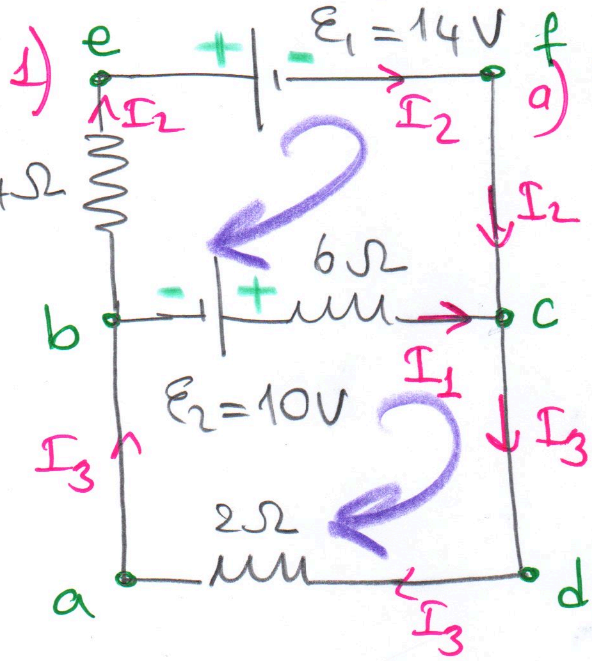
Exercises About DC Circuits

1)



a) Find the currents I_1, I_2, and I_3 for the circuits

We have two closed loops in the circuits. So we can apply Kirchhoff's rules to obtain I_1, I_2, and I_3 currents.

To describe the loops, let's use letters shown in the figure. In this question, the directions of the currents flow are given. So point c can be considered as a junction to write 1st Kirchhoff's rule:

$$\sum I_{in} = \sum I_{out} \Rightarrow \boxed{I_1 + I_2 = I_3} \quad (1)$$

for the 2nd Kirchhoff's rule, let's consider the direction of circulation as clockwise!

for abcda loop $\quad \sum \Delta V = 0$

$$\underbrace{\varepsilon_2}_{} - I_1 . 6 - I_3 . 2 = 0 \rightarrow \boxed{10 - 6I_1 - 2I_3 = 0} \quad (2)$$

we leave the battery from its positive terminal. So that $\varepsilon_2 > 0$

The direction of circulation and the directions of the I_1 and I_3 are the same. Due to this reason, the electric potential across these resistance must be negative!

for befcb loop

$$\underbrace{-4I_2}- \varepsilon_2 + 6I_1 - \varepsilon_2 = 0 \rightarrow \boxed{6I_1 - 4I_2 = 24} \quad (3)$$

on the befc branch there is only I_2 current

By using Eqs. (1), (2), and (3) we can find the related electric currents.

$$6I_1 + 2\overbrace{(I_1 + I_2)}^{I_3} = 10$$

$$6I_1 - 4I_2 = 24$$

$I_1 = 2A \qquad I_2 = -3A \qquad I_3 = I_1 + I_2$

$I_3 = -1A$

Please do not change the signs of the related currents for our following calculations!

we found negative values for the I_2 and I_3 currents. It means that I_2 and I_3 currents flow in the opposite direction to what we describe in the question.

b) Find the electric potential difference of $V_b - V_c$:

$V_b + \varepsilon_2 - 6I_1 = V_c$ from bc path

$V_b - V_c = 6I_1 - \varepsilon_2 = 6(2) - 10 = 2V$

$V_b - I_2 4 - \varepsilon_1 = V_c$ from befc path

$V_b - V_c = \varepsilon_1 + 4I_2 = 14 + 4\cdot(-3) = 2V$

$V_b + 2I_3 = V_c$ from badc path

$V_b = -2I_3 = -2(-1) = 2V$

c) find the powers supplied by the batteries and consumed by the resistors.

$P_{\varepsilon_1} = \varepsilon_1 I_2 = 14.3 = 42$ Watt

$P_{\varepsilon_2} = \varepsilon_2 . I_1 = 10.2 = 20$ Watt

$P_{supplied} = 42 + 20 = 62$ Watt

$P_{R_1} = I_1^2 6 = 2^2.6 = 24$ Watt

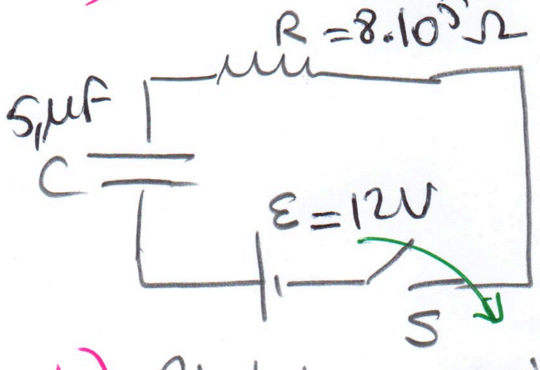
$P_{R_2} = I_2^2 4 = (-3)^2 4 = 36$ Watt

$P_{R_3} = I_3^2 2 = 1^2.2 = 2$ Watt

$P_{consumed} = 24 + 36 + 2 = 62$ Watt

2) An uncharged capacitor and resistor are connected in series to a battery as shown in the figure. If $\varepsilon = 12V$, $C = 5\mu F$ and $R = 8.10^5 \Omega$, find

a) the time constant of the circuit



Time constant $= \tau = RC$

$\tau = 8.10^5 . 5.10^{-6} = 40.10^{-1}$

$\boxed{\tau = 4s}$

b) find the maximum charge on the capacitor.

when the capacitor is fully charged, $I = 0$

$$\varepsilon - IR - \frac{q}{C} = 0$$

$$\varepsilon = \frac{Q_{max}}{C} \Rightarrow Q_{max} = \varepsilon C = 12.5.10^{-6}$$

$$\boxed{Q_{max} = 60\mu C}$$

c) find the maximum current in the circuit.

