/y^3 \end{vmatrix}} = \frac{x^2y^2}{3} = \frac{1}{3u^2v^2}$$

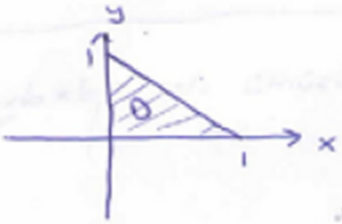
Cevap A

$$A=\iint_{D'} \frac{1}{3u^2v^2}\,du\,dv = \frac{1}{3}\int_1^2\int_1^3 \frac{1}{u^2v^2}\,dv\,du$$

(18) $\displaystyle\int_0^1\int_0^{1-x} \sqrt{x+y}\cdot(y-2x)^2\,dy\,dx$ integralini $u=x+y$ $v=y-2x$ dönüşümü ile yeniden yazarsak yeni int.?

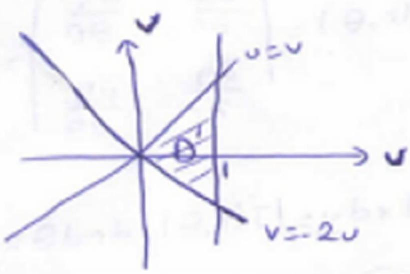
(1)

$D: \left. \begin{array}{l} x=1 \\ x=0 \\ y=1-x \\ y=0 \end{array} \right\}$



$u=x+y$ $v=y-2x$

D				D'
$x+y=1$	$\rightarrow$			$\boxed{u=1}$
$x=0$	$\rightarrow$	$u-v=0$	$\rightarrow$	$\boxed{u=v}$
$y=0$	$\rightarrow$	$2u+v=0$	$\rightarrow$	$\boxed{v=-2u}$



② $u = x+y \quad v = y-2x$

$$J = \frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix}} = \frac{1}{3} \Rightarrow dx\,dy = \frac{1}{3}\,du\,dv$$

③ $$\iint_D \sqrt{x+y}\,.\,(y-2x)^2\,dx\,dy = \iint_{D'} \sqrt{u}\,.\,v^2\,.\,\frac{1}{3}\,dv\,du$$

$$= \int_0^1 \int_{-2u}^{u} \sqrt{u}\,.\,v^2\,\frac{1}{3}\,dv\,du$$

Cevap B

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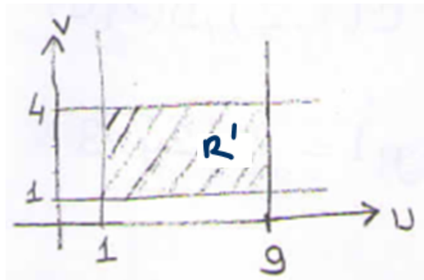
⑲ $R: \left\{ \begin{array}{l} xy=1 \\ xy=9 \\ y=x \\ y=4x \end{array} \right\}$ → 1. Bölgedeki R bölgesinin alanını veren integrali $xy=u$, $\frac{y}{x}=v$ dön. ile yazınız.

$xy = u \qquad \frac{y}{x} = v$

①

R		R'
$xy=1$	→	$u=1$
$xy=9$	→	$u=9$
$\frac{y}{x}=1$	→	$v=1$
$\frac{y}{x}=4$	→	$v=4$



② $$J = \frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}} = \frac{1}{\begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix}} = \frac{1}{2\frac{y}{x}} = \frac{1}{2v} \rightarrow dx\,dy = \frac{1}{2v}\,dv\,du$$

③ $$A = \iint_R dx\,dy = \int_1^9 \int_1^4 \frac{1}{2v}\,dv\,du$$

Cevap B

(20) R: $y=x$, $y=x^3$, $x\geq 0$, $y\geq 0$ olsun. $f(x,y)=e^{2x^2-x^4}$ fonksiyonunun R bölgesindeki ortalama değeri?