when $I = I_{max}$ $\quad q = 0$ $(\Delta V_C = 0)$ (just after closing switch S)

$$\varepsilon - IR - \cancelto{0}{\frac{q}{C}} = 0$$

$$\varepsilon = I_{max}R \rightarrow I_{max} = \frac{\varepsilon}{R} = \frac{12}{8.10^5}$$

$$I_{max} = 15.10^{-6}\,A$$

$$I_{max} = 15\,\mu A$$

d) find an expression for the charge and current as a function of time.

for charging process $\quad q(t) = Q_{max}\left[1 - e^{-t/RC}\right]$

$$q(t) = 60.10^{-6}\left[1 - e^{-t/4}\right]\ (C)$$

$$I = I_{max}\,e^{-t/RC}$$

$$I = 15.10^{-6}\,e^{-t/4}\ (A)$$

e) find the ratio of the charge on the capacitor at $t = 18\,s$ to the final charge on it.

final charge at the end of charging process must be Q_{max}.

$\frac{q(t=18s)}{Q_{max}} \rightarrow$ we will find this ratio.

$$q(t) = Q_{max}\left[1 - e^{-t/\tau}\right]$$

$$\frac{q(t=18s)}{Q_{max}} = 1 - e^{-18/4} = 1 - 0,011 = 0,988$$

when $t = 18\,s$, the 98,8 % of the capacitor is charged !

3) A capacitor C discharges through a resistor R

a) after how many time constants does its charge fall to one-half its initial value?

$$q = Q_{max}\, e^{-t/RC} \quad \text{for discharging process}$$

$$\frac{\cancel{Q_{max}}}{2} = \cancel{Q_{max}}\, e^{-t/\tau}$$

$$\frac{1}{2} = e^{-t/\tau} \Rightarrow \ln\left(\frac{1}{2}\right) = -\frac{t}{\tau}$$

$$t = -\tau \underbrace{\ln\left(\frac{1}{2}\right)}_{-0,69} = 0,69\,\tau$$

b) After how many time constants does the stored energy drop to half its initial energy?

At the beginning of the discharging process $U = U_{max}$

$$U = U_{max} = \frac{1}{2}\frac{Q_{max}^2}{C}$$

for any t during the discharging process q decreases.

$$U = \frac{1}{2}\frac{q(t)^2}{C} = \frac{1}{2}\frac{\left[Q_{max}\, e^{-t/\tau}\right]^2}{C}$$

$$U = \underbrace{\frac{1}{2}\frac{Q_{max}^2}{C}}_{U_{max}}\, e^{-2t/\tau} = U_{max}\, e^{-2t/\tau}$$

the energy stored on the capacitor during discharging process

$$U = \frac{\cancel{U_{max}}}{2} = \cancel{U_{max}}\, e^{-2t/\tau}$$

$$\frac{1}{2} = e^{-2t/\tau}$$

$$t = -\frac{\tau}{2}\ln\left(\frac{1}{2}\right) = 0,35\,\tau$$

c) After how many time constants, does the current in the RC circuit fall to one-half its initial current.

$$I = -I_{max} e^{-t/\tau}$$

$$\frac{\cancel{I_{max}}}{2} = \cancel{I_{max}} e^{-t/\tau}$$

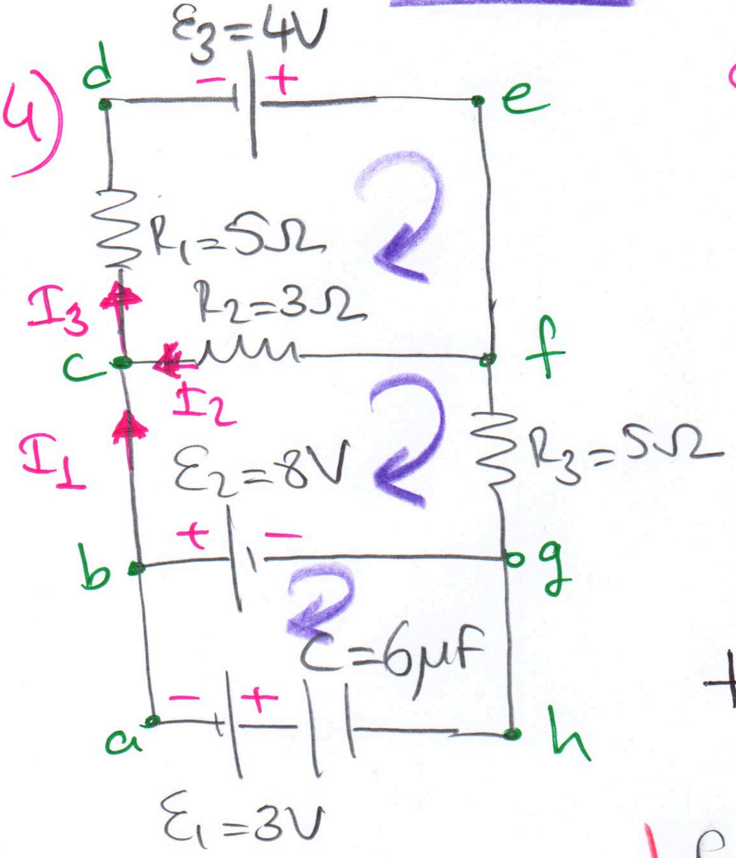
$$\frac{1}{2} = e^{-t/\tau}$$

$$t = -\tau \ln\left(\frac{1}{2}\right)$$

$$t = 0{,}69\,\tau$$

When we want to find a particular time to decrease the current, we consider the absolute value of this equation, Because negative sign corresponds to the direction of the current!

4)



a) find the steady currents of I_1, I_2, and I_3 when the circuit is running for a long time.

When the circuit is running for a long time, the capacitor is fully charged. So that there will be no current flowing through ah path.

$\boxed{\text{for ah branch } I=0}$

for junction c $\sum I_{in} = \sum I_{out}$

$$I_1 + I_2 = I_3 \quad (1)$$

for bcfgb loop $\sum \Delta V = 0 \rightarrow +I_2R_2 - I_1R_3 + \varepsilon_2 = 0$

$$3I_2 - 5I_1 = -8 \quad (2)$$

for cdefc loop $\Sigma \Delta V = -I_3 R_1 + \varepsilon_3 - I_2 R_2 = 0$

$$-5I_3 - 3I_2 = -4$$

$$5I_3 + 3I_2 = 4 \quad (3)$$

By using Eqs.(1),(2), and (3), we can find

$$I_1 = 1{,}38A \quad I_2 = -0{,}364A \quad I_3 = 1{,}02A$$

b) find the charge on the capacitor.

the charge on the capacitor can be calculated by using $C = \frac{Q}{\Delta V_c} \Rightarrow Q = C \Delta V_c$

the electric potential difference between the capacitor's plate.

We can determine ΔV_c by writing 2nd Kirchhoff's rule for the closed loop of abgha.

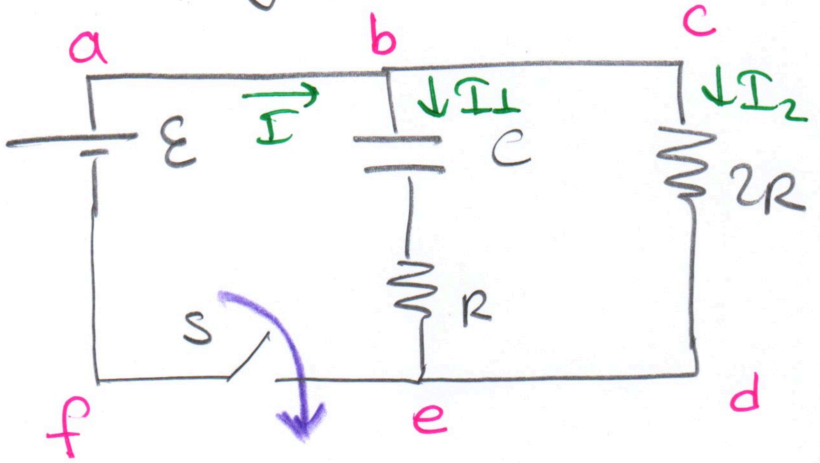
for abgha loop $\Sigma \Delta V = 0$

$$-\varepsilon_2 + \Delta V_c - 3 = 0$$

$$\Delta V_c = 3 + 8 = 11V$$

$$Q = Q_{max} = 6.10^{-6} . 11 = 66 \mu C$$

5) In a circuit given below, the capacitor is uncharged. The switch S is closed at $t=0$. (8)



a) Find an expression for current I as a function of time and draw $I=f(t)$ graphic when the capacitor is charging.

As you know, when the switch S is closed, the battery is connected to the circuit so that the capacitor starts to charge. During this process,

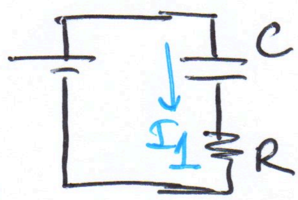
for junction b $\quad I = I_1 + I_2$

for acdfa loop $\quad \sum \Delta V = -I_2 2R + \mathcal{E} = 0$

$$I_2 = \frac{\mathcal{E}}{2R}$$

We have already learned the change in the electric current on the brach at which the capacitor is located:

$$I_1 = I_{max} e^{-t/\tau} \qquad \tau = RC$$



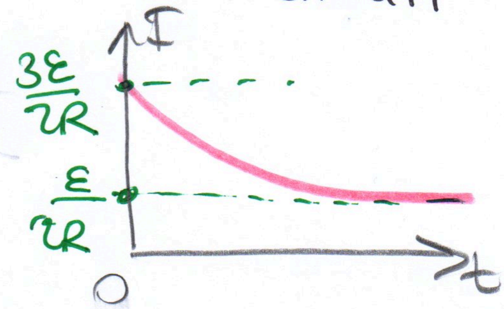
$$I_1 = \frac{\mathcal{E}}{R} e^{-t/RC} \qquad I_{max} = \frac{\mathcal{E}}{R}$$

charging circuit

Here $I = I_1 + I_2 = \frac{\mathcal{E}}{R} e^{-t/RC} + \frac{\mathcal{E}}{2R}$

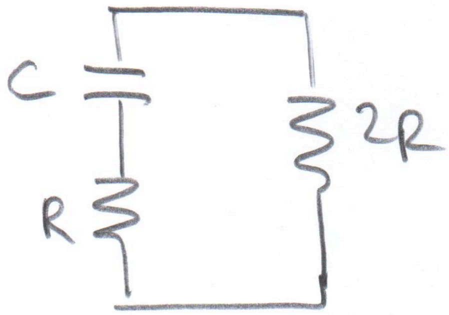
for $t=0$ $\quad I = \frac{\mathcal{E}}{R} + \frac{\mathcal{E}}{2R} = \frac{3\mathcal{E}}{2R}$

for $t \to \infty$ $\quad I = \mathcal{E}/2R$



b) After the circuit becomes a steady-state, the switch S is opened. Find the time required for the charge on the capacitor to fall one-half its initial value.

When the switch S is opened, the circuit becomes a discharging circuit. Let's draw the remaining circuit.