a) e   b) e-1   c) e-2   d) e+1

$$\text{Ort. Değer} = \frac{1}{R\text{nın Alanı}} \iint_R f\,dA$$

$$\iint_R dA = \int_0^1 \int_{x^3}^{x} dy\,dx = \int_0^1 \int_y^{\sqrt[3]{y}} dx\,dy = 1/4$$

$$\iint_R e^{2x^2-x^4}\,dA = \int_0^1 \int_{x^3}^{x} e^{2x^2-x^4}\,dy\,dx = \int_0^1 e^{2x^2-x^4}(x-x^3)\,dx = \int^1 \frac{1}{4} e^u du = \frac{1}{4}(e-1)$$

$$\text{Ortalama Değer} = \frac{1}{\cancel{1/4}} \cdot \frac{\cancel{1}}{4}(e-1) = e-1$$

Cevap B

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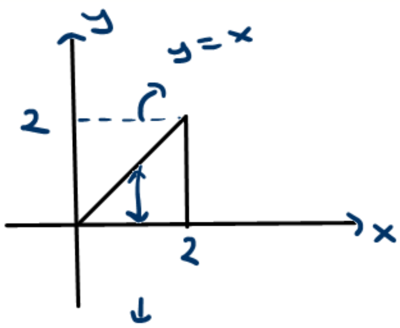
(21) Köşe noktaları $(0,0)$, $(2,0)$ ve $(2,2)$ olan üçgen bölge üzerinde $f(x,y)=x$ fonksiyonunun ortalama değeri nedir?



$$\bar{f} = \frac{\iint_D f\,dx\,dy}{D'\text{nin alanı}}$$

D'nin alanı $= \frac{2.2}{2} = 2$

$$\iint_D f\,dx\,dy = \int_0^2 \int_0^x x\,dy\,dx$$

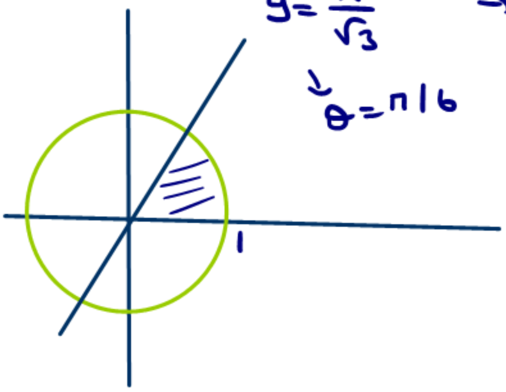
$$= \int_0^2 xy\Big|_0^x dx = \int_0^2 x^2 dx$$

$$= \frac{x^3}{3}\Big|_0^2 = \frac{8}{3}$$

$$\bar{f} = \frac{\frac{8}{3}}{2} = \frac{4}{3}$$

Cevap A

22) D bölgesi; 1. bölgede, $y = \frac{1}{\sqrt{3}}x$ doğrusu, x-ekseni ve $x^2 + y^2 = 1$ çemberi ile sınırlı bölge olmak üzere D bölgesi üzerinde $\int\int \cos(x^2 + y^2)dA$ integralinin kutupsal formu aşağıdakilerden hangisidir?



$y = \frac{x}{\sqrt{3}} \rightarrow \frac{y}{x} = \frac{1}{\sqrt{3}} \rightarrow \tan\theta = \frac{1}{\sqrt{3}} \rightarrow \theta = \frac{\pi}{6}$

$\theta = \pi/6$

$x = r\cos\theta \qquad y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta \qquad x^2 + y^2 = r^2$

$$I = \int_0^{\pi/6}\int_0^1 \cos r^2 \cdot r\,dr\,d\theta$$

Cevap A

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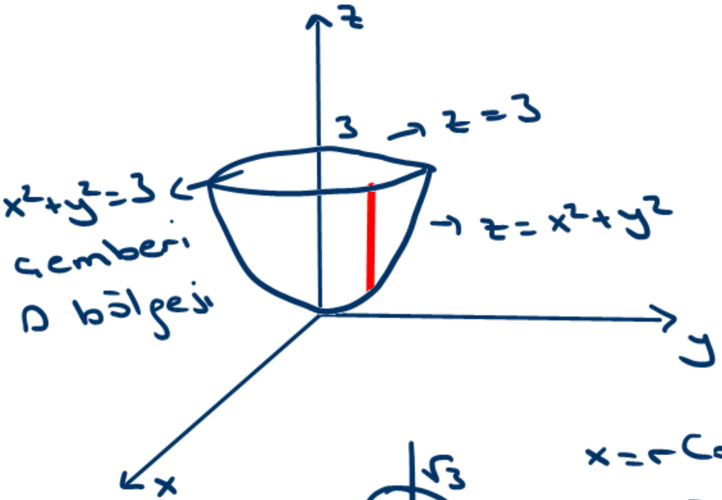
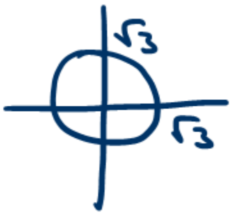
23) $z = x^2 + y^2$ yüzeyi ve $z = 3$ düzlemi arasında kalan hacmi veren integral aşağıdakilerden hangisidir?