For the discharging circuit

$$\tau = R_{eq} C_{eq} = 3RC$$

$$R_{eq} = \underbrace{R + 2R}_{} = 3R$$

these resistors are connected in series!

for the discharging circuit

$$q(t) = Q_{max} e^{-t/\tau} = Q_{max} e^{-t/3RC}$$

from the question $q(t) = \frac{Q_{max}}{2}$

$$\frac{\cancel{Q_{max}}}{2} = \cancel{Q_{max}} e^{-t/3RC}$$

$$\frac{1}{2} = e^{-t/3RC}$$

$$\ln\left(\frac{1}{2}\right) = -\frac{t}{3RC} \Rightarrow t = -3RC \ln\left(\frac{1}{2}\right)$$

$$t = -3RC(-0{,}69)$$

$$t = 2{,}07\, RC$$

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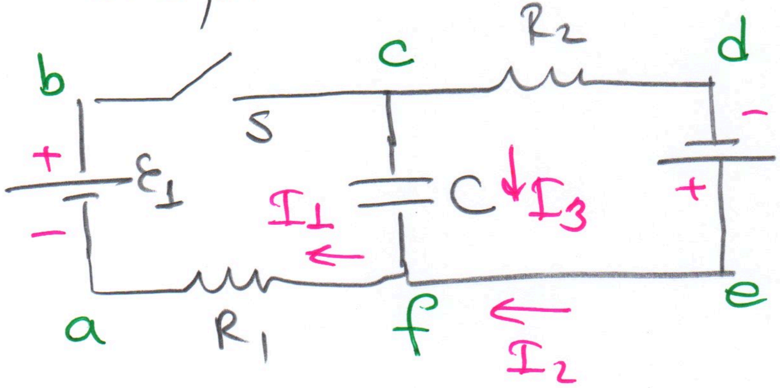
6) Midterm Question!

The switch S in the circuit is initially open and the capacitor is uncharged. In here, $\varepsilon_1 = 1V$ $\varepsilon_2 = 3V$, $R_1 = 0{,}2\,\Omega$, $R_2 = 0{,}3\,\Omega$, and $C = 5\,\mu F$



a) Find the currents in the circuit and the charge on the capacitor after a longtime while S is still open.

When S is opened for a long time there will be no current on each branch, and the capacitor is charged fully through ε_2 battery.

$I_1 = 0$ → since S is open

$I_3 = 0$ → the capacitor is fully charged.

$I_2 = 0$ → since the other currents are zero.

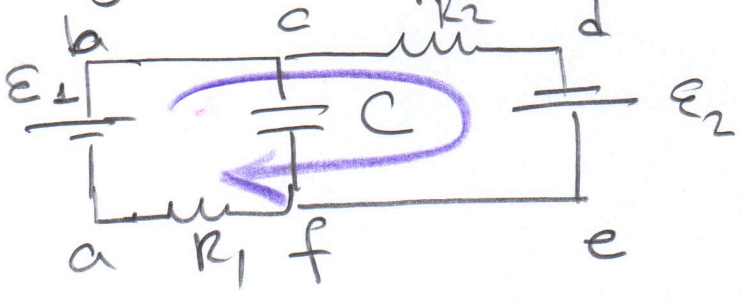
$$Q_{max} = C\Delta V_c = C\Delta V_{cf} = C\Delta V_{de} = C\varepsilon_2$$

$$Q_{max} = 5.10^{-6} \cdot 3 = 15\,\mu C$$

b) Now, the switch S is closed, Find the currents and the charge on the capacitor after a long time after S is closed.

Again the capacitor is charged fully. ($I_3 = 0$)

So that cf brach is open circuit.



for abdea loop

$$\varepsilon_1 + \varepsilon_2 - I(R_1 + R_2) = 0$$

$$I = \frac{\varepsilon_1 + \varepsilon_2}{R_1 + R_2} = \frac{1+3}{0,2+0,3} = 8A$$

let's find ΔV_C to calculate the charge on the capacitor.

for abcfa loop $\quad \varepsilon_1 + \Delta V_C - IR_1 = 0$

$$1 + \Delta V_C - 8.(0,2) = 0$$

$$\Delta V_C = 1,6 - 1 = 0,6V$$

or for edefe loop $\quad -IR_2 + \varepsilon_2 + \Delta V_C = 0$

$$\Delta V_C = 8.0,3 - 3$$

$$|\Delta V_C| = 0,6V$$

$$Q = C\Delta V_C = 5.10^{-6}.0.6$$

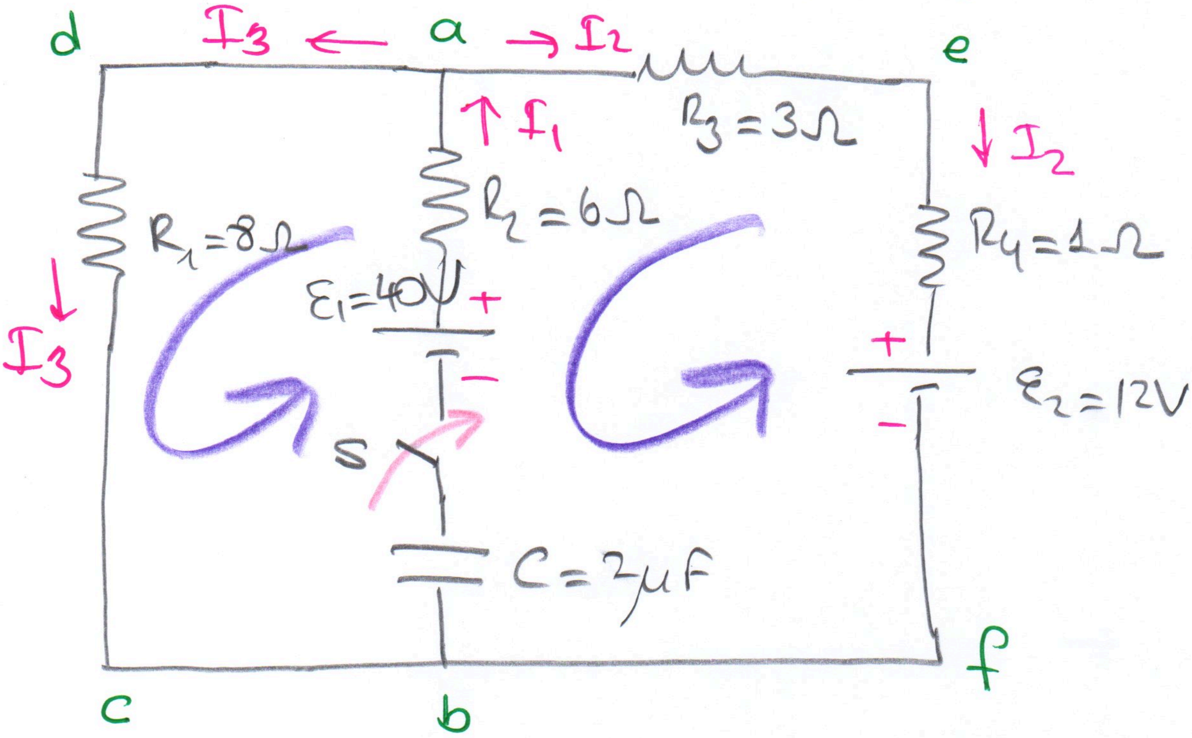
$$Q = 3\mu C$$

c) find powers supplied by the batteries and consumed across the resistors after switch S is closed for a long time.

$$\left.\begin{aligned} P_{R_1} &= I^2R_1 = 8^2.0,2 = 12,8\,W \\ P_{R_2} &= I^2R_2 = 8^2.0,3 = 19,2\,W \end{aligned}\right\} \sum P_{consumed} = 32\,W$$

$$\left.\begin{aligned} P_{\varepsilon_1} &= I\varepsilon_1 = 8.1 = 8\,W \\ P_{\varepsilon_2} &= I\varepsilon_2 = 8.3 = 24\,W \end{aligned}\right\} \sum P_{supplied} = 32\,W$$

7) Initially the capacitor is uncharged in the circuit. and switch S is closed at $t=0$.



a) find the currents of I_1, I_2, and I_3 just after the switch S is closed.

Just after the closing of the switch S, there will be no charge on the capacitor, hence $\Delta V_c = 0$

1st Kirchhoff's rule for junction a: $\Sigma I_{in} = \Sigma I_{out}$

(1) $\boxed{I_1 = I_2 + I_3}$

2nd Kirchhoff's rule

for adcba loop $\Sigma \Delta V = 0$

$$-I_3 R_1 + \varepsilon_1 - I_1 R_2 = 0$$

$$-8I_3 + 40 - 6I_1 = 0$$

$$\boxed{6I_1 + 8I_3 = 40} \quad (2)$$

for abfea loop $\Sigma \Delta V = I_1 R_2 - \varepsilon_1 + \varepsilon_2 + I_2(R_4 + R_3) = 0$

$$\boxed{6I_1 + 4I_2 = 28} \quad (3)$$

By using Eqs. (1), (2), and (3)

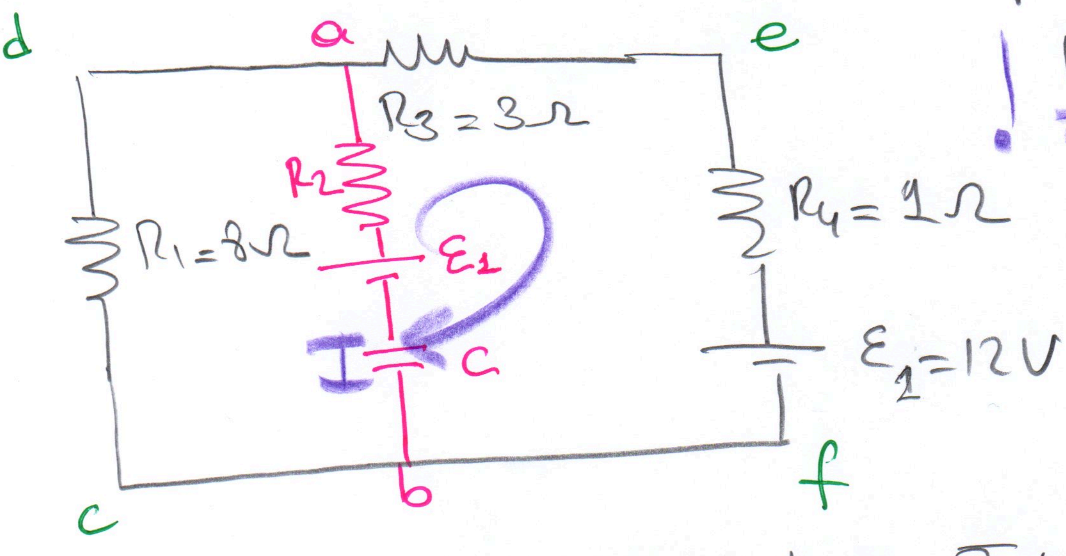
$I_1 \cong 3{,}7A$ $\quad I_2 \cong 2{,}2A$ $\quad$ and $I_3 \cong 1{,}5A$

b) find the currents of I_1, I_2, and I_3 after for a long time after the switch S was closed.

when the switch S was closed for a long time, since the capacitor is fully charged, there will be no current on the ab branch. Due to this reason;

$I_1 = 0$ hence $\cancel{I_1}^{0} = I_2 + I_3 \Rightarrow I_2 = -I_3$

and the circuit becomes one loop circuit



$|I_2| = |I_3| = I$

$I_2 = -1A$
$I_3 = 1A$

for cdefc closed loop $\Sigma \Delta V = 0$

$$-IR_1 - IR_3 - IR_4 - \varepsilon_1 = 0$$

$$I(R_1 + R_3 + R_4) = \varepsilon_1 \Rightarrow I = \frac{12}{8+3+1} = 1A$$

c) find the electric potential difference between points a and b for a long time after switch S was closed for a long time.