$$V = \iint_D (\text{yük.})\,dx\,dy$$

$$= \iint_D (3 - x^2 - y^2)\,dx\,dy$$

$$= \int_0^{2\pi}\int_0^{\sqrt{3}} (3 - r^2)\,r\,dr\,d\theta$$



$x = r\cos\theta$

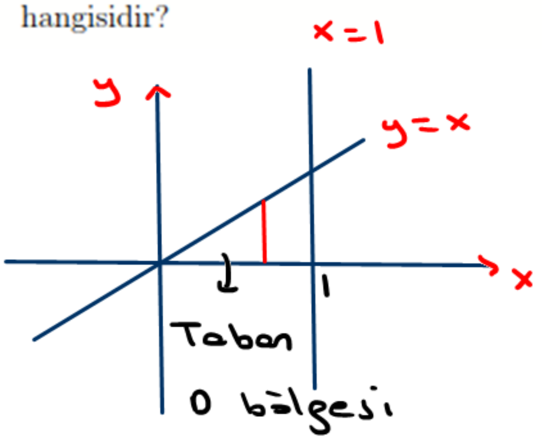
$y = r\sin\theta$

$r^2 = x^2 + y^2$

$dx\,dy = r\,dr\,d\theta$

Cevap D

24 Alttan xy-düzlemindeki $x \le 1$, $y \ge 0$, $y \le x$ eşitsizlikleri ile tanımlanan bölge ve üstten $z = 4 - 4(x^2 + y^2)$ yüzeyi ile sınırlanan katı cismin hacmini veren integral aşağıdakilerden hangisidir?



$$V = \iint_D (4-4(x^2+y^2))\,dx\,dy$$

$$= \int_0^1 \int_0^x (4-4(x^2+y^2))\,dy\,dx$$

Cevap A

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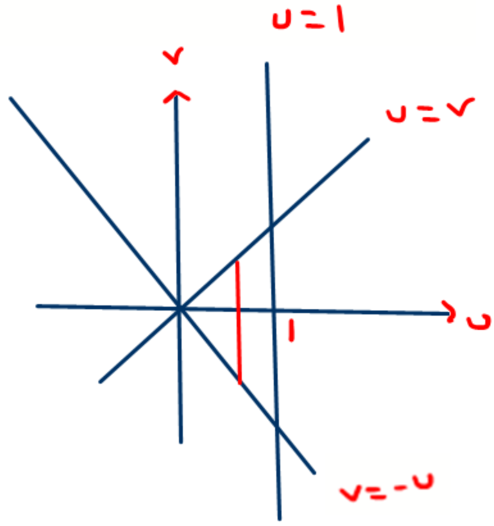
25 R bölgesi, $x = 0$, $y = 0$ ve $x - y = 1$ doğruları ile sınırlı bölge olmak üzere $\int\int_R e^{(x-y)(x+y)}dxdy$ integrali, $x - y = u$, $x + y = v$ dönüşümleri altında aşağıdaki integrallerden hangisiyle ifade edilebilir?

$u = x - y \qquad v = x + y$

1

R		R'			
$x-y=1$	$\to$	$u=1$			
$x=0$	$\to$	$u=-y$	$v=y$	$\to$	$u=-v$
$y=0$	$\to$	$u=x$	$v=x$	$\to$	$u=v$



2

$$J = \frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{1}{2} \quad \Rightarrow dx\,dy = \frac{1}{2}du\,dv$$

3

$$\iint_R e^{\overbrace{(x-y)}^{u}\overbrace{(x+y)}^{v}}dx\,dy = \int_0^1 \int_{-u}^{u} e^{uv}\frac{1}{2}dv\,du$$

Cevap B