$V_a - \varepsilon_1$

By using adcb path, I can write $V_a - V_b$.

$$V_a + I R_1 = V_b \Rightarrow V_a - V_b = -I_2 R_1 = -(-1).8$$

$$V_a - V_b = 8\,(V)$$

By using aefb path $V_a - I_2 R_3 - I_2 R_4 - \varepsilon_2 = V_b$

$$V_a - V_b = 12 + I_2 (R_3 + R_4)$$

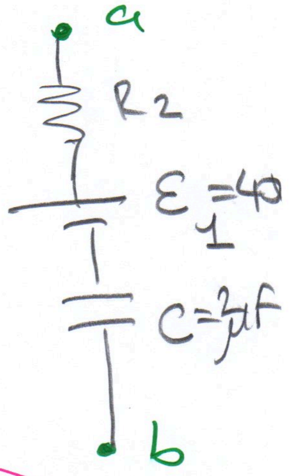
$$= 12 + (-1)[3+1] = 8V$$

At this moment we can not use ab path for writing $V_a - V_b$, because we don't know the electric potential between the capacitor's plate

d) find the charge on the capacitor after the switch S was closed for a long time.

⇓

the capacitor is fully charged
there is no current on the ab branch



$$V_a - \varepsilon_1 + \Delta V_C = V_b$$

we don't write any electric potential difference for the R_2 resistor, since there is no current on the ab branch!

$$V_a - V_b = \varepsilon_1 - \Delta V_C$$

$$8 = 40 - \Delta V_C$$

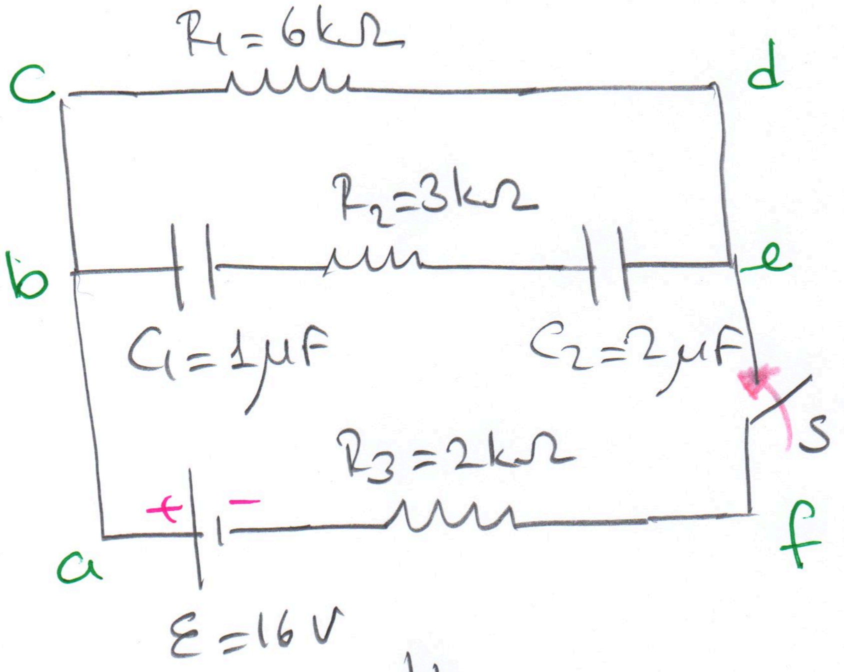
we found 8V in the part (c)

$$\Delta V_C = 32V$$

$$Q_{max} = C \Delta V_C$$

$$Q_{max} = 2.10^{-6} . 32 = 64 \mu C$$

8) a) For the circuit given below find the electric currents across each resistors for a long time after switch S was closed.



when the circuit is run for a long time after closing the switch S

⇓

capacitors are fully charged

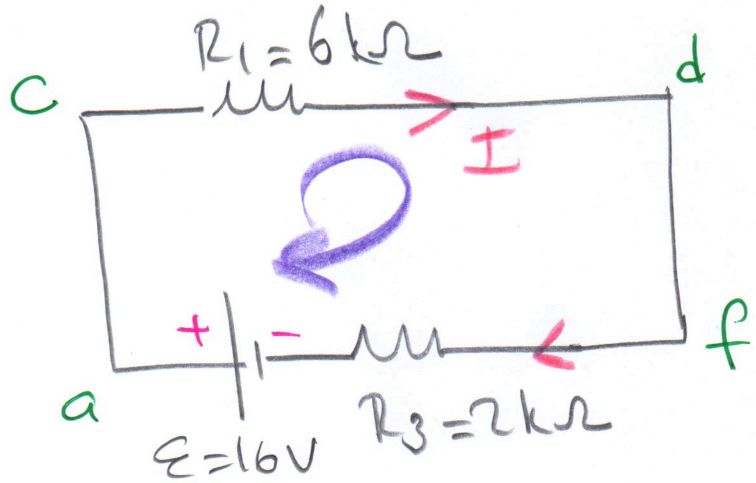
⇓

no current flow on the be branch.

⇓

Hence, the circuit becomes one loop circuit

⇓



for acdfa closed loop

$$\sum \Delta V = -I(R_1+R_3)+\mathcal{E}=0$$

$$-I(6+2)\cdot 10^{3} = -16$$

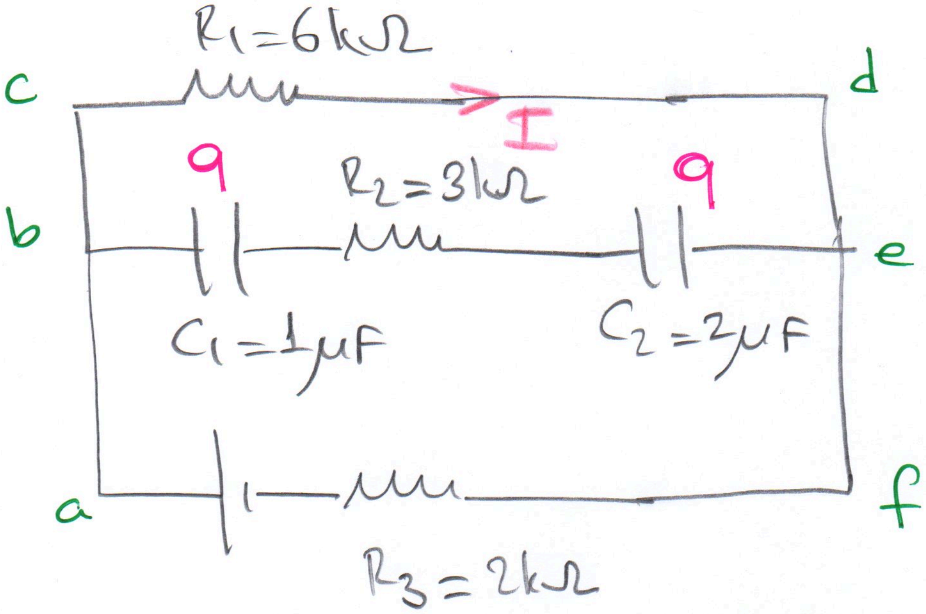
$$I = 2\cdot 10^{-3}\,A$$

$$I = 2\,mA$$

$$I_{R_2}=0,\ I_{R_1}=I_{R_3}=2\,mA$$

b) find the each capacitor's charge and the power consumed on R_2.

C_1 and C_2 capacitors are connected in series.
So their charges are the same $\Rightarrow q_1 = q_2 = q$



$I_{be} = 0$

for bcdeb loop $\Sigma \Delta V = 0$

$$-IR_1 + \Delta V_c + \Delta V_c = 0$$

$$-2.10^{-3}.6.10^{3} + \frac{q}{C_2} + \frac{q}{C_1} = 0$$

$$-12 + q\underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}_{\frac{C_1 + C_2}{C_1 . C_2}} = 0 \Rightarrow -12 + q\left[\frac{(1+2) . 10^{-6}}{1 \times 2 . 10^{-12}}\right] = 0$$

$$\Downarrow$$

$$12 = \frac{3q . \cancel{10^{-6}}}{2 . \cancel{10^{-12}}^{-6}}$$

$$\frac{24}{3} 10^{-6} = q$$

$$q = 8 . 10^{-6} C$$

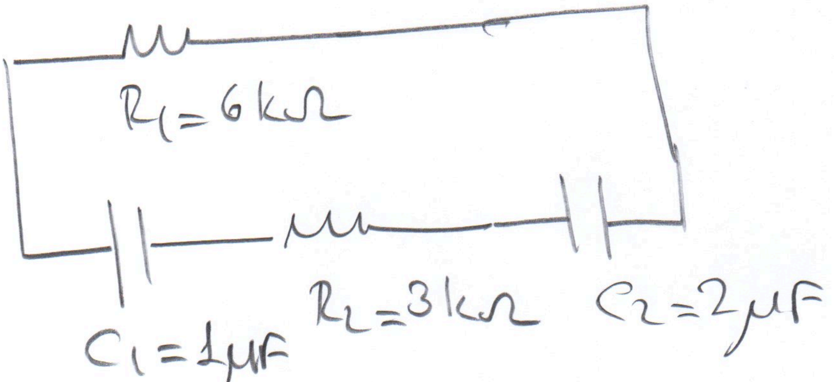
$$q = 8 \mu C$$

* $P_{R_2} = 0$; because there is no current flowing through R_2 resistor

$$(P = \cancel{I^2}^{0} R)$$

c) find the time constant of the discharging circuit when switch S is opened. 17

Let's draw the remaining circuit when the switch S is opened



$\tau = R_{eq} C_{eq}$

$R_{eq} = R_1 + R_2$

$R_{eq} = (6+3)\,10^3$

$R_{eq} = 9.10^3\,\Omega$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{1.2}{1+2} = \frac{2}{3}\,\mu F = \frac{2}{3}.10^{-6}\,F$$

$$\tau = \cancel{9}^{3}.10^3 . \frac{2}{\cancel{3}}.10^{-6} = 6.10^{-3}\,(s) \qquad \tau = 6\,ms$$

d) ~~After~~ the switch S is opened, obtain an expression for the current on R_1 as a function of time

For the discharging circuit $I = -I_{max}\, e^{-t/\tau}$

$$I_{max} = \frac{Q_{max}}{\tau} = \frac{8.10^{-6}}{6.10^{-3}} = \frac{4}{3}.10^{-3}\,(A)$$

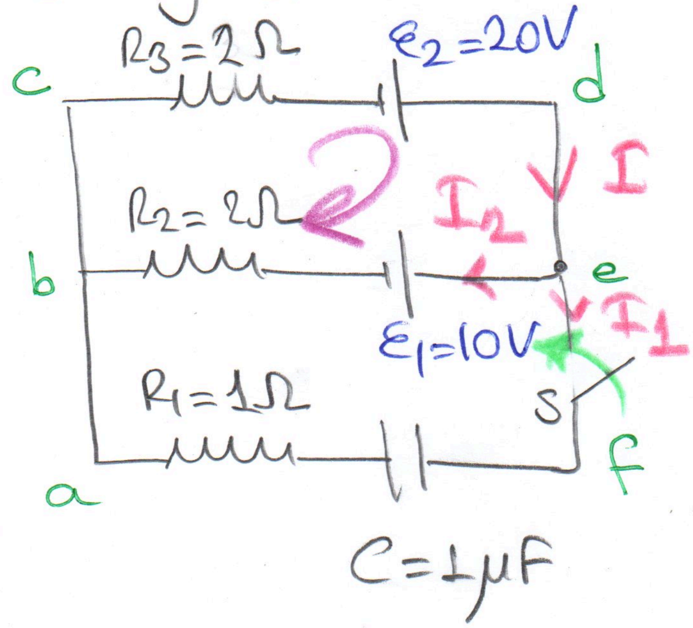
$$I = \frac{4}{3}.10^{-3}\, e^{-t/6.10^{-3}} = \frac{4}{3}.10^{-3}\, e^{-1000t/6}\,(A)$$

$$q = Q_{max}\, e^{-t/\tau}$$

$$I = \frac{dq}{dt} = -\frac{Q_{max}}{\tau}\, e^{-t/\tau} = -I_{max}.\, e^{-t/\tau}$$

g) Initially the capacitor given in the figure is uncharged. The switch S is closed at $t=0$.

a) Find I, I_1, and I_2 when the circuit becomes steady state.



Steady state of the circuit corresponds to a fully charged capacitor.

When the switch S is closed for a long time, the capacitor becomes fully charged and $I_1=0$

Junction Rule $\sum_b I_{in} = \sum_b I_{out}$ $\quad I = \cancel{I_1}^{0} + I_2 \quad I = I_2$

Loop Rule for bcdeb closed loop $\sum \Delta V = 0$

$$-IR_3 + \mathcal{E}_2 - \mathcal{E}_1 - I_2R_2 = 0$$

$$\downarrow$$

$$I_2 = I$$

$$-I(R_2+R_3) + \mathcal{E}_2 - \mathcal{E}_1 = 0$$

$$-I(2+2) + 20 - 10 = 0 \Rightarrow \boxed{I = 2{,}5\,A}$$

$I_1 = 0 \quad I_2 = 2{,}5\,A \quad I = 2{,}5\,A$

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b) For the steady state find the charge on the capacitor and electrostatic potential energy stored in the capacitor.

For the closed abefa loop:

$$\Sigma \Delta V = 0$$

$$+I_2 R_2 + \varepsilon_1 + \Delta V_C = 0$$

$\Delta V_{R_1} = 0$ (because $I_1 = 0$)

$$(2{,}5 \times 2) + 10 + \Delta V_C = 0$$

$$\Delta V_C = -15V \qquad |\Delta V_C| = 15V$$

$$C = \frac{Q}{|\Delta V_C|} \Rightarrow Q = C|\Delta V_C| = 1.10^{-6} \times 15 = 15\mu C$$

$$U = \frac{Q^2}{2C} = \frac{(15.10^{-6})^2}{2 \times 1.10^{-6}} = 112{,}5.10^{-6} \text{ Joule}$$

c) After closing switch S, find the I_1 current as a function of time while the capacitor is charging.

for the bcdeb closed loop $\Sigma \Delta V = 0$

$$-I R_3 + \varepsilon_2 - \varepsilon_1 - I_2 R_2 = 0 \qquad I = I_1 + I_2$$

$$I \rightarrow (I_1 + I_2)$$

$$-(I_1 + I_2).2 + 20 - 10 - I_2.2 = 0$$

$$-2I_2 - 2I_1 - 2I_2 + 10 = 0$$

$$-4I_2 - 2I_1 = -10$$

$$4I_2 + 2I_1 = 10$$

$$\boxed{2I_2 = 5 - I_1}$$

for abefa closed loop $\Sigma \Delta V = 0$

$$+I_2 R_2 + \varepsilon_1 - \Delta V_C - I_1 R_1 = 0$$

$$2I_2 + 10 - \Delta V_C - I_1 = 0$$

$$(5 - I_1) + 10 - \frac{q}{C} - I_1 = 0$$

$$15 - 2I_1 - \frac{q}{C} = 0 \Longrightarrow I_1 = \frac{dq}{dt}$$

$$2I_1 + \frac{q}{C} = 15$$

$$2 \cdot \frac{dq}{dt} + \frac{q}{C} = 15$$

$$2\frac{dq}{dt} = 15 - \frac{q}{C} = \frac{15C - q}{C}$$

$$\int_{q=0}^{q=q} \frac{dq}{15C - q} = \int_{t=0}^{t=t} \frac{dt}{2C}$$

$15C - q = u$

$-dq = du$

$q = 0 \quad u = 15C$

$q = q \quad u = 15C - q$

$$\int_{15C}^{15C-q} \frac{(-du)}{u} = \frac{1}{2C} t \Rightarrow -\ln u \Big|_{15C}^{15C-q} = \frac{1}{2C} t$$

$$\ln\left(\frac{15C - q}{15C}\right) = -\frac{t}{2C}$$

$$\frac{15C - q}{15C} = e^{-t/2C}$$

$$15C - q = 15C\, e^{-t/2C}$$

$$q(t) = 15C\left(1 - e^{-t/2C}\right)$$

$$I_1 = \frac{dq}{dt} = 15\cancel{C}\left(-\left(-\frac{1}{2\cancel{C}}\right) e^{-t/2C}\right)$$

$$I_1 = 7{,}5\, e^{-t/2C}$$

$$\tau = 2C = 2 \times 1.10^{-6}$$

$$\tau = 2.10^{-6}\,(s)$$

$$\tau = 2\,(\mu s)$$